

Using Geostatistical Approach to Assess the Hydrogeological Model Uncertainty on Groundwater Flow and Land Subsidence in Huwei Township, Taiwan

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Abstract: The distribution of hydrogeological materials is an important characteristic in a three-dimensional hydrogeological system. Simulations of groundwater flow and land subsidence in a heterogeneous aquifer system are challenging because the hydrogeological uncertainty embedded in the natural sedimentary sequences. How to quantify the hydrogeological model uncertainty to mitigate the danger in civil engineering is another challenge. Huwei Township is selected as the study area because it has sufficient geological borehole data, hydrological data, and sink and source data for a numerical model study. Forty-six boreholes are used to construct the three-dimensional heterogeneously hydrogeological models (HHMs). The transition probability/Markov-Chain model is one of the commonly used geostatistical approaches, which is capable of conducting simulations of the 3D heterogeneously hydrogeological aquifer system. The outcome of thirty-six realizations of HHM with Monte Carlo simulation reports how the heterogeneous system reflects the hydrogeological model uncertainty in groundwater flow and land subsidence simulations. To quantify the HHM uncertainty, the coefficient of variation (CV) is adopted, which is defined as the ratio of the standard deviation to the mean. The mean CV values are 0.8% and 25% for hydraulic head and subsidence simulations, respectively, which indicate that the estimated land subsidence has significant uncertainty due to the realizations of HHM. The results show that the CV values at 90th percentile (P_{90}) of the cumulative distribution function is 0.295 in subsidence simulation. The structure of HHM is shown to largely influence groundwater flow and land subsidence behaviors. The study results provide an important reference for groundwater resources and land subsidence investigations.

Keywords: Hydrogeological model uncertainty, Heterogeneous model realization, Groundwater flow, Land subsidence, Markov-Chain approach.

1 Introduction

The distribution of hydrogeological materials in a three-dimensional aquifer system of a numerical model is essential. The path of groundwater flow mainly follows the coarse-grained material (e.g., gravel and medium to coarse sand) but it is constrained by the fine-grained material (e.g., clay, silt, and mud). The primary compaction material for land subsidence is the fine-grained material (Liu et al., 2001; Tran et al., 2022). Simulations of groundwater flow and land subsidence in a heterogeneous aquifer system are challenging because of the hydrogeological uncertainty embedded in the natural sedimentary sequences. Zhu et al. (2020) reported the effect of aquifer heterogeneity on pressure/stress changes and land subsidence distribution. Tran et al. (2022) further discussed the uncertainty of heterogeneously hydrogeological model (HHM) on groundwater flow and land subsidence which is helpful for the quantification of hydrogeological model uncertainty. Therefore, HHMs play an important role in engineering geology, petroleum engineering, groundwater hydrology, and geohazard survey (Elfeki, 2006; Feyen & Caers, 2006; Tran et al., 2022; Wang et al., 2022; Zhu et al., 2020) and should be carefully considered in a numerical simulation.

Transition Probability Geostatistical Software (TPROGS) is a geostatistical technique to compute the transition probability in a Markov chain model, which is adopted to generate the HHMs in a spatially stochastic simulation. Markov chain models have been used in geological field to simulate stratigraphic sequences since 1960s. It includes the transition probability/Markov chain approach, coupled Markov chain, Markov random field, and machine learning methods (Carle & Fogg, 2020; Gong et al., 2020; Krumbein & Dacey, 1969; ; Qi et al., 2016; Shi & Wang, 2021). Modular finite-difference flow model (MODFLOW) with the subsidence and aquifer-system compaction (SUB) package are employed in groundwater flow and land subsidence simulations. Then, the coefficient of variation (CV), defined as the ratio of the standard deviation to the mean, is adopted to assess the uncertainty in the simulation results. The study results provide the reference for hydrogeological models in hydrogeological investigations and engineering geology studies.

