

System Reliability Analysis of Unsaturated Soil Slopes under Rainfall: Rotational or Translational Failure?

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Abstract: Most of the rainfall-induced landslides reported in the literature were shallow translational, with depth usually less than two meters. But occasionally deep rotational failures also occurred. A deep failure is likely to be more destructive than a shallow failure due to its greater volume of failure mass. Conventional unsaturated slope stability analysis usually focuses on a particular failure mode. For example, limit equilibrium analysis of slices with a circular slip surface is most likely to capture the deep rotational failure, while infinite slope analysis implicitly assumes the failure mechanism is shallow translational. However, either failure mode is possible. Considering the uncertain geological parameters, this study performs system reliability analysis on unsaturated slopes under rainfall considering both deep rotational and shallow translational failure mechanisms. Through parametric studies the factors that could control the failure mechanism of an unsaturated soil slope under rainfall are identified and investigated. The results show that the system failure probability is obviously different from that of a single failure mode when the rainfall infiltration exceeds a certain depth (more than 1m in this study) and the slope angle is between 50° and 75°. The research finding is useful to guide the selection of the right method for a slope stability problem in engineering practice.

Keywords: Landslide; Translational and rotational failure; Unsaturated soil slope; Random variables; System reliability.

1 Introduction

Most of the rainfall-induced landslides reported in the literature were shallow translational, with depth usually less than two meters (Toll 2001, Dai et al. 2003). But occasionally deep rotational failures could also occur (Huat et al. 2006, Wartman et al. 2014). A deep failure is likely to be more destructive than a shallow failure due to its greater volume of failure mass. The two most common failure mechanisms of unsaturated soil slopes under rainfall are illustrated in Figure 1. Conventional unsaturated slope stability analysis usually focuses on a particular failure mode. For example, limit equilibrium analysis of slices with a circular slip surface is most likely to capture the deep rotational failure (Rahardjo et al. 2007, Lu et al. 2012), while infinite slope analysis implicitly assumes that the failure mechanism is shallow translational (Cho and Lee 2002, Lu and Godt 2008). More advanced approaches such as finite element methods (FEM) (Griffiths and Lu 2005, Le et al. 2015) are free from the prior assumption on the shape and location of slip surfaces. However, application of advanced numerical methods to unsaturated slope stability assessment in practice is limited due to the difficulties in obtaining the required input parameters (e.g., soil-water retentive curve, unsaturated permeability function) and the complexity involved in the analysis (Huang et al. (2018a)). Hence, it is meaningful to investigate the factors that control the failure mechanisms, which could provide guidance on the selection of most appropriate method for unsaturated slope stability analysis.

Rahardjo et al. (2007) investigated the factors that controlling the instability of unsaturated slopes under rainfall and identified that the ratio of rainfall intensity to saturated permeability of the soil is most critical. The effects of soil properties and slope geometry were not clear, as those multiple parameters were investigated individually. Huang et al. (2018b) developed dimensionless slope stability charts for homogeneous unsaturated soil slopes under rainfall, as indicatively shown in Figure 2. They found that for a given slope there is a critical dimensionless parameter group $c'/\gamma H \tan \phi'$, below which the slope would fail in translational mode and above which rotational mode. The critical $c'/\gamma H \tan \phi'$ is affected by infiltration depth, contribution of soil suction, and slope angle. The increase in infiltration depth and contribution of suction in unsaturated zone could increase the critical $c'/\gamma H \tan \phi'$, therefore promote the occurrence of translational failure. Moderately steep slopes (e.g., 45°) tend to fail in translational mode (critical $c'/\gamma H \tan \phi'$ is large), while gentle (e.g., 15°) and steep (e.g., 75°) slopes tend to fail in rotational mode (critical $c'/\gamma H \tan \phi'$ is small).

However, the research work by (Huang et al. 2018b) was limited for deterministic analysis. As a slope under rainfall condition may have two potential failure modes, its failure probability as a system is usually different from that of a single failure mode. Ji and Low (2012) proposed a stratified response surface method (RSM) to efficiently handle system reliability analysis based on the first order reliability method (FORM). Liu and Cheng (2016)

presented a system reliability analysis approach for layered soil slopes based on multivariate adaptive regression splines and Monte Carlo simulation. Duan et al. (2020) presented a method coordinated with the shear strength reduction method to identify representative slip surfaces for system reliability analysis of soil slopes.

