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Investigation of Stratigraphic Uncertainty for Three-Dimensional Geological Modelling

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Abstract: Regarding the variability of subsurface geo-data, investigating uncertainty in geological strata has generally been ignored. However, because geological strata reveal the regularities of geo-data distribution, assessing stratigraphic uncertainty with a universal method in three-dimensional (3D) geological modelling is beginning to attract attention. As an advanced stochastic simulation method, the random field method has been used widely to estimate geo-data in one and two dimensions, but this method is always challenging because of the difficulty in interpreting the scale of fluctuation (SoFs), which are the key parameters regarding the outcome performance. Furthermore, the determination of SoFs proposed in previous research is no longer applicable because there is no numerical meaning for spatially discretized stratum points. To address this problem, the present study investigates the adaptability of the random field method in stratigraphic uncertainty 3D geological modelling with a series of possible inputs, such as smoothing coefficient and distance and depth factors. The horizontal SoFs are obtained based on the borehole coordinates, while the vertical SoF is defined according to the stratum thickness distribution. Each voxel in the space is assigned a probability distribution obtained from the Gaussian autocorrelation function, where the concept of information entropy can be used for uncertainty quantification. The proposed method is applied in a hypothetical case and an actual field case, which are illustrated by solid-voxel-based 3D models. The outcomes regarding different inputs are presented, and the uncertainty of the underground strata is well-quantified.

Keywords: Stratigraphic uncertainty; 3D geological modelling; random field; scale of fluctuation; information entropy

1 Introduction

Explaining subsurface characteristics by using boreholes has always been difficult throughout the development of geotechnical engineering, this being because of the inherent heterogeneity in soil and rock strata. Because of budget and scheduling constraints, the geotechnical information gathered from site investigations is limited, and with less information, engineers are likely to overdesign foundations to satisfy safety requirements, thereby increasing material and labor costs unnecessarily.

To better manage and interpret the underground characteristics of a specific site using limited boreholes, a three-dimensional (3D) geological model is developed to fill in the gaps with possible geological configurations. However, the existing filling techniques in 3D geological models—such as interpolation (Marinoni, 2003; Pan et al., 2020)—lead to large uncertainties when the borehole data are insufficient. Therefore, quantifying these uncertainties in geological models for superstructure design is a critical issue.

To date, a series of studies has trailed the advanced filling technique of a random field to explain the uncertainties of soil properties among unknown strata in both one-dimensional (1D) and two-dimensional (2D) modelling (Gong et al., 2013, 2019; Li et al., 2016; Simos &Costantinoa, 2004; Zhang et al., 2021). There has been much previous investigation of how to determine the scale of fluctuation (SoF) as the key parameter, which is rather difficult to interpret in stratigraphic uncertainty modelling when the mean and variance are no longer applicable. To address this problem, Wang et al. (2016) considered quantifying the uncertainties of multiple strata by using a Markov random field with an energy function. To reduce the uncertainties with additional information, Gong et al. (2020) recently introduced the idea of a random field with additional information, namely stratigraphic dip, which reduced the uncertainties at the stratum interfaces significantly. However, when extended to three dimensions, their way of interpreting SoFs is problematic, and its practicality for actual geological projects remains questionable.

To enhance the interpretability of SoFs for stratigraphic uncertainties in 3D geological modelling, a random-field-based approach is proposed based on borehole data. Herein, a series of possible inputs for the horizontal SoFs is given based on the borehole coordinates, while the vertical SoF is defined according to the characteristics of stratum thickness distribution within selected boreholes. In 3D space, the voxel can be used as the smallest geological unit that stores values in a regular grid, so voxel-based 3D geological modelling is adopted, where the continuous borehole data are discretized into voxels and assigned to the model. The information entropy is introduced as a quantitative index to measure the uncertainty in every voxel point, and the stratum uncertainty modelling outcomes regarding different inputs are presented in both a hypothetical case and an actual field case.

