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Multi-Block Failure Mechanism Approach with Broken Lines in Bearing Capacity Estimation of Spatially Variable Soil

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Abstract: This study presents the novel use of kinematical failure mechanisms to analyze the shallow foundation bearing capacity for spatially variable soil. The approach preserves the upper bound limit theorem. For this reason, the straight dissipation lines resulting from the multi-block failure mechanism are no longer straight lines but become broken lines. The broken lines are adjusted to the actual value of the friction angle, which is considered spatially variable. In the study, the application of the idea to bearing capacity problems is presented and discussed. The motivation for the study is to ensure numerical efficiency and the ability of failure geometry to adapt to weaker zones in spatially variable soil. The numerical efficiency comes from the similarity of the proposed approach to Vanmarcke's spatial averaging applied to dissipation regions resulting from the failure mechanism. However, the ability of the failure mechanism to adapt to weaker zones is a result of the broken lines method used together with an optimization procedure based on a subset simulation-based approach. In numerical analysis, the eleven-block failure mechanism is assumed as a reasonable compromise between numerical efficiency and accuracy. In addition, double-sided and one-sided failure mechanisms are analyzed and results obtained for both cases are compared and discussed in the study. A comparison with random finite limit analysis (RFLA) is provided; a very good agreement in the results for both methods is observed. The numerical implementation of the proposed algorithm was used to analyze a variety of cases with different values of the horizontal scale of fluctuation. Two general cases of isotropic and anisotropic correlation structures are analyzed. The proposed method is very convenient for determining the volume of the failure mechanism, for this reason, the volume of the corresponding failure mechanism is also discussed together with the corresponding bearing capacities. The proposed method is promising for applications in shallow foundation reliability analyses.

Keywords: Upper Bound; Failure Mechanism; Scale of Fluctuation; Random Field; Reliability; Shallow Foundation.

1 Introduction

Soil spatial variability is one of the main uncertainties that impacted the discipline of geotechnical engineering. For this reason, many numerical approaches have been developed in recent years to facilitate the reliable consideration of this natural phenomenon. The increase in computer power in recent years caused an acceleration in the development of this branch. This is also visible in a field of bearing capacity estimation for shallow foundations in the case of spatially variable soil where many methods were proposed: e.g., random finite element method (RFEM) by Griffiths and Fenton (2001), Fenton and Griffiths (2003), random finite difference method by Kawa and Puła (2020), random finite limit analysis by Simoes et al. (2014), random failure mechanism method by Chwała (2019) and other approaches that handle soil spatial variability in foundation bearing capacity analyses, i.e., for three-dimensional analysis by Li et al. (2021a), for Prandtl-type failure mechanism by Li et al. (2021b) or with kriging metamodeling usage by Soubra et al. (2019). RFEM seems to be the most commonly used approach; however, due to relatively low numerical efficiency in the case of three-dimensional applications, its use is very limited. The motivation for this work is to propose a method that will result in sufficient numerical efficiency and ability to find the weakest zones in spatially variable soils. The method uses the kinematical failure mechanism approach together with explicitly represented random fields. The idea was first proposed by Chwała (2021) and very recently its adaptation to bearing capacity was proposed by Chwała and Zhang (2022). In this work, the performance of the proposed method is compared with that of random finite limit analysis (RFLA) by using OptumG2 software. Moreover, the impact of the horizontal scale of fluctuation on the results is analyzed. The study indicates that the proposed approach can be successfully used to assess the bearing capacity in spatially variable soils.

2 Method Overview

The main objective of the method described below is to ensure high numerical efficiency subject to high flexibility in adapting the failure mechanism to existing soil conditions. To make the objective feasible a connection of the multi-block failure mechanism approach of the upper bound theorem is used together with explicitly generated random fields that describe soil spatial variability. This spatial variability forced the failure mechanism slip lines (energy dissipation lines) to become broken lines. This is due to the upper bound theorem, which states that to ensure the kinematical admissibility of the failure mechanism, the velocity jump vector must be inclined to the slip line by friction angle (Chen, 1975). On the other hand, the block of soil moves as a rigid body, and the velocity jump vector needs to be the same on each dissipation line. The idea was first proposed by Chwała (2021) and applied to slope stability problems. Recently, Chwała and Zhang (2022) extended this latter approach to shallow foundation analysis. The geometry of the broken line is shown in Fig. 1. It is worth noticing, as indicated earlier, that the geometry of line AB (which separates rigid block 1 from rigid block 2) is a result of friction angle spatial variability, i.e., in each cell a different friction value is assigned.

