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Have We Underestimated the Influence of Geological Uncertainty on Tunnel Deformational Performance in the Uncertain Stratum?

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Abstract: The geological uncertainty and the inherent spatial variability of soil properties are two main uncertainties in geotechnical engineering that will significantly affect the embedded geo-structures. The influence of spatial variability of soil properties on tunnel deformation has been widely studied, while geological uncertainty is usually not considered. Is this simplification reasonable? Have we underestimated the influence of geological uncertainty on tunnel deformational performance? This study aims to present a method for tunnel probabilistic analysis by considering both the geological uncertainty and spatial variability of soil properties. The comparisons of tunnel deformational performance between only considering soil spatial variability and coupling considering these two types of uncertainty are presented to reveal the role of geological uncertainty in tunnel probabilistic analysis. An improved coupled Markov chain (CMC) model is used to simulate geological uncertainty based on spare limited boreholes. Moreover, the spatial variability of soil properties within each soil layer is modelled using random fields. The results indicate that the proposed method can effectively evaluate the tunnel structural response considering these two types of uncertainty. With the borehole number increases, the influence of geological uncertainty gradually decreases. The uncertainty of calculations only considering the soil spatial variability may be overestimated if the geological uncertainty is ignored.

Keywords: Tunnel; geological uncertainty; spatial variability; probabilistic analysis.

1 Introduction

The structural safety of the tunnel has always been one of the most concerned issues of the governor and engineers (Huang and Zhang 2016). Once the tunnel accident occurs, it will cause enormous casualties, economic losses, and social severe adverse effects. Meanwhile, the embedded environment of geo-structures is complex and uncertain. The uncertainty can be mainly divided into two categories (Elkateb et al. 2003): spatial variability of soil properties which is the variation of soil properties from one point to another in space, and geological uncertainty in the heterogeneous layer appears in the form of thin soil layers embedded in another layers within a more uniform soil mass.

The spatial variability is often modeled by random field theory. Vanmarcke (1977) discussed the effect of soil spatial variability on geotechnical systems using random field method. Currently, the random field analyses primarily focus on slope stability, foundation settlement, excavation and tunnel (Zhang et al. 2021c). On the other hand, geological uncertainty also exists in reality and plays a significant role in geo-structure performance (Li et al. 2016). Due to the limitation of geotechnical investigation techniques and project budgets, only a few boreholes can be afforded in a practical project (Gong et al. 2019). Recently, Markov random field and Coupled Markov chain (CMC) model are two popular methods to simulate geological uncertainty. Subsequently, many efforts have been devoted to analyzing the deformation and stability of geotechnical structures (Zhang et al. 2021a; Zhang et al. 2021b). However, it is relatively rare to comprehensively consider the influence of the two uncertainties on the geotechnical structure. Deng et al. (2017) explored the influence of both the inherent variability of soil parameters and geological uncertainty on slope reliability analysis. It is still worth investigating both types of uncertainty on the tunnel performance.

This paper aims to develop a probabilistic model to reveal both geological uncertainty and soil spatial variability on the tunnel deformational performance. This paper is organized as follows. Firstly, the methods of simulating the both uncertainties are introduced. Next, a case study is presented to reveal the influence of these two uncertainties on the tunnel deformational performance. Finally, some conclusions are given.

2 The proposed framework considering geological uncertainty and soil spatial variability

2.1 Simulation of geological uncertainty using improved coupled Markov chain

Coupled Markov chain model has the ability to characterize the heterogeneity of geological formations (Elfeki and Dekking 2001). It is easy to explain, has few parameters, and high applicability (Elfeki and Dekking 2005). The two-dimensional CMC model is more suitable for simulating the geological uncertainty than the one-dimensional CMC model, which can only characterize one direction. As shown in Figure 1, the domain is divided into $Nx \times Ny$ cells of the same size, and each cell corresponds to its state. The basic idea of the CMC model is that the state of the current step depends only on the state in the previous step. This means that the state X_{ij} of the cell (i, j) depends on states $X_{i\text{-}1,\,j}$ and $X_{i,\,j\text{-}1}$ of the cells on the left (i-1, j) and on the top (i, j-1) of the current cell in the domain. For simplicity, let the state of cells (i, j), (i-1, j), (i, j-1) and (N_x, j) is S_k, S_l, S_m, S_q, respectively. The conditioning formula can be expressed by

