Proc. of the 8th International Symposium on Geotechnical Safety and Risk (ISGSR)
Edited by Jinsong Huang, D.V. Griffiths, Shui-Hua Jiang, Anna Giacomini and Richard Kelly

doi: 10.3850/978-981-18-5182-7_06-003-cd

ISGSR 2022

Multi-Objective Inverse Reliability Design Approach in Geotechnical Engineering

Jian Ji[1*], Zhe-Ming Zhang[1] and Zhao-Cheng Wang[1]

[1]Key Laboratory of Ministry of Education for Geomechanics and Embankment Engineering, Hohai University, Nanjing, China
*Correspondence: Professor Jian Ji, Email: ji0003an@e.ntu.edu.sg

Abstract: In order to further expand the application of inverse reliability methods in geotechnical engineering, a multi-objective inverse reliability design method is presented in this paper. Based on the HLRF algorithm, the approach can easily realize the traditional single-objective inverse reliability design. Further, multi-objective inverse reliability design is achieved by combining several single-objective inverse reliability analysis modules in a matrix directly without any other mapping functions. The design parameters can be independent of the basic random variables or associated with those random variables, such as their means or standard deviations. Verifications of the proposed algorithm are illustrated through several examples to show the feasibility and accuracy. Based on the proposed approach, the multi-objective inverse reliability design of a strip footing and retaining wall are performed.

Keywords: Geotechnical uncertainty; Inverse reliability analysis; Reliability-based Design; Implicit performance function.

1 Problem Background

Geotechnical analysis and design are often faced with a large amount of uncertainties. Traditional deterministic design methods accommodate uncertainty by means of empirical safety margins. However, these safety factors do not quantify the margin of safety of the design, nor do they consider the impact of different design variables and their uncertainties on the overall performance of the system. In the last few decades, probabilistic methods have been developed to quantify this uncertainty in various geotechnical systems. In this context, the concept of Reliability-Based Design (RBD) is gaining importance in geotechnical engineering (Lopez and Beck 2012). The basic idea is to calculate design parameters or safety factors directly at a given level of reliability.

A great deal of research on RBD of geotechnical engineering has been conducted. Zhang et al. (2011) present a preliminary indirect method for solving the RBO of geotechnical systems based on the Mean First Order Reliability Method (MFORM). Langford et al. (2013) used a modified point estimate method to design the tunnel lining. Zhao et al. (2015) employed an Artificial Bee Colony (ABC) algorithm for reliability-based optimization retaining walls and spread footings. Ji et al. (2019) proposed an inverse reliability algorithm in the original parameter space based on HLRF and performed RBD for shallow foundations and slope angles of side slopes. Wang and Owens (2021) improved the RBD algorithm for conventional structures and applied it successfully on examples of tunnels, retaining walls and shallow foundations. For a given geotechnical structure, it may have multiple failure modes, which requires a multi-objective reliability design approach. Most of the above studies have been limited to a single objective reliability design. Moreover, when it comes to multi-objective reliability design, the above methods need to establish additional restriction functions and cannot be targeted for a certain failure mode. This paper proposes a multi-objective reliability design approach by combining the processes of single-objective reliability design.

2 Inverse Reliability Method

2.1 Hasofer–Lind index and FORM algorithm

The matrix form of the Hasofer-Lind reliability index for correlated normals and FORM for correlated non-normals can be recast as follows (Low and Tang 2007):

$$\beta = \min_{x_i \in F} \sqrt{\left[\frac{x_i^* - \mu_i^N}{\sigma_i^N}\right]^T \mathbf{R}^{-1} \left[\frac{x_i^* - \mu_i^N}{\sigma_i^N}\right]} = \min_{x_i \in F} \sqrt{\mathbf{n}^T \mathbf{R}^{-1} \mathbf{n}} \tag{1}$$

Where x_i^* is the design point value of the ith random variable being defined in the original space, μ_i^N and σ_i^N are equivalent normal mean and standard deviation of the ith variable, respectively, $\mathbf{R}$ is the correlation matrix, $\mathbf{n}$ is the equivalent standard normal vector, and F is the failure domain. The probability of failure can then be inferred from the reliability index as:

$$P_f \approx 1 - \Phi(-\beta) = \Phi(-\beta) \tag{2}$$

2.2 Inverse FORM algorithm

2.2.1 single-objective reliability design problem

For a target reliability index β, the inverse problem can be stated as (Li Hong 1998):

Given β
Find d
Subject to: $\min(\mathbf{u}^T\mathbf{u}) = \beta^2$ and $G(\mathbf{x}) = g(\mathbf{u}, d) = 0$

where **u** is uncorrelated standard normal variables (i.e. in u-space), **x** is the corresponding non-normal random variable in x-space, G(.) and g(.) are performance functions in x-space and u-space, d is the target design parameter to be sought. Based on FORM, the design point **u** must satisfy Eq. (3) at the target reliability level.

