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Reliability-Based Design Optimization of Pile Stabilized Earth Slopes Aided by Pareto Optimality

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Abstract: Stabilizing piles are known as promising structures for earth slope reinforcement and landslide mitigation, and stability analysis for slopes reinforced with piles under 3D conditions has been widely carried out. However, the optimization design of stabilizing piles for achieving a balance between safety and cost efficiency is still challenging and rarely reported. In this paper, under 3D slope conditions and geological uncertainties, a framework for the multi-objective optimization design of stabilizing piles based on the first-order reliability method (FORM) is presented. The study firstly conducts limit analysis to build a deterministic pile stabilized slope stability model. By accounting for the soil shear strength uncertainties, reliability analyses of the 3D reinforced slopes are carried out based on the prescribed stabilization patterns. In the end, a probabilistic multi-objective design procedure using the Pareto optimality theory combining the reliability analysis results are presented. An illustrative example of a 3D earth slope reinforced with piles is given to explain the feasibility of reliability-based design method which helps to attain the trade-off between the two conflict objectives, i.e., the cost and safety, for practical engineering.

Keywords: Pile stabilized slope; 3D limit analysis; reliability-based design; multi-objective optimization.

1 General Aspects

Stabilizing piles are widely used to improve the stability of earth slopes, and to prevent landslides (Lirer 2012; Zhang et al. 2017). In the traditional design of stabilizing piles, factor of safety (F_s) is conventionally used to evaluate the stability of earth slope reinforced with piles (Hassiotis et al. 1997; Gao et al. 2013). In the deterministic analysis, a series of representative soil parameters are adopted to calculate the F_s. However, it could ignore the variability of soil properties (Oguz et al. 2017). Many investigations reported that some slopes with high F_s value still ultimately fail (Li et al. 2015; Zhang et al. 2017). Therefore, evaluating the stability of slope only by F_s method is not rigorous. The variability of soil properties has a significant influence on the stability of slope and it cannot be ignored.

The variability of soil properties has attracted considerable attention in recent years. Reliability methods include Monte Carlo Simulation (MCS), first-order reliability method (FORM), and second-order reliability method (SORM) (Ji et al. 2021). These methods provide efficient access to consider the variability of soil parameters in evaluating the stability of earth slope (Hasofer and Lind 1974; Low and Tang 2007).

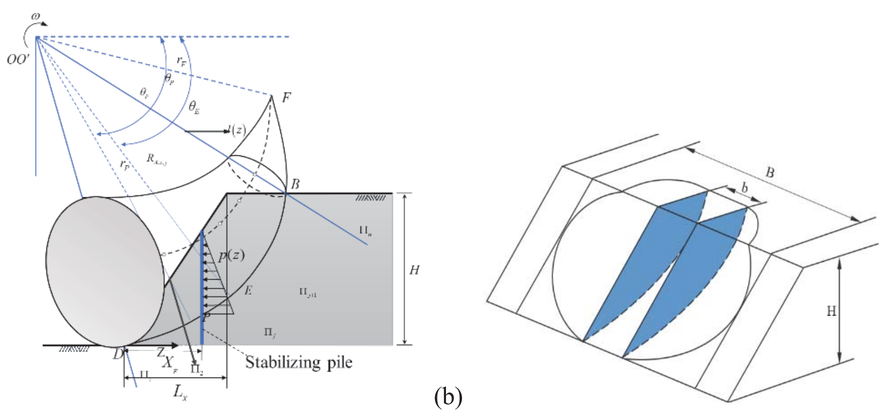
However, despite these efforts, the optimization design of stabilizing piles based on reliability analysis has been seldom studied in previous literature. Traditionally, stabilizing piles are often designed based on factor of safety approach (Kourkoulis et al. 2011; Poulos 1995; Wang et al. 2020). In the conventional design procedure of stabilizing piles, the stability of the landslide-stabilizing pile system within a prescribed engineering budget will be considered as the primary or the only performance objective. However, a stabilizing pile system is always expected to be cost-effective (Tang et al. 2019). The total cost including costs of construction and the unpredictable landslide failure loss are crucial, which must be taken into consideration during the design of stabilizing piles. As soil mass exhibits inherent geotechnical uncertainties, imprecise estimation or neglect of these uncertainties may lead to improper evaluation of landslide stability and cost, and further affects the decision making of stabilizing pile design. Consequently, it is essential for geological and geotechnical engineers to optimize the stabilizing piles design and to assess the related project cost in a probabilistic context (Phoon 2017). A landslide-stabilizing pile system is expected to be safe, cost-effective and robust against the uncertainties of geotechnical parameters(Gong et al. 2019), which are actually conflicting criteria since a higher investment often leads to a safer and more robust system response. With the help of the multi-objective optimization technique, an optimized design can be achieved considering relevant design objectives(Sankaran and Manne 2013; Tang et al. 2019). From the practical point of view, the optimization design of stabilizing piles will be greatly promoted if the widely studied method of stabilizing piles design based on F_s is extended to reliability analysis.