In this study, the controlling factors on the failure mode of unsaturated soil slope under rainfall are investigated in the framework of system reliability analysis based on FORM. In the following sections, the methodology for system reliability analysis is reviewed first, followed by parametric studies.

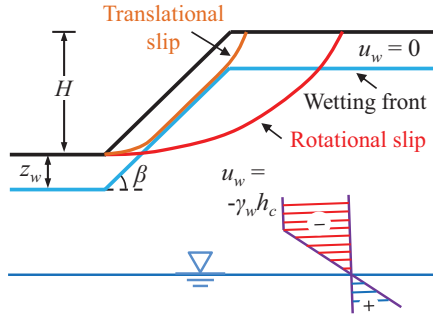


Figure 1. Translational and rotational failure of an unsaturated soil slope under rainfall (modified from Huang et al. (2018a))

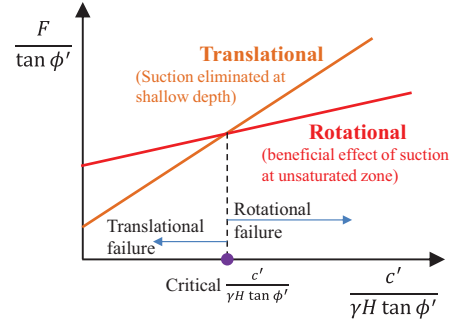


Figure 2. Slope stability for translational and rotational failure mechanisms (modified from Huang et al. (2018b))

2 Methodology

2.1 Stability analysis of slope under rainfall

Stability of an unsaturated soil slope under rainfall can be accurately assessed through a numerical seepage and stability analysis. However, stability analysis using advanced methods (e.g., FEM) only predict the safety level for the most critical failure mechanism and give no information on the other potential failure modes. However, it is essential to consider multiple failure modes in parallel for system reliability analysis. Huang (2018) develop a practical framework and regression models for unsaturated slope stability analysis considering both rotational and translational failure mechanisms, which are adopted in this study. The methodology is briefly described below.

Water pressure is negative above the water table, which contributes to the soil strength and enhance the slope stability (Rahardjo and Fredlund 1993). Infiltration of rainwater produces a wetting front, and it is assumed to be parallel to the slope surface, as shown in Figure 1. Above the wetting front water pressure is assumed to increase to zero, which is reasonable for coarse-grained soils and conservative for fine-grained soils (Huang et al. 2016). The depth of wetting front z_w can be estimated by (Li et al. 1998)

$$z_w = \begin{cases} \frac{k_{sat} t}{n(s_f - s_0)} & I \geq k_{sat} \\ \frac{It}{n(s_f - s_0)} & I < k_{sat} \end{cases} \quad (1)$$

where I and t are the rainfall intensity and rainfall duration, respectively; k_{sat} is the saturated permeability coefficient of the soil; s_f and s_0 are the final degree of saturation of the soil after rainfall infiltration and initial degree of saturation of the soil before rainfall infiltration, respectively; n is the porosity of the soil.

Stability of unsaturated soil slopes with sharp wetting fronts (Figure 1) can be assessed by upper bound limit analysis (UBLA) (Huang et al. 2018b), and the stability solutions are commonly presented as charts (Figure 2). Huang (2018) carried out a large number of UBLA for a wide range of slope geometry and soil properties, and instead of presenting the results as charts, developed regression models that are more convenient to use. For a slope with deep water table, the regression model for rotational failure mechanism can be expressed as:

$$F_{rot} = A_{est} \left(\frac{c'}{\gamma H \tan \phi'} - \frac{\gamma_w h_p \tan \phi'}{\gamma H \tan \phi'} + \zeta \frac{\gamma_w h_c \tan \phi^b}{\gamma H \tan \phi'} \right)^{B_{est}} \times \tan \phi' + \frac{\tan \phi'}{\tan \alpha} \quad (2)$$