2 Study area

The Taiwan High Speed Rail (THSR) and a major expressway pass through Huwei Town, which suffers the severe land subsidence in Jhuoshuei River Alluvial Fan (JRAF) (Figure 1). JRAF covers approximately 2000 km², bordered on the north by the Wu River, on the south by the Beigang River, on the east by the Bagua and Douliu foothills, and on the west by the Taiwan Strait. Water Resources Agency (WRA) (2020) reported a subsidence rate of 55.0 mm/year from 1991 to 2020. The WRA has set up hundreds of groundwater observation wells and dozens of multi-layer compaction monitoring wells for hydraulic head and land subsidence observations, respectively. 87 remote water meters were installed in a project (WRA, (2017) to monitor groundwater pumping quantity in Huwei Town. Huwei Town has sufficient geological borehole data and hydrological data for the study on assessing the effect of a hydrogeological model on land subsidence assessment, and thus was selected as the study area. Data from 46 boreholes were collected from three sources, namely the Central Geological Survey (CGS), the WRA, and the Irrigation Association (IA). The depths of the adopted boreholes are 63-330 m, as presented in Tran et al. (2022). Three kinds of material were classified based on the basic interpretation of the core sample, namely gravel, sand, and clay. Geological borehole data were used for the construction of the HHMs.

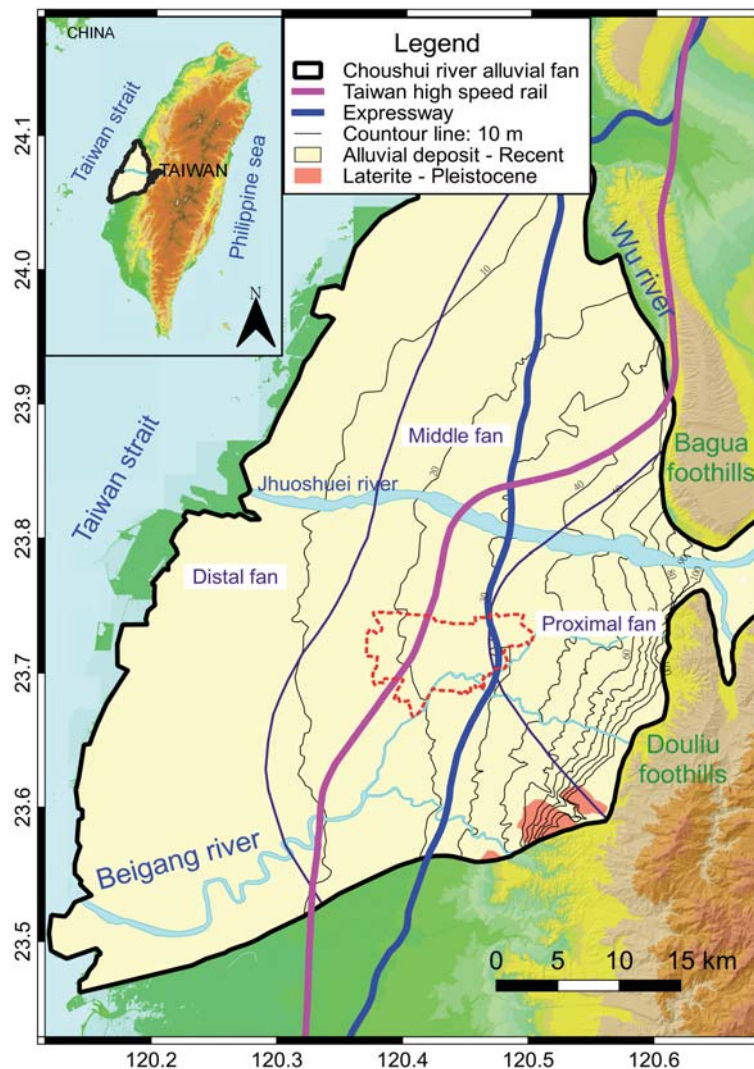


Figure 1. JRAF and location of Huwei Township in Taiwan.

3 Methodology

3.1 Transition probability geostatistical software

TPROGS (Carle, 1999) has become a package in the software of Groundwater Modeling System (GMS) since 2011. This package is used to implement a transition probability/Markov chain approach to generate realizations under the conditional stochastic simulation. There are three basic steps for TPROGS implementation after the borehole logs have been inputted to get the realizations (Carle, 1999). Firstly, transition probability measurements are calculated by GAMEAS program. Secondly, one-dimensional (1D) and three-dimensional (3D) Markov chain models of spatial variability are developed by MCMOD program. Finally, TSIM program is used to generate 3D HHMs, which ensured that the spatial cross-correlation and juxtapositional tendencies of the hydrogeological materials were

fully considered in the conditional stochastic simulation. The detail information for TPROGS and relative programs can be found in Carle (1999). A simple definition of transition probability according to a conditional probability can be written as:

$$t_{j,k}(\Delta h) = \Pr\{k(x + \Delta h) | j(x)\}; 0 \leq t_{j,k}(\Delta h) \leq 1 \quad (1)$$

where x is the spatial location [L], Δh is the lag distance [L], and j, k denotes mutually exclusive materials (e.g., gravel, sand, and clay).