$$\begin{aligned} P_{lm,k|q} &= P(X_{i,j} = S_k \left| X_{i-1,j} = S_l, X_{i,j-1} = S_m, X_{N_x,j} = S_q \right.) \\ &= \frac{p_{lk}^{h} \cdot p_{kq}^{h(N_x-i)} \cdot p_{mk}^{v}}{\sum_{f=1}^{n} p_{lf}^{h} \cdot p_{fq}^{h(N_x-i)} \cdot p_{mf}^{v}} \end{aligned} \tag{1}$$

Where $p_{kq}^{h(N_x-i)}$ is the (N_x-i)-step horizontal transition probability from S_k to S_q, p_{mk}^{v} is the vertical transition probability from S_m to S_k.

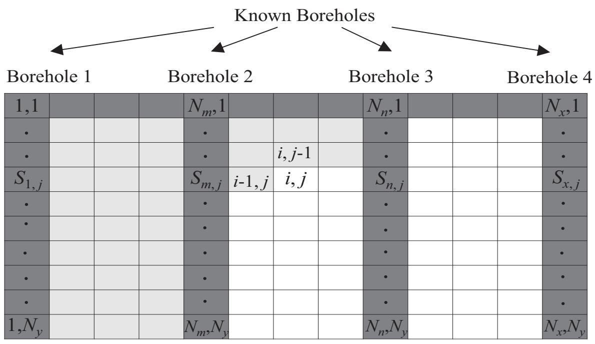

Figure 1. The schematic of using coupled Markov chain to simulate two-dimensional domain.

Recently, Zhang et al. (2022) proposed an improved CMC model to enhance the rationality of determining the horizontal transition probability matrix. The steps of this method are summarized as below:

(1) All boreholes are discretized and utilized to calculate the vertical transition count matrix (VTCM) and estimate the vertical transition probability matrix(VTPM). The initial horizontal transition count matrix (HTCM') is also calculated using the same method. It is worth noting that the initial HTCM' is not the final HTCM, which is only an intermediate matrix to give extra transition information in the horizontal direction. Taking three soil types revealed by boreholes as an example, the VTCM and initial HTCM' (T^v and $T^{h'}$) are expressed in Eq. (2).

$$T^{v} = \begin{pmatrix} T_{11}^{v} & T_{12}^{v} & T_{13}^{v} \\ T_{21}^{v} & T_{22}^{v} & T_{23}^{v} \\ T_{31}^{v} & T_{32}^{v} & T_{33}^{v} \end{pmatrix} \quad T^{h'} = \begin{pmatrix} T_{11}^{h} & T_{12}^{h} & T_{13}^{h} \\ T_{21}^{h} & T_{22}^{h} & T_{23}^{h} \\ T_{31}^{h} & T_{32}^{h} & T_{33}^{h} \end{pmatrix} \tag{2}$$

(2) The $T^{h'}$ determined using the above method will be inaccurate due to the discreteness between boreholes. Both T^v and $T^{h'}$ are considered to determine the realistic HTCM in this study. Let T^{hv} represents the larger values of T^v and $T^{h'}$. Through this process, the transfer information revealed at $T^{h'}$ (especially for transition not revealed by the T^v) can be expressed at the upper and lower triangles of T^{hv}. Thus, T^{hv} can better reflect the conversion between soil types in the horizontal direction. Meanwhile, based on Walther's law, the final HTCM can be denoted in Eq. (3), having an unknown value $K = T_{ii}^{h} / T_{ii}^{v}$.

$$T^{h} = \begin{pmatrix} KT_{11}^{hv} & T_{12}^{hv} & T_{13}^{hv} \\ T_{21}^{hv} & KT_{22}^{hv} & T_{23}^{hv} \\ T_{31}^{hv} & T_{32}^{hv} & KT_{33}^{hv} \end{pmatrix} \tag{3}$$