2 Methodology

As an advanced stochastic simulation method, the random field method has long been used to explore the unknown areas among boreholes. However, it is challenging to use this method in classification problems because the samplescontain no numerical feature, i.e. neither mean nor variance is available. In this case, the SoF plays a major role in describing the correlation between elements.

Clearly, the correlation between voxels should be high when they are close to each other. In this study, the Gaussian correlation function is used to calculate the correlation between the voxels and the boreholes. The terms I_X, I_Y and I_Z are known as the SoFs, which present the features of the specific field. For any voxel i in 3D space and the borehole voxel j in stratum m, the correlation function $\rho(i, j)$ and its SoFs are illustrated as follows:

$$\rho_m(i,j) = e^{\left(-\frac{\pi d_X^2}{I_X^2}-\frac{\pi d_Y^2}{I_Y^2}-\frac{\pi d_Z^2}{I_Z^2}\right)} \tag{1}$$

$$I_X(i) = (X_{max} - X_{min}) \times C_X \tag{2}$$

$$I_Y(i) = (Y_{max} - Y_{min}) \times C_Y \tag{3}$$

$$I_Z(i) = D \tag{4}$$

where X_{max} and X_{min} are the maximum and minimum coordinates, respectively, on the X axis, Y_{max} and Y_{min} are those on the Y axis, C_X andC_Y are the distance factors for I_X and I_Y, respectively, and$D$is the depth factor defined by the characteristics of stratum thickness distribution.

After obtaining the correlation for each stratum, the probability of which stratum the voxel belongs to can be calculated. As mentioned above, a distant voxel should have a lower correlation, resulting in a more averaged probability. Herein, we use Laplace smoothing to average the probability with larger distance. The probability function and the smoothing coefficient are defined as

$$P_m(i) = \frac{\sum_{n_b}[\rho_m(i,l)\cdot Index(l,m)]+\lambda\sigma(i)}{\sum_{n_m}\left\{\sum_{n_b}[\rho_m(i,l)\cdot Index(l,m)]+\lambda\sigma(i)\right\}} \tag{5}$$

$$\sigma(i) = \frac{1}{\sum_{n_m}\rho_m(i,j)} \tag{6}$$

where λ is the smoothing coefficient and $Index(l, m)$ is an indicator function to identify whether stratum l matches stratum m. To better visualize the results, the set of probability is interpreted as the information entropy, which is indicated as

$$Q(i) = -\sum_{n_m}[P_m(i)\cdot log\, P_m(i)] \tag{7}$$

3 Modelling Performance in Hypothetical Case

To demonstrate the performance of the proposed method in three dimensions, four prescribed boreholes located at (5, 5), (5, 25), (25, 5) and (25, 25) are simulated for a 30 m × 30 m site with a depth of 20 m. Three strata with nonuniform thickness are observed in the boreholes. As in Equations (1)–(6), the smoothing coefficient λ,the distance factors C_X andC_Y and the depth factor D are the key parameters for the modelling performance and are tuned manually in this section.

Figure 1(a) shows the effect of the smoothing coefficient λ when the other parameters are held constant. As can be seen, the center of the site where no borehole is assigned has high entropy (i.e. high stratigraphic uncertainty). The borehole information is extended when λ is increased, resulting in lower entropy. Figure 1(b) shows the effect of the distance factors C_X andC_Y where they are set to be equal to have the same influence on both the X and Y axes. The low-entropy areas are restricted to around the boreholes when the distance factors are low, and they extend significantly when the distance factors are increased, ultimately becoming a trend. Using the parameter from the stratum thickness distribution, Figure 1(c) shows the results of modelling using the depth factor with the minimum value (i.e. 2 m), the first quartile (i.e. 6 m) and the average of these two (i.e. 4 m). The results show a tendency similar to that with previous models, in which the total entropy decreases as the values increase. Note that the depth factor has a significant influence on the uncertainty at the stratum interfaces. Upon increasing the depth factor from 2 to 6, the entropy of the interfaces increases dramatically, resulting in an overestimation of the uncertainty (see Figure 1(c) when $D = 6$). This is especially problematic when a thin stratum is observed, because its effect will be neglected when a high depth factor is applied. However, a low depth factor also overestimates the uncertainty at the gap (see Figure 1(c) when $D = 2$). Therefore, it is reasonable to use the average of the minimum value and the first quartile for balancing purposes. Consequently, the aforementioned parameters have considerable impact on the outcome performance and so should be chosen with care.