3.2 Numerical model of groundwater flow and land subsidence

MODFLOW, developed by McDonald and Harbaugh (1988), is a commonly used finite difference code for groundwater flow simulation. In this research, the well (WEL1), recharge (RCH1), Subsidence and Aquifer-System Compaction (SUB), Preconditioned Conjugate-Gradient (PCG), Hydrogeologic-Unit Flow (HUF), and output (OC) packages are used. With the following governing equation, MODFLOW represents the 3D movement of groundwater flow through porous media (Rushton & Redshaw, 1979):

$$\frac{\partial}{\partial x} \left(K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial h}{\partial z} \right) - W = S_s \frac{\partial h}{\partial t} \quad (2)$$

where h is the hydraulic head [L], W is the volumetric flux that represents sources and sinks of water [1/L], t is time [T], S_s is the specific storage of the porous material [1/L], and K_x, K_y, K_z are the hydraulic conductivities along the x -, y -, and z -directions, respectively [L/T].

SUB package is based on the Interbed Storage Package version 2 (IBS2) (Leake, 1990) and includes a delay in the release of groundwater from compressible interbeds. The compaction mechanisms of confined unconsolidated aquifer systems in response to a hydraulic head decline were determined in a series of model, field, and laboratory studies (Terzaghi, 1925). The change in thickness of the sediment layer can be defined as:

$$\Delta b = S_{sk} b \Delta h \quad (3)$$

where Δb is the change in the thickness of a control volume with the initial thickness b of the sediment layer [L], S_{sk} is the skeletal specific storage [1/L], and Δh is the change in hydraulic head [L].

4 Results and discussion

4.1 Heterogeneously hydrogeological model

Tran et al. (2022) generated an ensemble of 40 HHMs with a uniform depth of 300 m using T-PROGS with the TSIM program for Huwei Town. Figure 2 shows the cumulative distribution function (CDF) results of the clay material with the outcome of 5, 10, 20, 30, 36, and 40 realizations. The cumulative distribution function of the mean thickness of each material becomes more stable as the number of realizations increases. When the number of realization is greater than 36, the CDFs of the mean thickness of clay become stable. Thus, 36 were chosen as the realization number for Monte Carlo simulations to reduce the computational burden.

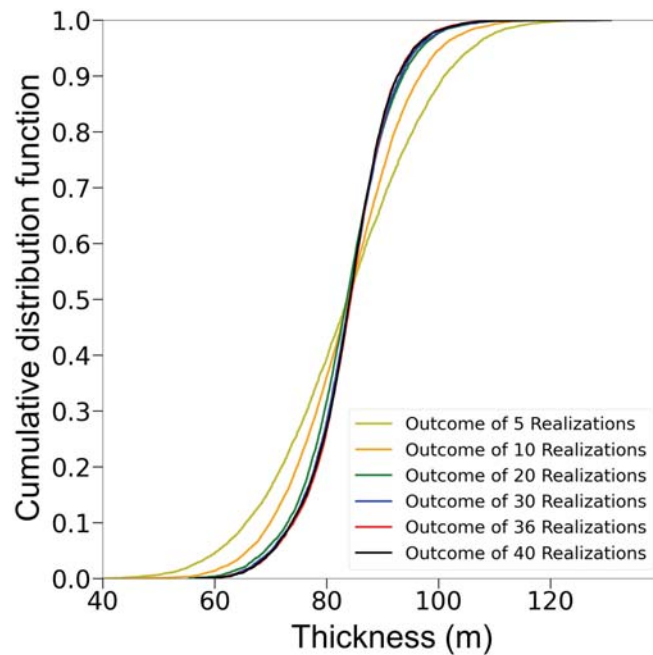


Figure 2: Cumulative distribution function for the mean of clay thickness under outcome of various numbers of realizations. The cumulative distribution function became stable after 36 realizations [modified from Tran et al. (2022)].