This study aims to develop an efficient method of optimization design for pile stabilized earth slopes in 3D conditions, which helps to attain the trade-off between the two conflict objectives, i.e., the cost and safety. This paper is organized as follows. First, the deterministic analyses of 3D earth slope reinforced with stabilizing piles are carried out to calculate the F_s value considering the pile spacing, pile diameter, and pile location based on limit analysis method. By accounting for the soil shear strength uncertainties, reliability analyses of the 3D reinforced slopes are carried out based on the prescribed stabilization patterns. In the end, a probabilistic multi-objective design procedure using the Pareto optimality theory combining the reliability analysis results are presented. An illustrative example of a 3D earth slope reinforced with piles is given to explain the reliability-based design method and presents a series of reliability-based design charts for practical engineering.

2 Methodology

2.1 Limit analysis of 3D slopes reinforced with piles

In the framework of limit analysis, it is essential to establish a failure mechanism to obtain the upper bound. A kinematically admissible velocity field (failure mechanism) needs to be established to obtain an upper bound. (Michalowski and Drescher 2009) proposed a kind of horn-like rotational failure mechanism for slope under 3D conditions in both frictional/cohesive and purely cohesive soils, as illustrated in Figure 1(a).Gao et al. (2015) adopted the postulated mechanism to analyze the stability of 3D slope reinforced with a row of piles. The failure of the 3D slope mentioned above has distinctive 3D characteristics. However, with the increase of with width of the slope B, the failure will transit to 2D failure (Ito and Matsui 1975). In order to reflect the influence of slope width B on failure characteristics, a block with width B was inserted into the horn-like mechanism, as shown in Figure 1(b). The three-dimensional failure characteristics of slopes with different widths can be more accurately reflected by the failure mechanism after inserting by blocks. More details of 3D failure mechanism are presented in Gao et al. (2015).



(a) (b)

Figure 1. 3D rotational failure mechanism (a) and 3D failure mechanism with plane insert (b)

Based on the upper bound of limit analysis, an energy balance equation of work rate is written for the failure mechanism and an upper-bound solution can be derived from the balance equation. To account for the presence of the piles, an additional rate of energy dissipation done by the resistance of the piles must be taken into consideration in the balance equation of work rate. Finally, the F_s of earth slope is calculated by strength reduction method.

2.2 Reliability theory and first order reliability method

There are many uncertainties existing in engineering, and the deterministic methods, such as the safety factor method, are relatively inaccurate in dealing with these uncertainties, which cannot consistently reflect the risk level of different projects. In contrast, the reliability theory can consider the uncertainty in engineering more reasonably, and can provide valuable risk level information as a judgment basis for engineering decision-making, and it is always regarded as an effective supplement to the deterministic method.

In reliability calculation, the uncertainty of parameters in engineering structure or component are regarded as random variables, i.e. $X = (x_1, x_{2,\dots,} x_n)$, which are assumed to obey some probability distributions. For engineering structures or components with a performance functions g(X), the probability of failure P_f can be calculated by reliability method, such that:

$$P_f = P[g(x<0)] = \int_{\Omega} f_x(x)dx \approx \Phi(-\beta) \quad (1)$$

where, $f_X(X)$ is the joint probability density function of X; Ω is the integral space, corresponding to the failure domain $g(x<0)$; $\Phi(\cdot)$ =cumulative distribution function of the standard normal variable, and β is the reliability index which is well known in engineering reliability analysis.

However, in practical engineering, it is challenging to calculate the multidimensional integral represented by Eq. (1). The joint probability density function $f_X(X)$ is sometimes hard to acquire. To overcome this difficulty, many studies proposed approximate methods to solve Eq. (1). using the MCS methods and/or the FORM/SORM methods. Particularly, the FORM is widely used in engineering due to its simplicity and effectiveness. The basic algorithm of FORM is the Hasofer-Lind-Rackwits-Fiessler (HLRF) algorithm. In carrying out slope reliability analysis, the performance function defining the margin of safety is

$$g(x) = F_s - 1 \tag{2}$$

where, the factor of safety F_s can be acquired from the aforementioned limit analysis.