(3) Proposed method for determining K value. As shown in Figure 1, firstly, assuming N_k different K values, the horizontal transition probability matrix (HTPM) can be obtained using Eqs. (2) and (3). Then, N times CMC simulations can be performed according to the input VTPM and HTPM. Thirdly, the back analysis calculated HTPM based on simulated strata (i.e., from result to input) can be calculated. Next, the error matrix for input input and back analysis calculated HTPM can be obtained. The estimation goal is the HTPM, thus, the error matrix of input and back analysis calculated HTPM can well reflect the performance of the selected K value. Finally, the final error under this K value is the mean square error (MSE) of N times CMC simulations. The K value corresponding to the minimum MSE value is the optimal K value. The proposed method of determining K value makes full use of all borehole information and can better present the influence of K on HTPM.

(4) Finally, simulating the geological uncertainty using the VTPM and final HTPM with optimal *K* value.

2.2 Simulation of soil spatial variability using random field

After simulating the geological uncertainty, the next step is to generate the soil spatial variability using the random field theory. Firstly, the soil type of each grid unit of finite difference model (FDM) need to obtain. Then, the center coordinates of different soil type units can be extracted. Next, the K-L expansion technique is adopted to discrete the random field for each soil layer. Finally, the random fields of soil parameter are mapped into FDM to calculate the tunnel performance under the uncertain stratum.

As mentioned above, the elastic modulus E_s within the soil mass is spatially varied. The lognormal distribution is adopted because it avoids the generation of negative values of soil parameters. The lognormal distribution of E_s means that ln_{Es} is normally distributed and the standard deviation ζ_{lnEs} and mean λ_{lnEs} of the normal distribution of ln_{Es} are given by:

$$\zeta_{lnEs} = \sqrt{\ln(1 + COV_{Es}^2)} \tag{4}$$

$$\lambda_{\ln Es} = \ln \mu_{Es} - \frac{1}{2}\sigma_{\ln Es}^2 \tag{5}$$

Scale of fluctuation is an important concept of geotechnical parameters in the random field modeling. It can well reflect the spatial variability of the soil. In this study, the correlation matrix is built with the anisotropic exponential autocorrelation function:

$$\rho(\Delta_h, \Delta_v) = \exp\left[-2\left(\frac{(\Delta_h)^2}{\delta_h^2} + \frac{(\Delta_v)^2}{\delta_v^2}\right)\right] \tag{6}$$

where $\square_h$ and $\square_v$ are horizontal and vertical distances between the two points, respectively, $\square_h$ and $\square_v$ are scale of fluctuation in the horizontal and vertical direction, respectively, and $\square\square(\square_h, \square_v)$ is the correlation coefficient between two points; The scale of fluctuation quantifies the distance within which the soil properties exhibit relatively strong correlation. A smaller correlation distance indicates a stronger spatial variability. The Karhunen-Loeve expansion technique is used to discretize the random field in this study. The random field of soil domain is discretized by the mid-point method, in which the interested parameter for each element is equal to the value given by the random field value at the centroid of each element.

2.3 Implementation procedure

Figure 1 shows the proposed framework considering geological uncertainty and soil spatial variability in this study. It mainly includes two parts: geological uncertainty simulation and soil spatial variability simulation. The process of combining the geological uncertainty and soil spatial variability is shown as follows. Firstly, the improved coupled Markov chain model is adopted to simulate geological uncertainty. Secondly, the random field theory is used to simulate the soil spatial variability for each simulated stratum. The soil Young's modulus is highlighted and simulated with a horizontally stratified anisotropic random field that is discretized by the Karhunen-Loeve expansion. Random finite difference modelling (RFDM) is performed to calculate the influence of these two types of uncertainty on the tunnel.