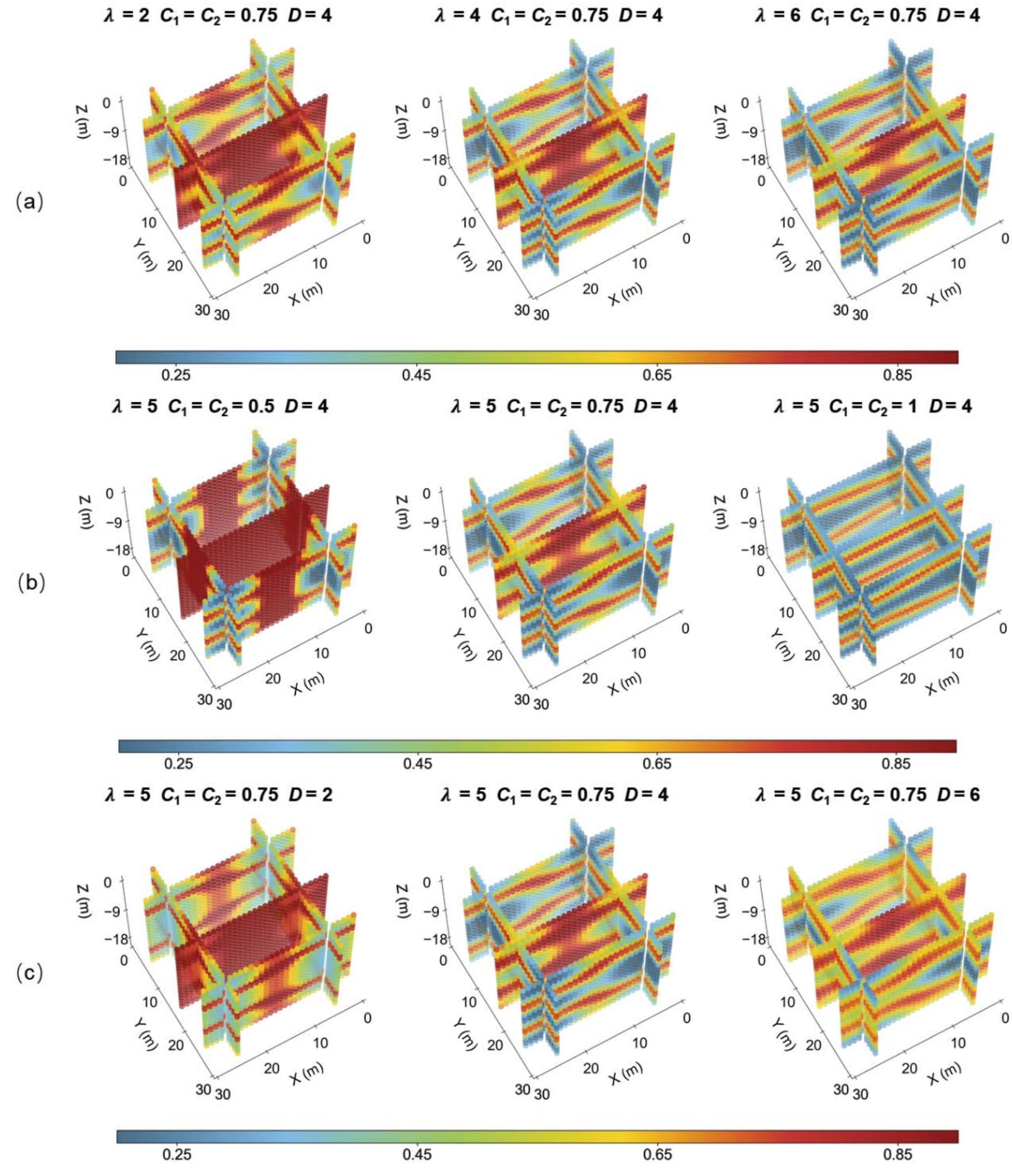

Figure 1. Hypothetical geological entropy models with effects of selected (a) smoothing coefficients, (b) horizontal scale of fluctuation (SoFs) and (c) vertical SoFs

4 Modelling Performance in Field Case

4.1 Geological data

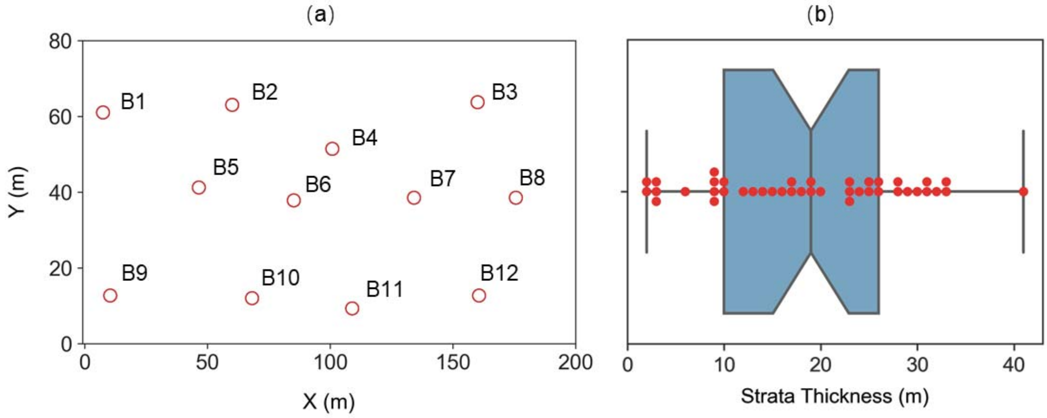

Figure 2.(a)Borehole layout and (b) stratum thickness distribution.

To verify further the practicality of the proposed method, boreholes labelled B1–B12 in an actual tunnelling project are selected and scattered in a voxel-based 80 m × 200 m 3D model with a depth of 60 m (from −20 m to −80 m), as shown in Figure 2(a). Four different strata are observed in the boreholes, i.e. Layer 1, Layer 2, Layer 3, and Layer 4. Note that Layer 1, Layer 3, and Layer 4 are thick strata whereas Layer 2 is regarded as an interlayer observed in only a few boreholes. The borehole data are discretized into 1-m^3 voxels and assigned to the voxel-based model for computation. The thickness of these strata was extracted and displayed in boxplot format (Figure 2(b)), where the minimum thickness is 2 m and the first quartile is 10 m. For illustration, only one group of parameters is used for the 3D geological uncertainty modelling.