where F_{rot} is factor of safety for rotational failure mechanism; c' and ϕ' are the effective cohesion and effective internal friction angle of the soil, respectively; γ and γ_w are the unit weight of the soil and water, respectively; H is the height of the slope; α is the slope angle; h_c denote the suction head cutoff in the unsaturated zone; ϕ^b is an angle which indicates the rate of increase in shear strength relative to matric suction; ζ denotes the degree of contribution of matric suction to slope stability under rainfall and it can be approximated by Eq. (3); parameters A and B can be calculated by Eq. (4) and Eq. (5), respectively.

$$\zeta = 1 - 1.4 \frac{z_w}{H} \quad (3)$$

$$A_{est} = 10.50 \exp(-0.009\alpha) \quad (4)$$

$$B_{est} = \begin{cases} 0.72 - 3.5 \times 10^{-5} \alpha^2 + 0.0031\alpha, & 0 \leq \frac{c'}{\gamma H \tan \phi'} \leq 1 \\ 0.83 - 2.2 \times 10^{-5} \alpha^2 + 0.0026\alpha, & 0 < \frac{c'}{\gamma H \tan \phi'} \leq 3 \end{cases} \quad (5)$$

As for the shallow translational failure mode, the factor of safety (F_{trl}) can be calculated by:

$$F_{trl} = \frac{c'}{\gamma z_w \sin \alpha \cos \alpha} + \frac{\tan \phi'}{\tan \alpha} + \frac{5c'}{\gamma H} \exp(-0.008\alpha) \quad (6)$$

It should be noted that the slope angle α in Eq. (4), Eq. (5) and the exponential function [i.e., $\exp(-0.008\alpha)$ in Eq. (6)] is input as degree ($^\circ$). The difference between Eq. (6) and the infinite slope model lies in the last term, which captures the boundary effect and was obtained through regression analysis (Huang 2018). The accuracy of Eq. (2) and Eq. (6) has been validated by back analyses of slope failures (Huang et al. 2018a). Both Eq. (2) and Eq. (6) calculate the factor of safety explicitly with minimal computational effort, therefore they can readily be incorporated into the system reliability analysis.

2.2 System reliability analysis

As mentioned, translational and rotational are the two failure modes that are considered in this study. For this reason, reliability-bounds theories are available to handle the rainfall-induced unsaturated slope failure problems. The bimodal bounds for system reliability can be calculated by (Ji and Low 2012):

$$P_{F_1} + \sum_{i=2}^m \max \left[\left\{ P_{F_i} - \sum_{j=1}^{i-1} P(E_i E_j) \right\}; 0 \right] \leq P_F \leq \min \left[\left\{ \sum_{i=1}^m P_{F_i} - \sum_{i=2}^m \max_{j < i} P(E_i E_j) \right\}; 1 \right] \quad (7)$$

where P_{F_1} is the largest failure probability among the potential failure modes and P_{F_i} is the failure probability of the i th failure mode, $P(E_i E_j)$ is the joint probability of events E_i and E_j , and P_F is the system failure probability. $P(E_i E_j)$ can be calculated according to (Ditlevsen 1979):

Eq.(8) and Eq. (9) represent the two events are positively correlated and negatively correlated, respectively. The terms $P(A)$ and $P(B)$ are defined by Eq.(10) and Eq. (11).

$$\max [P(A), P(B)] \leq P(E_i E_j) \leq P(A) + P(B) \quad (8)$$

$$0 \leq P(E_i E_j) \leq \min [P(A), P(B)] \quad (9)$$

$$P(A) = \Phi(-\beta_i) \Phi \left(-\frac{\beta_j - \rho \beta_i}{\sqrt{1 - \rho^2}} \right) \quad (10)$$

$$P(B) = \Phi(-\beta_j) \Phi \left(-\frac{\beta_i - \rho \beta_j}{\sqrt{1 - \rho^2}} \right) \quad (11)$$