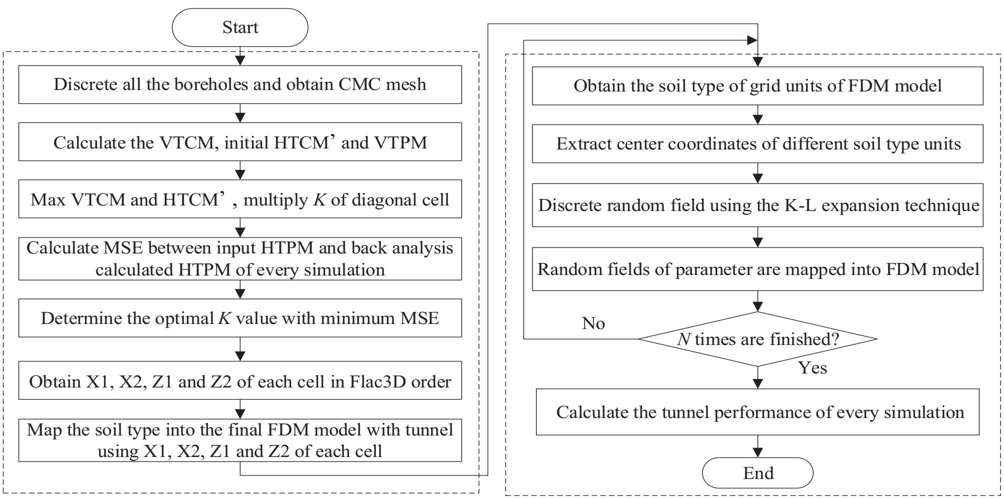

Figure 2. The proposed framework considering geological uncertainty and soil spatial variability.

3 Case study

3.1 Borehole data of collected site

The site information revealed by collected borehole data is shown in Figure 3. A total of four soil types have been identified, namely topsoil, clay, quick clay and sand, which are represented by red, yellow, blue and cyan colors in Figure 3. The x and z coordinates represent the horizontal and vertical directions, respectively. The strata at the depth between 0 and -30 m of this site are addressed in this study. The geological profile is 70 m long and 30 m deep. The first step of the simulation of the stratum uncertainty is to discrete the geological profile into cells. The proper size of the cell in the CMC model should be less than or equal to the minimum thickness of the geologic unit in the corresponding direction. The minimum thickness of the strata revealed by boreholes is 0.5 m of a thin sand layer at the B5 borehole. Therefore, the sampling intervals in the vertical direction is 0.5 m in this study. The sedimentation scale of the stratum in the horizontal direction is often more significant than that in the vertical direction so that the horizontal sampling interval can be appropriately larger than the vertical interval. To balance the calculation efficiency, the sampling interval in the horizontal direction is taken as 1 m in this study.

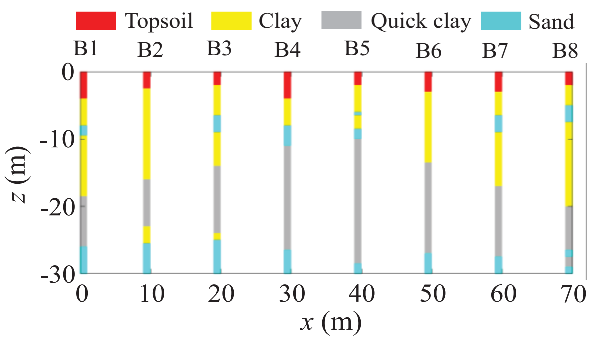

Figure 3. Site information revealed by the collected borehole data.

3.2 Determination of the runs number of Monte Carlo simulation

Figure 4 investigates the impact of the number of Monte Carlo simulations on the mean value and coefficient of variance (COV) of tunnel horizontal convergence (ΔD_h) using the differnet boreholes (BH). It can be seen that the mean value and COV is convergent when the number of Monte Carlo simulations is set to 300, as shown in Figure 4 (a) and (b) using three boreholes (3BH) and eight boreholes (8BH).