4.2 3D geological modelling

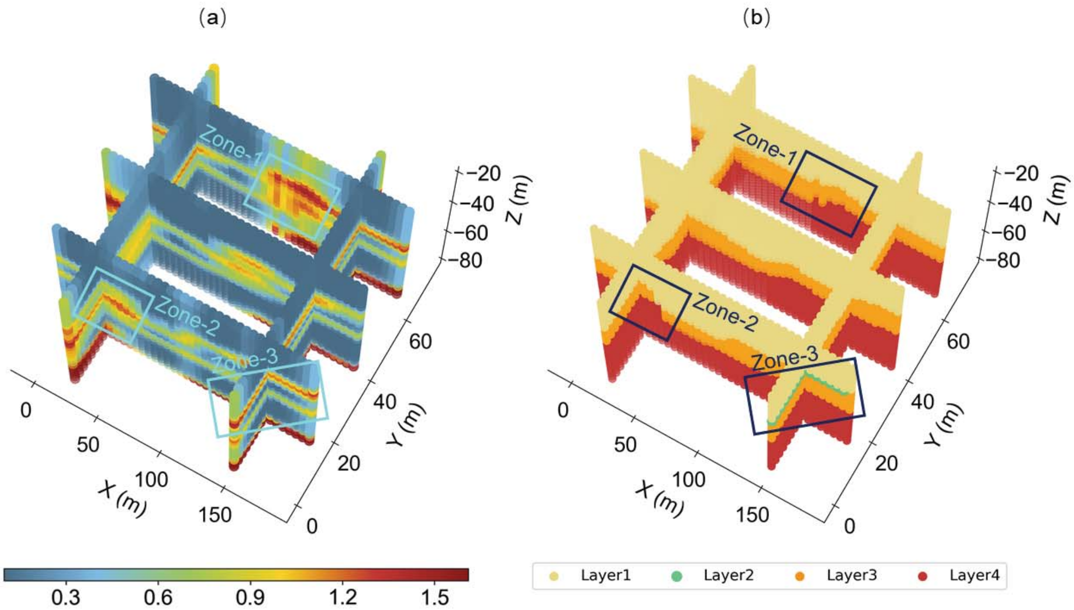

Figure 3.(a) 3D geological entropy models with fixed parameter and(b)most probable stratigraphic model

In this section, we select only one group of parameters ($\lambda = 5$, $C_1 = C_2 = 0.75$, $D = 6$) for the proposed approach. According to the stratum thickness distribution in Section 4.1, the depth factor is obtained as the average of the minimum and first quartile. During the computation, it is inefficient to consider all the boreholes in the site because the correlation between distant boreholes could be negligible. In this field case, every voxel identifies the nearest four boreholes for uncertainty modelling. As shown in Figure 3(a), a 3D geological entropy model is generated using the proposed approach. The entropy model can capture the uncertainty among the strata, where high entropy can be detected at the interfaces. Note that the entropy along the interfaces is not simply a trend but fluctuates, as marked in Zone-1, Zone-2 and Zone-3. These zones contain potential high uncertainty according to the spatial characteristics of strata in surrounding borehole information. To better interpret the stratigraphic distribution, Figure 3(a) shows the most probable stratigraphic model according to the stratigraphic probability distribution. As can be seen, Layer 3 is observed between Layer 1 and Layer 4, which demonstrates that the proposed method can illustrate the spatial stratigraphic distribution of the site in 3D space. Moreover, complex stratigraphic changes are presented in Zone-1 and Zone-2, where the uncertainty should be high. In Zone-3, the thin stratum of Layer 2 is detected in between Layer 1 and Layer 3, and the uncertainty of this region is high as well.

5 Conclusion

This study proposed a novel method with the concept of a random field for stratigraphic uncertainty in 3D geological modelling. The borehole data are processed as voxels for correlation computation. The uncertainty is indicated as a probability distribution, which can be quantified using information entropy. The proposed method demonstrated its performance in a hypothetical case with four boreholes using different inputs for key parameters, such as smoothing coefficient, distance factor and depth factor. The borehole information spread farther as these parameters increased, leading to lower overall entropy. To verify the proposed method in an actual case, 12 boreholes from a tunnelling project were used, and the 3D geological entropy model with selected parameters was constructed. The model identified the interfaces among the strata and captured the zones with potential high uncertainty. As revealed in the most probable model, the uncertainty is usually high when complex stratigraphy is detected or a thin stratum is found. However, because the determination of the key parameters has a significant impact on the outcome performance, a proper criterion for parameter selection will be investigated in future work.

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