2.3 Basic concept of Pareto front optimization theory

Pareto optimality originates from the concept of Pareto efficiency, which was proposed to study economic efficiency and income distribution (Barr 2020). In economics, a Pareto improvement, given an initial allocation of goods among a set of individuals, is defined as a change to a different allocation that makes at least one individual better off without making any other individual worse off, and an allocation is called "Pareto efficient" or "Pareto optimal" when no further Pareto improvements can be made (Steuer 1986). This concept has been widely applied to multi-objective optimization, which aims to simultaneously optimize two or more often conflicting objectives subject to certain constraints. In multi-objective optimization problems, decisions have to be made in order to achieve the best tradeoffs between two or more conflicting objectives.

In multi-objective optimization problems, it always impossible to find one single solution which makes every single objective reach the optimal solution simultaneously. In such cases, improving an objective often worsens other objectives. Therefore, Pareto optimality becomes an essential concept for solving multi-objective problems. Among all the tentative solutions, if a solution can improve one objective without leading to another one suffers, it is called non-dominated solution. Therefore, the goal of multi-objective optimization is to find such non-dominated solutions and to find a tradeoff between the different objectives. All the Pareto-optimal solutions compose the Pareto-optimal set, and the projection of Pareto-optimal set in the objective space is called the Pareto front.

3 Illustrative example of stabilizing piles for 3D earth slopes

3.1 A brief introduction

To demonstrate the effectiveness of the proposed method for stabilizing piles design, an illustrative example is represented here. The geometric definitions of the pile stabilized slope in three dimensions (3D) are given by Figure 1. The width and height of the slope are 20m and 10m respectively. The inclination of the slope is 45°, and the soil parameters in this example are determined by experience from practical engineering and listed in the following Table 1. For the candidate design in the specific design space (Table 2), the design objective is to simultaneously maximize the reinforcement effectiveness and minimize the cost of stabilizing piles for the soil slope *C*.

Table 1. The soil parameters of the illustrative example.

Soil Parameter	Value
Cohesion	15 kPa
Friction angle	20°
soil bulk density	19.63 g/cm^3

Table 2. Design space adopted for the illustrative examples.

Design parameters	Design pool
Pile diameter, *D* (m)	{0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5}
Pile spacing, *S* (m)	{1.6, 1.8, 2.0, 2.2., 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 3.6, 3.8, 4.0}
Pile position, *XL* ($=\frac{X_F}{L_X}$)	{0.4, 0.5, 0.6, 0.7, 0.8, 0.9}

In the selected design space, a total of 847 discrete candidate pile designs are possible and will be analyzed, and the optimal pile design will be identified accordingly. According to (Tang et al. 2019), the relationship between cost and geometrical parameters of stabilizing piles can be represented by:

$$C = \eta \frac{\pi D^2 L}{4S} \tag{3}$$

where η represent cost per unit piling work.

3.2 Results of multi-objective optimization-based design of stabilizing piles

Within the specified design space, different pile diameters, pile spacing, and pile positions correspond to different safety factors. The specific calculation results are shown in Figure 2. It can be seen that there are 858 feasible design in the design space and the Pareto front consistent with the left and upper boundary of the feasible domain. The Pareto front consists of 25 non-dominated designs, all the non-dominated designs on the Pareto front are superior to the others in the feasible domain; while among these non-dominated designs on the Pareto front, no one is superior or inferior to the others when both design objectives are taken into consideration. Therefore, a trade-off relationship exists between the reinforcement effectiveness and the cost for better decision-making.