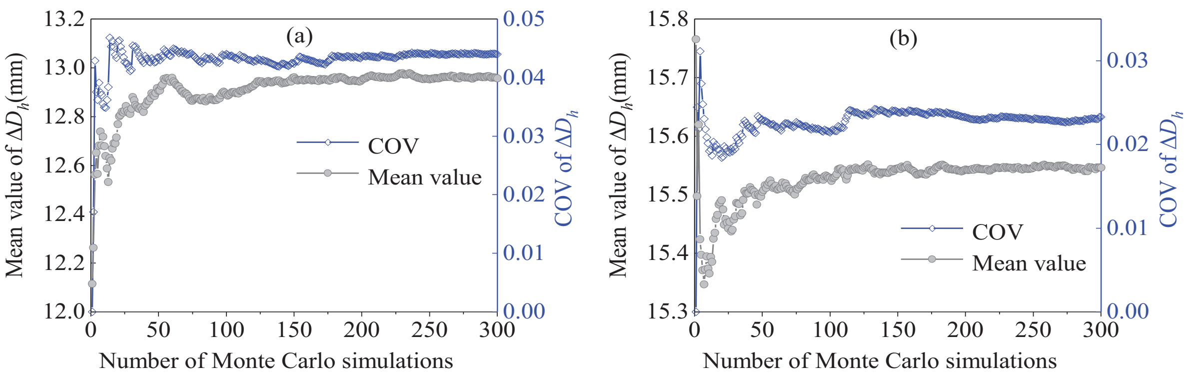

Figure 4. Effect of number of Monte Carlo simulations on mean value and COV of ΔD_h: (a) 3BH; (b) 8BH.

3.3 Typical realizations of the different borehole schemes

A tunnel is considered embedded in this geological uncertainty strata with its outer diameter D = 6.2 m, lining thickness t = 0.35 m. The elastic modulus and Poisson's ratio of the tunnel are set at 34.5 GPa and 0.2. Meanwhile, the effective rigidity ratio of the tunnel lining is set to 0.67 to consider the effect of the segment joints on the rigidity of the tunnel lining. A set of soil parameters including Young's modulus (E), Poisson's ratio (v), effective cohesion (c'), effective friction angle (φ') and unit weight (γ) for all soil types are summarized in Table 1.

Figure 5 shows some typical realizations of different borehole scheme layouts considering both the geological uncertainty and soil spatial variability. Two borehole scheme layouts of eight and three boreholes are considered, a typical realization only considering geological uncertainty (GU) is presented in Figure 5 (a) and (b). Figure 5 (c) and (d) illustrate the realizations of eight and three boreholes considering both GU and soil spatial variability (SV). The realizations of only considering the soil spatial variability are presented in Figure 5

(e) and (f). It is worth noting that the stratigraphic distributions in Figure 5 (e) and (f) are obtained by linear connection between each borehole. The vertical and horizontal scale of fluctuations are all set as 3 m and 60 m. The COV of E value of each soil layer is 0.3.

Table 1. Parameters for various types of soil for tunnel structural performance analysis

Soil types	E (MPa)	v	c' (kPa)	□’ (º)	□ (kN/m^3)
Topsoil	10	0.35	0	22	1800
Clay	30	0.33	6	28.5	1850
Quick clay	12	0.4	18	14	1850
Sand	40	0.3	0	35	1850