where β_i is the reliability index of the failure mode i and ρ is the correlation coefficient between modes i and j . Low (2017) obtained the correlation coefficient ρ of different failure modes by using Cholesky decomposition of the correlation matrix \mathbf{R} on n -space as follows:

$$\rho = \frac{1}{\beta_i \beta_j} \mathbf{n}_i^* \mathbf{R}^{-1} \mathbf{n}_j^* \quad (12)$$

where \mathbf{n}_i^* and \mathbf{n}_j^* are the vectors obtained by equivalence normalization of design points which can be gained by iHLRF- x algorithm. The reliability index β_i and design point on x -space can be calculated according to the iHLRF- x algorithm (Ji and Kodikara 2015, Ji et al. 2018).

3 Results and Discussions

Stability analysis and system reliability analysis of unsaturated soil slopes under intensive rainfall condition with deep water table are conducted and discussed. As shown in Eq. (1), the rainfall intensity and the rainfall duration can be directly characterized by the depth of the wetting front. A parametric study has been carried out to investigate the influence of slope angle (α), depth of wetting front (z_w/H) and the contribution of matric suction ($\tan(\phi^b)/\tan(\phi')$) on the FoS and system reliability. The adopted parameters are shown in Table 1.

Table 1. Parametric study to FoS and system reliability

α	z_w/H	H	h_c	k_{sat}	$\tan(\phi^b) / \tan(\phi')$
15°~85°	0.05, 0.10, 0.15, 0.20	10 m	3 m	10 ⁻⁶ m/s	0, 1/3, 2/3, 1
μ_c	$\mu_{\phi'}$	γ	COV_c	$COV_{\phi'}$	
10 kPa	30°	18 KN/m ³	0.3	0.3	

3.1 Deterministic analysis based on the mean values

The influence of rainfall and slope angle to the FoS is obtained by stability analysis as shown in Figure 3 (In this and subsequent figures, the letter R means the rotational failure mode, and the letter T means the translational failure mode). From Figure 3, it can be seen that shallow translational failure mode is more sensitive to rainfall compared to deep rotational failure mode. The reason is the entire translational slip surface is wetted by rain, while the rotational slip surface is only partially wetted by rain, as shown in Figure 1. The FoS decreases with the increase of wetting front depth. It can also be found that the FoS of slopes in rotational failure mode decreases gradually with the increase of slope angle, while for translational failure of slopes, the FoS tends to decrease gradually and then increase significantly with the gradual growth of the slope angle. This phenomenon was also observed in the finite element analyses carried out by (Griffiths et al. 2011). When $z_w/H > 0.15$, there are intersections of the FoS of two failure modes, which means that different slope angles may correspond to different failure modes when the rainfall holds longer. For cases where the intensity of rainfall is low or the duration of rainfall is short, the slope is more prone to deep rotational failure mode.

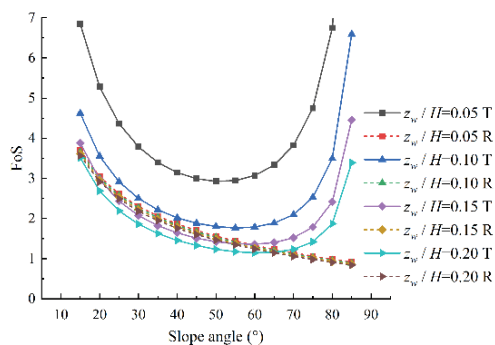


Figure 3. The influence of slope angle and z_w to FoS ($\tan(\phi^b) / \tan(\phi') = 2/3$)