$$\mathbf{u} = \frac{-\beta_t \nabla_u g}{(\nabla_u g^T \nabla_u g)^{1/2}} \tag{3}$$

A Taylor expansion of the performance function at ($\mathbf{u}_0$, d_0) yields the following equation:

$$g(\mathbf{u}_0, d) = g(\mathbf{u}_0, d_0) + \frac{\partial g(\mathbf{u}_0, d)}{\partial d}\Big|_{d=d_0}(d - d_0) = 0 \tag{4}$$

$$d = d_0 - \frac{g(\mathbf{u}_0, d_0)}{\left(\frac{\partial g(\mathbf{u}_0, d)}{\partial d}\right)\Big|_{d=d_0}} \tag{5}$$

In general, when the random variables x_i are correlated and non-normal distributed, a transformation is required prior to the execution of the procedure.

A mapping of **u** into ***x*** is achieved through Eq. (6) and Eq. (7)

$$x_i = F_{x_i}^{-1}(\Phi(n_i)) \tag{6}$$

$$\mathbf{n} = \mathbf{L}\mathbf{u} \text{ or } \mathbf{u} = \mathbf{L}^{-1}\mathbf{n} \tag{7}$$

where $F_{x_i}(.)$ is the cumulative distribution (CDF) for the variable x_i, $\Phi(.)$ is the standard normal function, the matrix **L** results from the Cholesky decomposition of $\mathbf{R_0}$. $\mathbf{R}_0$ is the result of the Nataf transformation of **R**.

The resulting basic procedure for calculating the inverse reliability is as follows:

Step 1: Determine the parameter distribution type and correlation. Specify the target reliability index βt and an initial input ($\mathbf{u}_0$, d_0).

Step 2: Calculate the gradient operator $\nabla_u g(\mathbf{u}_0, d_0)$.

Step 3: Update **u** by Eq. (3).

Step 4: Calculate g(**u**, d_0) and partial derivative $\frac{\partial g(\mathbf{u}_0, d)}{\partial d}$.

Step 5: Update d by Eq. (5).

Step 6: Repeated Step 2 to 5 until convergence for both **u** and d is achieved.

In this study, the above-mentioned algorithms and procedure are coded in Matlab.

2.2.1 Multi-objective reliability design problem

The multi-objective reliability problem requires a combination of multiple single-objective reliability problems by the matrix M. Starting from the idea of a single-objective reliability problem, any design parameter can be expressed with the other parameters fixed by the function *f*. This allows the relationship between design parameters d_i and arbitrary reliability constraints g_i to be established.

$$F_i = d_i - f_i(d_1, d_2, ..., d_{i-1}, d_{i+1}, ...d_n, \beta_i) \quad (i = 1, 2, ..., n) \tag{8}$$

$$\mathbf{d}_{new} = \mathbf{d} - \mathbf{M}^{-1}(\mathbf{d} - \boldsymbol{f}) \tag{9}$$

Where $\mathbf{d} = (d_1, d_2, d_3, \ldots, d_n)$, $\boldsymbol{f} = (f_1, f_2, f_3, \ldots, f_n)$, $M_{ij} = \frac{\partial F_i}{\partial d_i}\Big|_{x^*} \; i \neq j$, $M_{ij} = 1.0 \; i = j$ (i = 1,2,…,n), (j = 1,2,…,n).

Starting from an initial vector of design parameters d, Eq. (9) is used as an iterative formula until convergence.

3 Case Studies

3.1 CASE 1: Shallow Foundation Design

The RBD for shallow foundations is adapted from the literature (Wang and Kulhawy 2008), as shown in Figure 1. The foundations were placed on sand 1 m below the surface and subjected to a combination of dead and live loads, denoted Q_D and Q_L, respectively. The water table was assumed to be far enough away from the foundations to have a negligible effect on the foundation design.

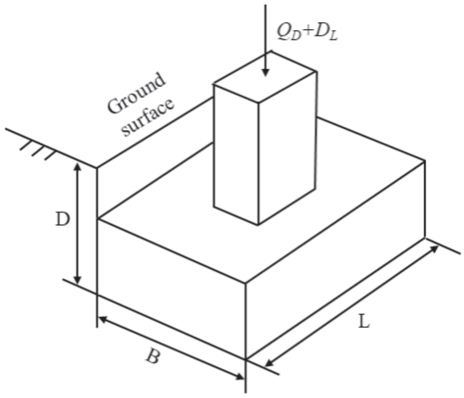

Figure 1. Reliability-based design of a shallow foundation

Two main failure modes are considered: (1) failure due to overload; (2) failure due to excessive settling. The limit state functions are:

$$g_1(\mathbf{x}) = q_u \mathrm{BL} - Q_L - Q_D \tag{10}$$

$$g_2(\mathbf{x}) = s_{\max} - s \tag{11}$$

Where q_u is the ultimate bearing pressure, which is calculated by Vesic (1975) 's method in this study using the friction angle of the sand φ, the effective weight of the soil γ_s and the geometry parameters of the foundation. B and L are the width and length of the foundation. $s_{\max}$ is the maximum tolerable settlement, which is taken as 25mm as recommended. s is the predicted settlement and computed using the method suggested by Whitman and Richart Jr (1967). The uncertain variables involved in shallow foundations can be represented by $\mathbf{x} = [Q_D, Q_L, \gamma_s, \varphi, E, v]$. Table 1 summarised the distribution types and statistics for the six uncertain variables.

Table 1. Statistical information of the variables for the shallow foundation.

	Q_D (kN)	Q_L (kN)	γ_s (kN/m^3)	φ (°)	E (MPa)	v
Distribution	Lognormal	Lognormal	Normal	Lognormal	Lognormal	Normal
Mean	1200	400	18.5	38	40	0.3
Standard deviation	120	72	1	4	8	0.05

The main task in the design of a shallow foundation is to determine the ground dimensions of the foundation for a given load and burial condition. When the foundation length L is determined, the foundation width B can be designed to ensure that the bearing capacity of the foundation meets the specified level of reliability. As shown in Figure 2, with only 4-5 iterations, the RBD will give design values that satisfy the bearing capacity requirements. For example, to achieve the target β of 3.0 (i.e. the collapse failure probability is 0.135%), RBD recommends the foundation width should be 2.14 m. When the shallow foundations meet the bearing capacity requirements as well as their settlement requirements, the multi-objective reliability design approach allows for the design of both B and L. In Table 2 the results of the previous study and some of the results of the different design combinations are given. The results of this paper are consistent with the results of the previous study. The advantage of this research approach is that instead of using a proxy model to correlate the reliability metrics, it is a straightforward combination of single-objective design approaches. From the results, it can be seen that different design parameters constrain and influence each other when multi-objective reliability design is used. In addition, some design combinations result in no solution because the most probable failure points of the two failure modes do not intersect.

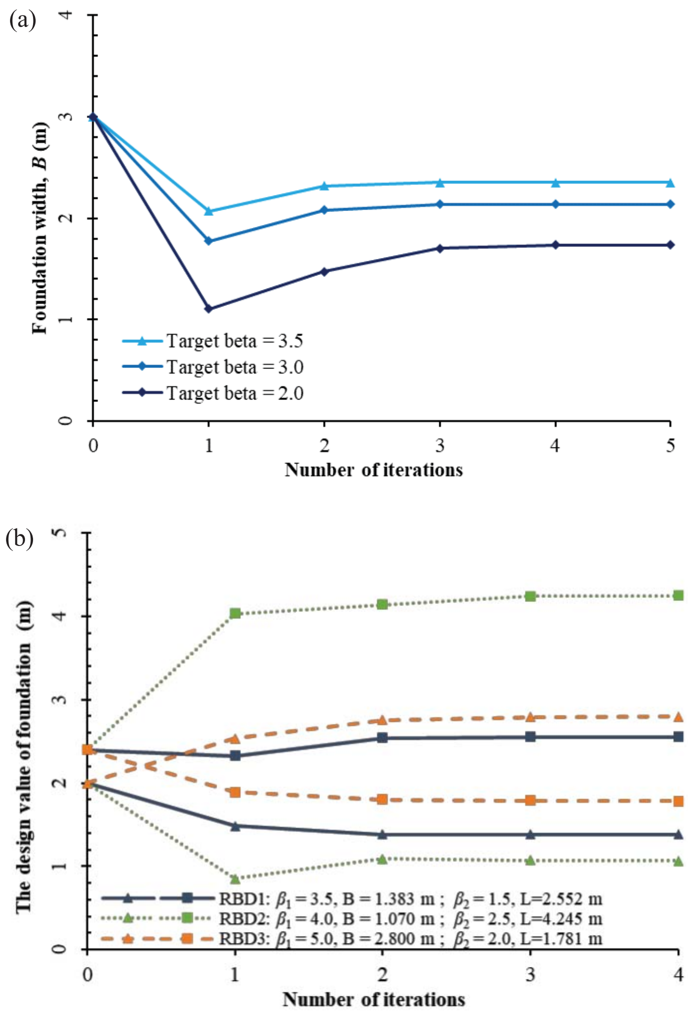

Figure 2. RBD for shallow foundation showing their iterative calculations: (a) With L fixed, design B; (b) Design B and L

Table 2. Comparison with past studies and some other design combinations.