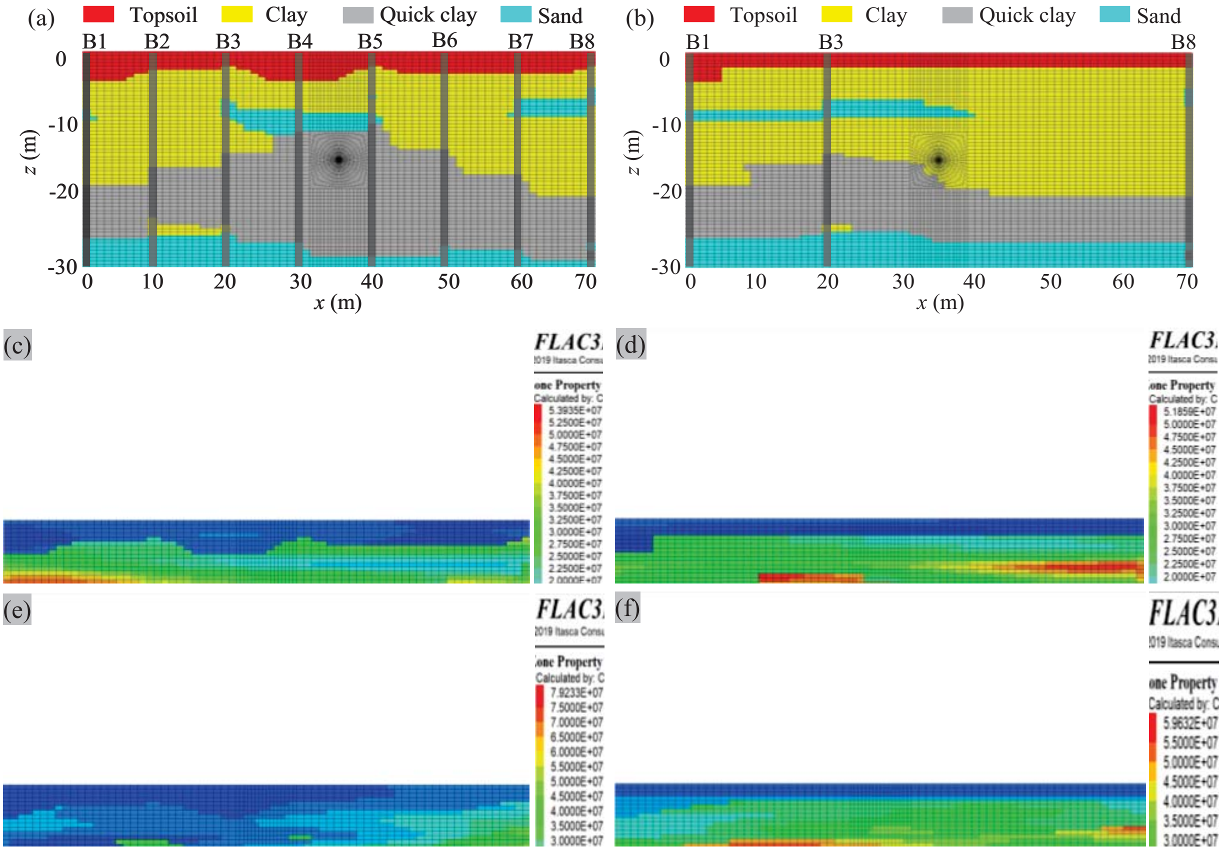

Figure 5. Some typical realizations using the proposed framework: (a) and (b) are realizations of eight and three boreholes only considering GU; (c) and (d) are realizations considering GU and SV; (e) and (f) are realizations only considering SV.

3.4 Results analysis

Figure 6 and 7 show the histogram results of 3BH and 8BH considering different types of uncertainty: only SV, only GU and both GU and SV. With the number of boreholes increase, the result of only considering GU is more concentrated, indicating the influence of GU on factor of safety of tunnel serviceability (0.4%$D/\Delta D_h$) gradually decreases. The lower bound of 95% confidence interval (CI) is adopted to evaluate the effect of uncertainty quantitatively. The lower bound of 95% CI of only considering SV is less than that of considering both GU and SV.

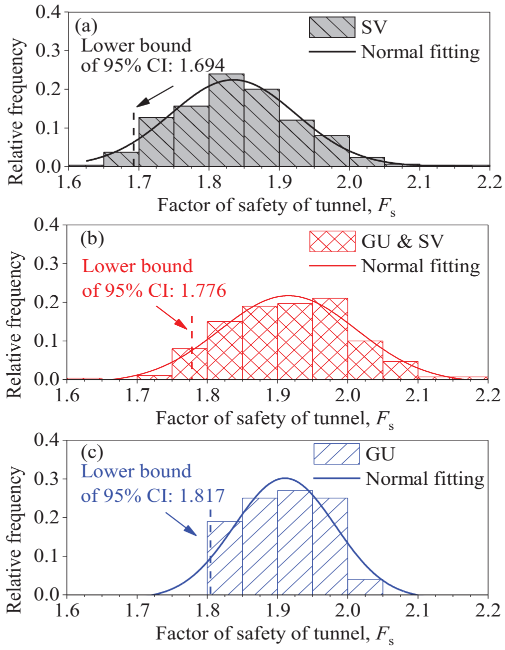

Figure 6. Histogram results of 3BH.