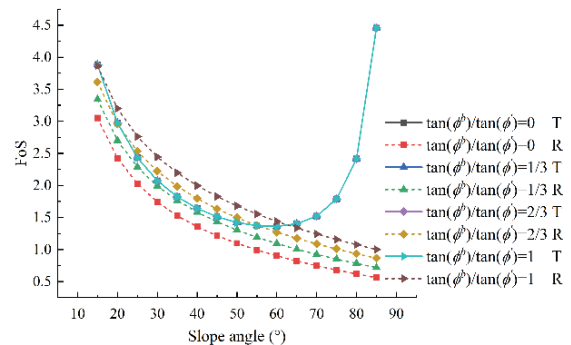


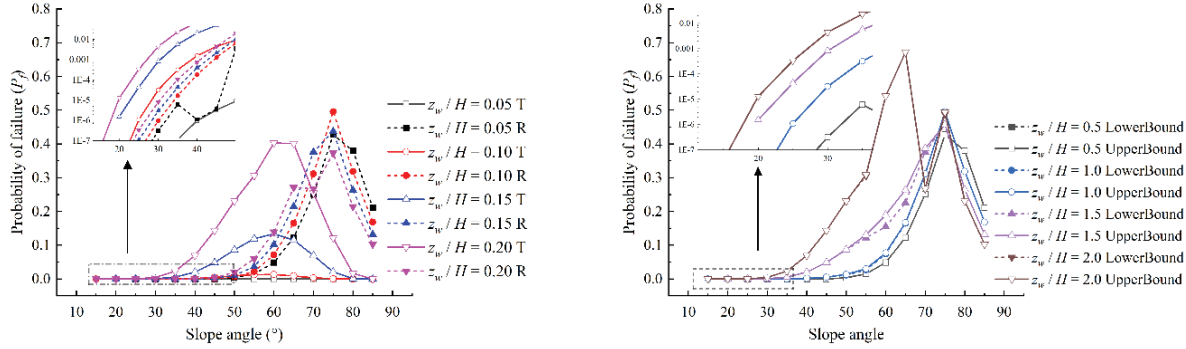
Figure 4. The influence of ϕ^b to FoS ($z_w/H = 0.15$)

As mentioned above, rainfall infiltration leads to the existence of different failure modes of slopes mostly in the middle and late stages of rainfall infiltration, in order to consider the influence of ϕ^b on slope stability, assuming $z_w/H = 0.15$, the influence of ϕ^b on slope stability is shown in Figure 4. From Figure 4, it can be seen that ϕ^b contributes more to the deep rotational failure mode, FoS gradually increases with the growth of ϕ^b , an increase in the value of $\tan(\phi^b)/\tan(\phi')$ implies a gradual growth in the effectiveness of contribution of matric suction to soil strength in the unsaturated zone, therefore the slope tends to be more stable.

3.2 Probabilistic analysis of unsaturated soil slopes under rainfall

This section analyzes the potential failure modes of unsaturated soil slopes under rainfall from the perspective of system reliability. Similar to the deterministic analysis process, the effect of wetting front depth on the probability of failure is first analyzed as shown in Figure 5. Unlike the deterministic analysis, the probability of failure of both the translational and rotational slope failure modes tend to increase and then decrease as the slope angle continues to increase. The extreme value of the slope failure probability in the translational failure mode is at a slope angle of about 60°, while the extreme value of the slope failure probability in the rotational failure mode is about 75°. Since the slope failure probability is very small when the slope angle is less than 50°, a local enlargement is made in Figure 5 (a), and the vertical coordinates in the local enlargement are taken as logarithmic axes to distinguish the relationship between the magnitude of failure probability corresponding to the two failure modes. When the depth of wetting front is small (e.g., $z_w/H = 0.05$), due to the high matric suction of soil inside the slope, the slope is prone to rotational failure. With the increase of rainfall infiltration depth, the slopes begin to gradually tend to translational failure, but for slope angles greater than 75°, the slope still inclines to rotational failure. The lower bound and upper bound of system reliability analysis are shown in

Figure 5 (b). The difference between the two bounds is small when the slope angle is less than 50° or more than 75°, which is due to the fact that the reliability indexes corresponding to the two failure modes differ significantly, and the failure probability values differ by a factor of ten or more, thus leading to a higher degree of dominance of a single failure mode. In the middle of rainfall ($z_w/H=0.15$), when the slope angle is greater than 50° and less than 75°, the two bounds are significantly different from each other and the system failure probability is obviously larger than each single failure probability, the systematic nature of slope failure cannot be ignored.