Figure 3 shows a 3D heterogeneous hydrogeological model and a fence diagram of realization #2. The hydrogeological materials with a full depth of 300 m in the borehole 03 (BH03) are the same in different realizations due to a conditional condition. The wells of BH17, BH30, BH34, BH37, and BH46, which do not have full 300 m stratigraphic sequences, may have different hydrogeological material beneath the borehole bottom. The greatest volumetric proportion of the geological material in the simulation results is gravel since it has the highest proportion in the hydrogeological data in the interested domain (i.e., it is the background material). Its HHM is used as an essential in the groundwater flow and land subsidence simulations, as presented in the next section.

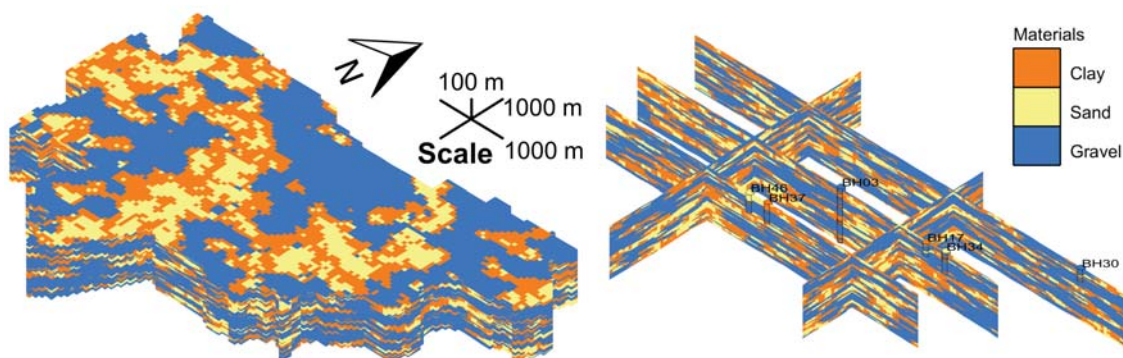


Figure 3. The 3D HHM of realization #2 in Huwei Township, Taiwan [modified from Tran et al. (2022)].

4.2 Uncertainty of HHMs on groundwater flow and land subsidence simulations

The CV values were used to assess the uncertainty distribution. The mean CV value of the hydraulic head is 0.8% (Figure 3), which is extremely low due to the assumed boundary conditions. The influence of the hydrogeological model uncertainty is thus not significant in the groundwater flow model under a steady state condition. According to Eq. (3), the thickness of clay controls the quantity of land subsidence (Tran et al., 2022). The uncertainty of the distribution of clay thickness from HHMs leads to a high uncertainty of the land subsidence results. The mean CV value of the land subsidence is 25%, which indicates that land subsidence is sensitive to HHM.

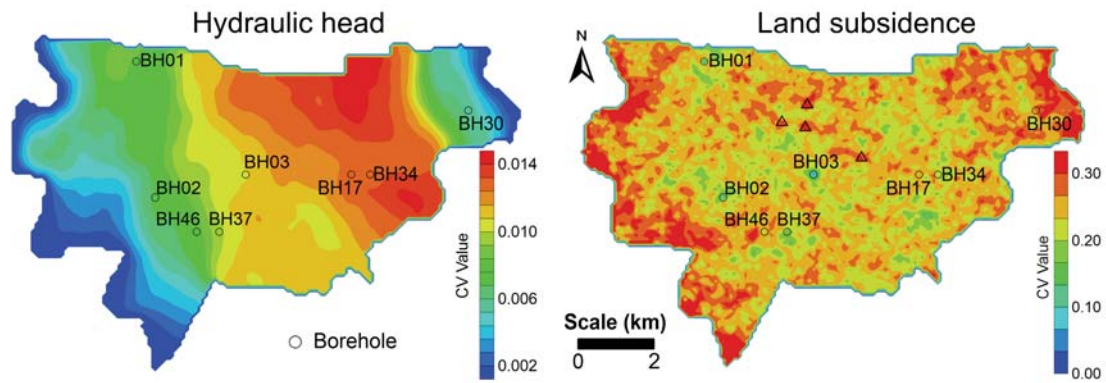


Figure 4. CV distribution of hydraulic head and land subsidence calculated from the simulation results of 36 HHMs [modified from Tran et al. (2022)].