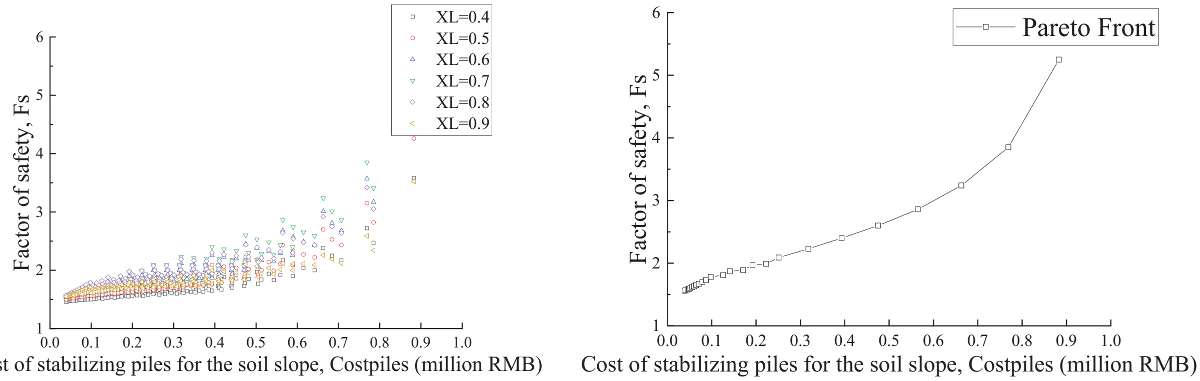


Figure 2. All candidate designs and Pareto front based on factor of safety

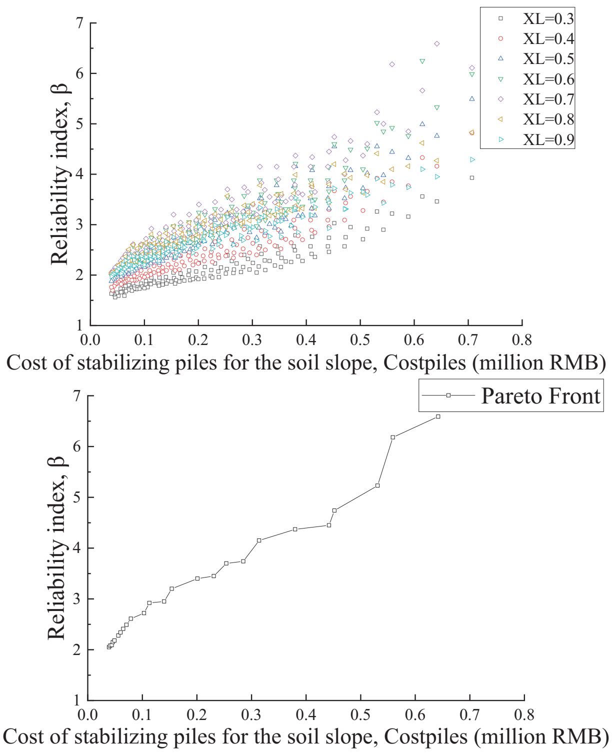


Figure 3. All candidate designs and Pareto front based on reliability

Figure 2 and Figure 3 illustrate 847 candidate deigns based on factor of safety and 858 candidate deigns based on reliability index, and their Pareto front respectively. For both multi-objective designs, the Pareto front consists of 25 optimal designs. The Pareto optimal also illustrates that the optimal pile position should be XL = 0.7 and large pile diameter and small pile spacing can be preferred. For multi-objective designs based on F_s, the Pareto front is smooth, and becomes steep gradually. For multi-objective designs based on reliability index, the middle section of Pareto front is gentle, which indicates that a huge increase in the cost of stabilizing piles can only lead to slight increase in reinforcement effectiveness, and with the increase of cost of stabilizing piles, Pareto front becomes sharp, which indicates that only a small cost of stabilizing piles increase can drastically increase the reinforcement effectiveness.

To investigate the influence of soil uncertainty on the optimal design of stabilizing piles, three levels of soil variation coefficients are specified in this study. The specific parameters are illustrated in Table 3:

Table 3. The table caption must be centered.

Level of uncertainty	c	φ
High COV	0.5	0.3
Medium COV	0.3	0.2
Low COV	0.15	0.1

The calculation results are presented in Figure 4. As expected, with the increase of cost of stabilizing piles, the reliability of slope witnesses an increase, and the stronger soil uncertainty will lead to smaller reliability index for slope. For low and medium soil uncertainty, the difference of reliability index is relatively small. When the cost of stabilizing piles of the stabilized piles is lower than 0.4 million RMB, the reliability of slope under high uncertainty soil properties are far below that under other two cases, and the increase of cost of stabilizing piles leads to smaller difference of these three cases.