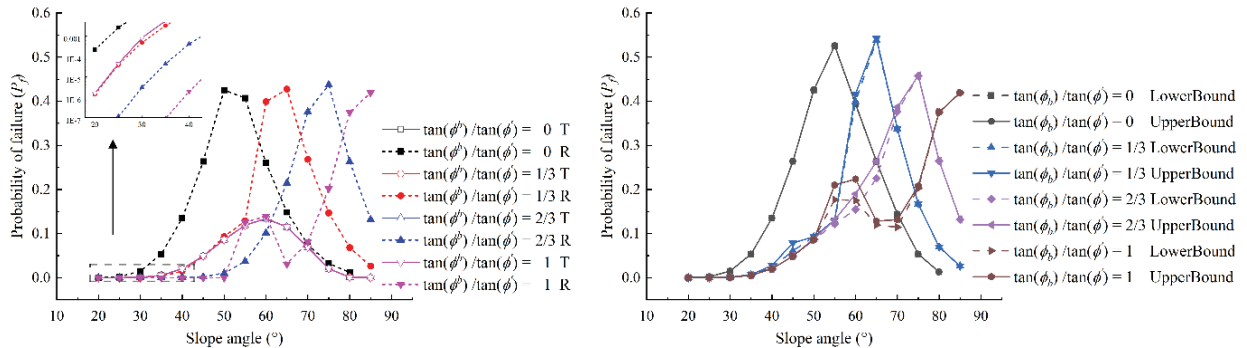


(a) Probability of failure of single failure modes

(b) System failure probability

Figure 5. The influence of slope angle and z_w to the probability of failure ($\tan(\phi^b)/\tan(\phi') = 2/3$)

The contribution of ϕ^b to the failure probability when z_w/H equals 0.15 is shown in Figure 6(a). Similar to the deterministic analysis, the failure probability of the slope rotation failure mode gradually decreases with the increase of ϕ^b . Meanwhile, it can be found that the smaller the value of ϕ^b , the more dominant the rotational failure of the slope is. The influence of ϕ^b to the two failure bounds is shown in Figure 6(b). The figure shows that when the slope angle is greater than 50° and less than 75°, there is a significant difference between the two boundaries, which is similar to the pattern shown in Figure 5(b) **Error! Reference source not found.**, and the slope failure problem needs to be considered by applying the system reliability theory.



(a) Single failure modes (Translational or Rotational)

(b) System failure

Figure 6. The influence of ϕ^b to the failure probability of two potential failure modes ($z_w/H = 0.15$)

4 Conclusions

This paper analyzes the contribution of two failure modes (shallow translational and deep rotational) of rainfall-induced slope instability to the slope failure probability from the perspective of system reliability, and the main conclusions are as follows:

(1) From the deterministic analysis, it can be seen that for the rainfall-induced slope failure problem, the very gentle slope or steep slope tends to rotational failure mode. With the gradual deepening of the wetting fronts, the slopes all gradually tend to be in the translational failure mode. As $\tan(\phi^b)/\tan(\phi')$ gradually increases, the safety factor of its corresponding rotational failure mode gradually increases.

(2) For the probabilistic analysis, if only a single failure mode is considered, when the depth of the wetting front is shallow, the slope is more inclined to rotational failure, and with the gradual deepening of the wetting front, the failure probability of slope translational mode gradually increases, but for steep slopes (slope angle >75°), the slope still tends to rotational failure. When the slope angle is less than 50° or the slope angle is more than 75°, the failure probability of the slope system is not much different from that of the dominant mode, but when the slope angle is between 50° and 75°, the system failure probability of the slope is obviously greater than that of the single

mode dominant, and when the slope angle of a target slope is in this range, a combination of prevention and control measures that take into account both shallow sliding and deep failure of the slope is required.

In this paper, rainfall-induced slope stability is analyzed from the perspective of system reliability. Rainfall intensity, rainfall holding time and soil saturation permeability coefficient are expressed uniformly by infiltration front depth, but for the actual unsaturated soil slope problem, soil saturation permeability coefficient is the main calculation parameter of soil infiltration. Further work needs to consider the soil cohesion, internal friction angle and the uncertainty of soil parameter saturation permeability coefficient from practical cases.

Acknowledgments

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