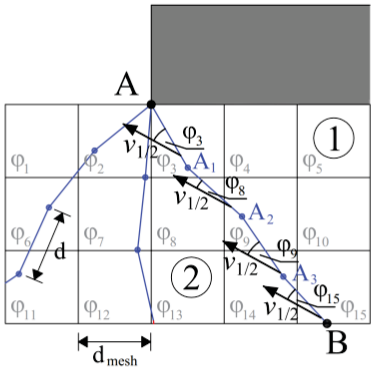

Figure 1. Idea for the construction of the dissipation broken line. Note that velocity jump vector $v_{1/2}$ is the same on dissipation line AB.

3 Application to Shallow Foundations

The above described idea is applied to the 6-block failure mechanism in the case of a one-sided failure mechanism and the 11-block failure mechanism in the case of a double-sided failure mechanism, the corresponding failure geometry examples are shown in Fig. 2a and Fig. 2b, respectively. Within the method, the friction angle is assigned to each straight section of the broken line by finding the corresponding mesh square closest to the center of the considered straight section. This approach imposes a method of generating a random field, i.e. the random field needs to be discretized to a square mesh. The square mesh grid is a background that never changes when the failure geometry is built. Fig. 2 presents the denoting convention of rigid blocks and the names of characteristic points used to describe failure geometry. As noted earlier, in this study, 11-block and 6-block failure mechanisms are assumed, this assumption originates from a compromise between accuracy and efficiency, based on earlier experiences of using kinematical failure mechanisms e.g., Puła and Chwała (2018).

According to the denoting convention of rigid blocks shown in Fig. 2, the velocity hodographs are constructed as shown in Fig. 3. Based on the geometrical relations, the lengths of the velocity jump vectors are determined and later used in the calculation of bearing capacity. It is worth noting that the velocity hodographs shown in Fig. 3 are constructed in the same way as for classical kinematical failure mechanisms (Michałowski, 1997).

On the basis of a failure geometry, the bearing capacity can be calculated. Generally, the calculation of the bearing capacity requires the total energy dissipation along the dissipation lines (see Fig. 1 and Fig. 2) and the gravity forces (in the case of considering the weight of the soil). To calculate dissipation for a given straight section of the broken line, the product of straight section length, cohesion on the straight section, and cosine of friction angle on the straight section need to be calculated. Next, the dissipations are summed for all straight sections in the broken lines. After that, the total dissipation on the broken line is multiplied by velocity jump vectors that are assigned to each broken line. To obtain a total gravity force, a sum of products for each rigid block needs to be calculated, i.e., the area of each rigid block is multiplied by the vertical component of velocity for each rigid block and the unit weight of soil. Finally, the bearing capacity is the sum of the total dissipation and gravity forces. The detailed procedure for determining bearing capacity can be found in the study by Chwała and Zhang (2022).

4 Method Description

The Monte Carlo framework is used to estimate the mean values and standard deviation of the bearing capacity probability distributions when soil parameters are assumed to be spatially variable. For generating random fields, any available generator can be used. There is one constraint that the final random field needs to be discretized to square cells. In this study, a generator developed by Jha and Ching (2013) is used. The failure geometry needs to be optimized in