Ref	Method	Reliability Constraint		Design Parameters	
		β_1	β_2	B	L
Zhang et al. (2011)	MFOSM	3.498	1.505	1.378	2.564
Present study	Inverse FORM	3.498	1.505	1.378	2.564
		5.0	2.0	2.800	1.781
		5.0	1.0	4.581	0.743
		4.0	3.0	NaN	NaN
		4.0	2.5	1.070	4.245
		4.0	2.0	1.391	3.053
		4.0	1.5	1.920	2.002
		3.0	3.0	NaN	NaN
		3.0	2.0	NaN	NaN
		3.0	1.0	1.351	2.153

3.2 CASE 2: Retaining Wall Design

A Gravity retaining wall design problem, adopted from (Mahmood, 2020), was used to illustrate the application RBD. A 6 m high concrete gravity retaining wall, as shown in Figure 3 is considered. Four failure modes are considered: (1) overturning failure, (2) sliding failure, (3) bearing capacity failure and, (4) eccentricity failure. The corresponding limit state functions are respectively：

$$
\begin{aligned}
&g_1(\mathbf{x}) = M_1 - M_2 \\
&g_2(\mathbf{x}) = R_V \tan\delta_b + Bc_b - F_H \\
&g_3(\mathbf{x}) = q_u - \frac{R_v}{B}(1+\frac{6e}{B}) \\
&g_4(\mathbf{x}) = 1 - \frac{6e}{B}
\end{aligned}
\tag{12}
$$

Where M_1 and M_2 are the restoring moment and the overrunning moment respectively, R_v is the vertical base resultant, B is the sum of b_1, b_2, b_3.

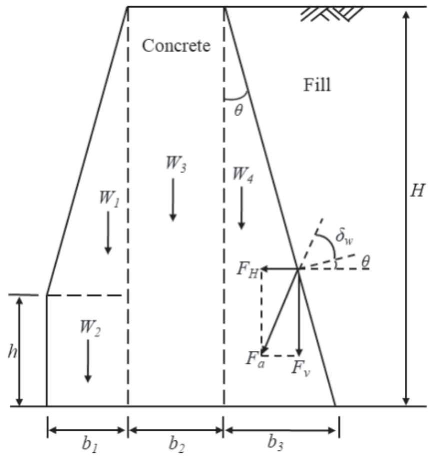

Figure 3. Reliability-based design of gravity retaining wall.

Six uncertain variables are considered: the friction angle of the fill material (φ); the friction angle between the fill material and the wall (δ_w); the unit weight of the fill material (γ_s); the cohesion along with the interface between the wall base and the founding soil (c_b); the friction angle along with the interface between the wall base and the founding soil (δ_b); and the maximum allowable bearing pressure of the foundation (q_u). The distributions and statistics of these uncertain variables are summarized in Table 3. The correlation matrix assumed for these variables are summarized in Table 4.

Table 3. Statistical information of the variables for retaining wall.

	c_b (kPa)	δ_b (°)	δ_w (°)	φ (°)	γ_s (kN/m^3)	q_u (kPa)
Distribution	Normal	Lognormal	Lognormal	Lognormal	Lognormal	Lognormal
Mean	20	26	24	34	18.5	350
Standard deviation	5	3	3	4	1	70

Table 4. Correlation matrix of the uncertain variables.

	γ_s	c_b	δ_b	q_u	δ_w	φ
γ_s	1	0	0	0	0	0
c_b	0	1	-0.4	0.4	0	0
δ_b	0	-0.4	1	0.4	0	0
q_u	0	0.4	0.4	1	0	0
δ_w	0	0	0	0	1	0.8
φ	0	0	0	0	0.8	1

The design values for partical combinations of reliability constraints are shown in Table 5.

Table 5. RBD results of retaining wall.

Reliability Constraint				Design Parameters			
β_1	β_2	β_3	β_4	b_1 (m)	b_2 (m)	b_3 (m)	h (m)
9.68	5.04	3.58	2.95	1.82	0.70	0.07	0.40.
9.39	4.85	3.85	3.00	2.09	0.50	0.00	0.40

4 Conclusions

To simplify the reliability analysis of engineering problems and facilitate practical applications, this study proposes a strategy for solving inverse reliability problems and illustrates the effectiveness and applicability of the method with several examples. The strategy can be applied to single or multiple design variables, which can be considered deterministic or stochastic.

In multi-objective reliability analysis, the method does not need to establish connections between state equations through mapping functions, but directly integrates single-objective methods through a matrix. Targeted design can be done for one of the multi-failure modes.

In future work, the algorithm can be combined with any nonlinear optimization method when the number of design parameters is greater than the number of performance functions, at which point the reliability-based design (RBD) is extended to reliability-based design optimization (RBDO).

Acknowledgments

This work is supported by the Natural Science Foundation of China [NSFC Grant Nos. 51879091].

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