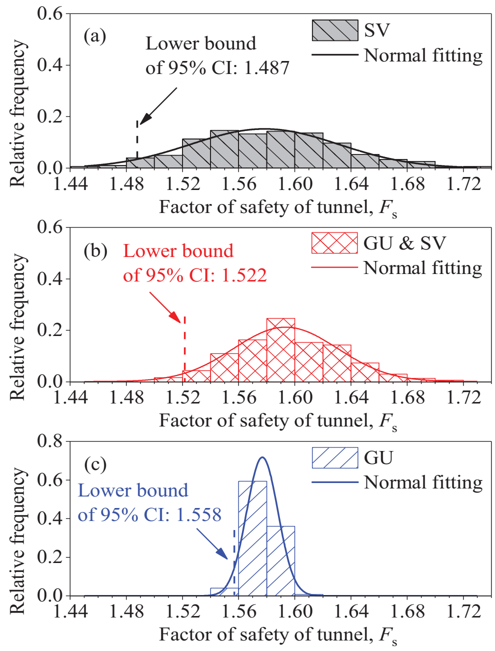

Figure 7. Histogram results of 8BH.

The boxplot comparison results of considering different types of uncertainty for 3BH and 8 BH are shown in Figure 8 (a) and (b). The COV of only considering GU is the smallest situation. When only the SV of soil properties is considered, the distribution range of the results is the widest. The variation of calculated result can be certainly reduced when both considering GU and SV.

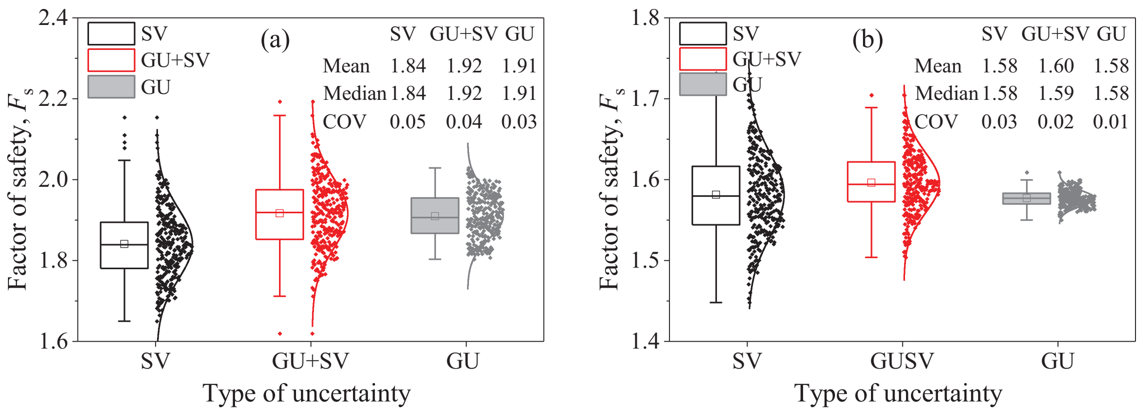

Figure 8. Boxplot comparison of considering different types of uncertainty: (a) 3BH; (b) 8BH.

4 Conclusion

This paper investigates both the geological uncertainty and spatial variability of soil properties on tunnel deformational performance. Based on the analysis result, the following conclusions are tentatively summarized.

(1) The proposed coupled probabilistic framework can well reflect the two types of uncertainty on tunnel deformation. The improved coupled Markov chain model is adopted to simulate the geological uncertainty. The random field theory is used to simulate the soil spatial variability.

(2) With the borehole number increases, the influence of geological uncertainty gradually decreases. The uncertainty of calculations only considering the soil spatial variability may be overestimated if the geological uncertainty is ignored.

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