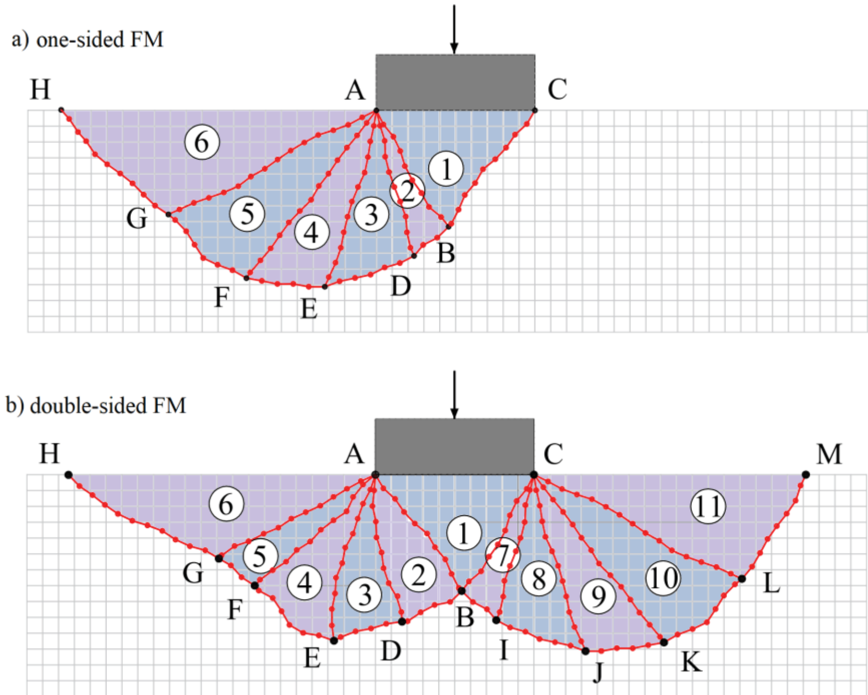

Figure 2. Exemplary failure geometries and denoting convention of rigid blocks for one-sided (a) and double-sided (b) failure mechanism.

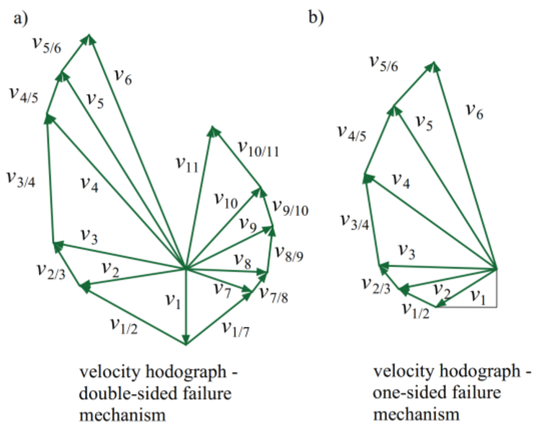

Figure 3. Exemplary velocity hodographs for double-sided (a) and one-sided (b) failure mechanism. Note that the indexes of velocity jump vectors denote the rigid block according to Fig.2.

order to provide the lowest possible bearing capacity estimation (since it is an upper bound approach). According to Chwała and Zhang (2022), in this work, a variation of a subset simulation-based optimization scheme is used. Generally, the optimization problem can be considered as a multi-parameter optimization, i.e., there are 10 design variables for one-sided failure mechanism and 20 design variables for double-sided failure mechanism. As design variables, five angles near points A and C are used, those angles described the inclination of broken lines originating in points A and C. The remaining five angles are the inclinations of broken lines that close the failure geometry from the bottom. For this reason in the double-sided failure mechanism there are 20 design variables. Within the optimization procedure for the assumed number of Monte Carlo realizations *N*, the resulting bearing capacities are sorted in ascending order and 10% of the lowest bearing capacities are used in the next step, where the values of the corresponding design parameters are used as seeds for generating 9 new failure geometries. Finally, after this process, in the next step there are *N* sets of designed parameters ready to be analyzed within Monte Carlo framework. The detailed algorithm for the generation of new failure geometry sets is described by Chwała and Zhang (2022); however, in general, the new set of design variable values is obtained by adding a uniformly distributed random number, where its range is controlled by the parameter r (its value is established through numerical tests). After repeating this approach T times, the final bearing capacity for a given random field realization is taken as the lowest value found. Chwała and Zhang (2022) show that the optimization procedure works efficiently for the concerned scenarios and provides a detailed analysis of method parameters. Note that the failure mechanisms used in this work are for a rough foundation base. It is also possible to extend the approach for a smooth foundation base by using different failure mechanisms (e.g., Hill-type mechanisms; see Chen, 1975).