4.3 Uncertainty assessment of HHM

In this study, the CDF curve was calculated to examine the uncertainty of the HHMs. The CDF curves of land subsidence, calculated from each cell in the numerical model for the outcome of 5, 10, 20, 30, 36, and 40 realizations, are shown in Figure 4. The outcome of 36 realizations shows no significant change in the CV value of land subsidence compared to those of 40 realizations. This result supports that 36 realizations are sufficient for the Monte Carlo simulation. In the figure, the CV values of land subsidence at the 50th percentile (P_{50}) are the median value. The CV values at P_{50} , P_{90} , and P_{95} are 0.248, 0.295, and 0.316 for the outcome of 36 realizations, respectively. Meckel et al. (2006) reported that P_{90} could be considered as the most probable representative quantification value. Based on the geological borehole data adopted in this study, the uncertainty of HHMs for Huwei Town in terms of the CV value of land subsidence at P_{90} is 0.295. Suppose a CV value of land subsidence at P_{90} can be defined as acceptable uncertainty (e.g., $CV \leq 0.3$). The assessment results of this study show that the adopted geological borehole data are sufficient for assessing land subsidence in the study area ($0.295 < 0.3$). Different scales of the study area and the numerical results from different countries should be used to determine a suitably acceptable uncertainty.

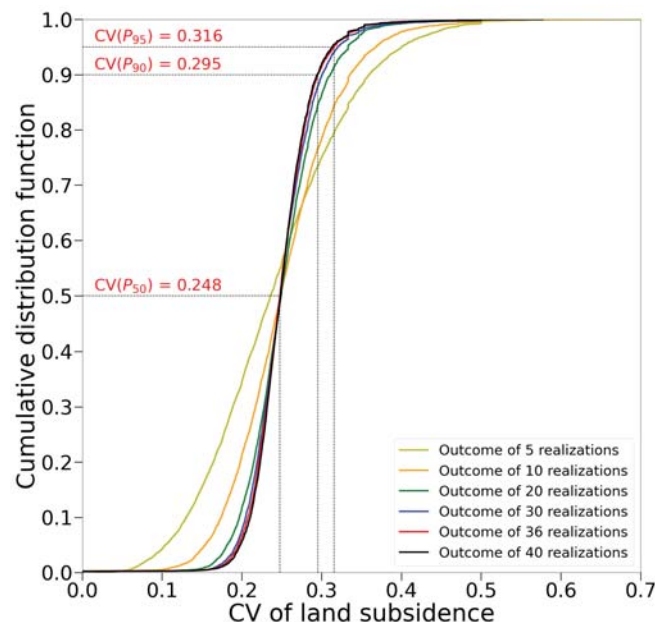


Figure 5. CDF of the CV values of land subsidence. The horizontal line indicates the values of CDF equal to 0.95, 0.90, and 0.50 percentiles for the outcome of 36 realizations, which were used to quantify the effect of hydrogeological model uncertainty on land subsidence simulations in Huwei Town.

5 Conclusions

In a spatial stochastic simulation, the distribution of hydrogeological materials helps quantify the hydrogeological model uncertainty. The distributions of mean hydraulic head and land subsidence provided reliable assessment results, which avoid a unique outcome of single hydrogeological model. The mean, variance, and CV were used to estimate the effect of hydrogeological model uncertainty on groundwater flow and land subsidence simulations. The CV value of the land subsidence results provides a reference of HHM uncertainty. An equivalent uncertainty evaluation using the CV value of land subsidence at P_{90} can be used to determine whether the model uncertainty is acceptable.

The results of groundwater flow and land subsidence simulations show that the uncertainty of HHMs significantly affects the reliability of the assessment results. This study proposed a method which can be used to understand and quantify the influence of the HHMs on groundwater flow and land subsidence simulations. It can provide the reference for estimating uncertainty under complex hydrogeological conditions.

This study adopted the assumed hydrological conditions and boundary conditions to decrease the uncertainty of them and focused on assessing the geological model uncertainty. Therefore, the simulation results did not represent the real situation. A real case model with the calibration of observation data is under investigation.

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