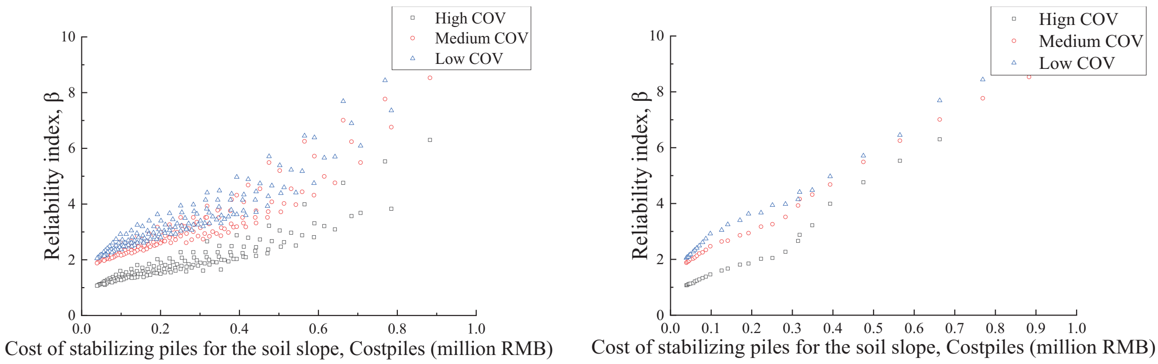


Figure 4. All candidate designs and Pareto front based on reliability considering different soil parameters COV

The design groups provided by Pareto front are all optimal designs, however, in order to find the most preferred design in the design space, not only the cost of stabilizing piles but also the damage loss after slope failure needs to be taken into consideration.

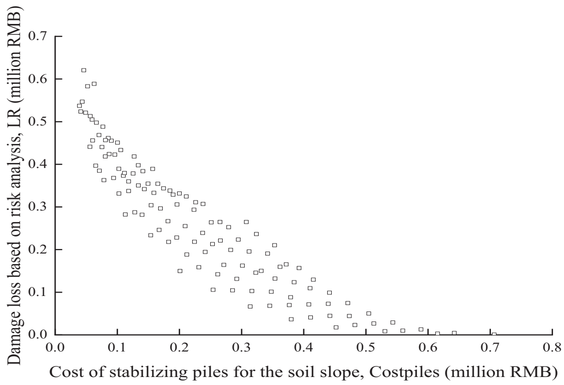


Figure 5. Relationship between cost of stabilizing piles and damage loss

Based on risk analysis, the damage loss L_R of the slope reinforced with stabilizing piles is given as:

$$L_R = P_f \times (Cost_{piles} + Cost_{slope}) \tag{4}$$

The $Cost_{slope}$ is determined by the engineers' experience. Herein, we have assumed nominal $Cost_{slope} = 10$ (million RMB), and the cost of stabilizing piles is calculated by Eq. (3).

Figure 5 illustrates the relationship between the cost of stabilizing piles for the soil slope and damage loss based on risk analysis. It can be seen that with the cost of stabilizing piles increase, the calculated risk level decreases on the whole. Particularly, when the cost of stabilizing piles ranges from 0 to 0.1 million RMB, the risk level declines very sharply; and when the cost increases to 0.5 million RMB, the risk level tends to remain constant and close to 0. This indicates that after the cost of stabilizing piles reaches a certain value, increasing investments on stabilizing piles will not bring much meaningful safety improvements.

4 Conclusions

This paper presents a framework for the multi-objective optimization design of stabilizing piles based on the first-order reliability method (FORM) under 3D slope conditions and geological uncertainties. Within the framework, two conflicting objectives: reinforcement effectiveness and cost efficiency are explicitly considered and optimized. The output of this multi-objective optimization design is a Pareto front which describes the trade-off between these two objectives. This Pareto front can provide information for better decision-making. The following conclusions are drawn.

1. An illustrative example demonstrated the effectiveness of the proposed framework, which provides a pile design with best compromise between cost of stabilizing piles and reinforcement effectiveness.

2. The Pareto fronts of optimization design based on factor of safety and reliability index both illustrated the optimal stabilizing piles position is at $XL = 0.7$. The tendency of the Pareto front indicated that when the cost of stabilizing piles is small, the increase of reinforcement effectiveness is slow. With the increase of the cost of stabilizing piles, the Pareto front became sharp, which indicates that only a small increase of cost of stabilizing piles will lead to a huge increase in reinforcement effectiveness.

3. With the cost of stabilizing piles increase, the risk level witnesses a decreasing trend. When the cost of stabilizing piles ranges from 0 to 0.1 million RMB, the risk level declines sharply. When the construction increases to 0.5 million RMB, the decrease of risk level slows down. After reach 0.5 million RMB, increasing investments on piles construction can hardly lead to risk level decrease.

Acknowledgments

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