5 Comparison with RFLA

The deterministic upper bound analysis performed in the OptumG2 software (Krabbenhoft, 2016) was used to validate the proposed method. As shown in Fig. 4 a very good agreement was obtained for a wide range of friction angles. It is worth noting that the calculation time for the proposed approach is shorter than the equivalent analysis performed by OptumG2 software (for comparable accuracy), please find more details on calculation time in the caption of Fig. 4.

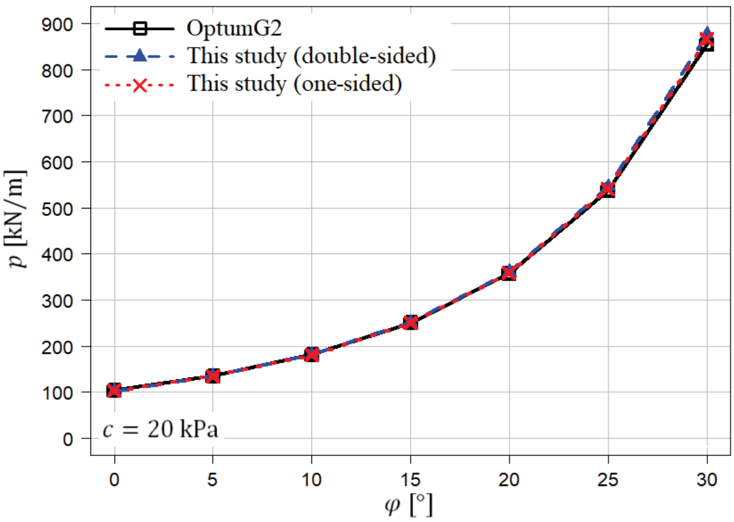

Figure 4. Bearing capacity comparison for OptumG2 (RFLA) and the method described in this work; 10,000 FE with 4 adaptive mesh iterations were used in OptumG2 (calculation time per realization is 35s); for one-sided and double-sided FMs, 100 subset simulation steps, r = 0.05, and 200 Monte Carlo realizations were used, which result in a calculation time of 5s and 15s, respectively.

To make possible the comparison between both approaches for spatially variable soil, one realization is shown in Fig. 5, where the failure geometries for RFLA and this work are compared. Comparison of exactly the same random field realizations required generation of random field within OptumG2 software, and after that the random field is mapped to the square grid used in this work. In Fig. 5a a random field representation is shown and in Fig. 5c its map to the grid mesh. As shown in Fig. 5 the failure geometries are almost exactly the same in terms of failure mechanism size. Moreover, as indicated in the red box, the failure mechanism locally takes a very similar shape determined by the area with a higher value of the friction angle. This observation indicates the ability of the failure mechanism to adapt to a weaker zone in the spatially variable soil. Another example that can be seen in Fig. 5c is a rigid triangular block directly under the footing. Its shape is stretched vertically due to the high friction angle region directly behind the foundation surface. The optimal failure geometry of dissipation lines avoids this area. The random field shown in Fig. 5 is generated for the anisotropic correlation structure for the vertical scale of fluctuation 1 m and the horizontal scale of fluctuation 5 m. The bearing capacity for RFLA is 549.4 kN/m vs 566.1 for this study. For 50 random field realizations analyzed by both approaches the mean value for RFLA (3 adaptive mesh iterations, each 10000 finite elements) is 343.1 with coefficient of variation 0.237 and for this study (r =0.2, 200 Monte Carlo realizations and 300 subset simulation steps) bearing capacity mean value is 339.48 with coefficient of variation 0.247. As a numerical example, the bearing capacity of a surface foundation of width $b = 1$ m is analyzed, the lognormal random fields were assumed to describe friction angle, and cohesion with the following parameters: $\mu_\varphi = 20°, v_\varphi = 0.25$ and $\mu_c = 20$ kPa, $v_c = 0.25$, respectively. The Markovian correlation function is assumed (Fenton and Griffiths, 2008). The results are shown in Fig.6, where the influence of horizontal scale of

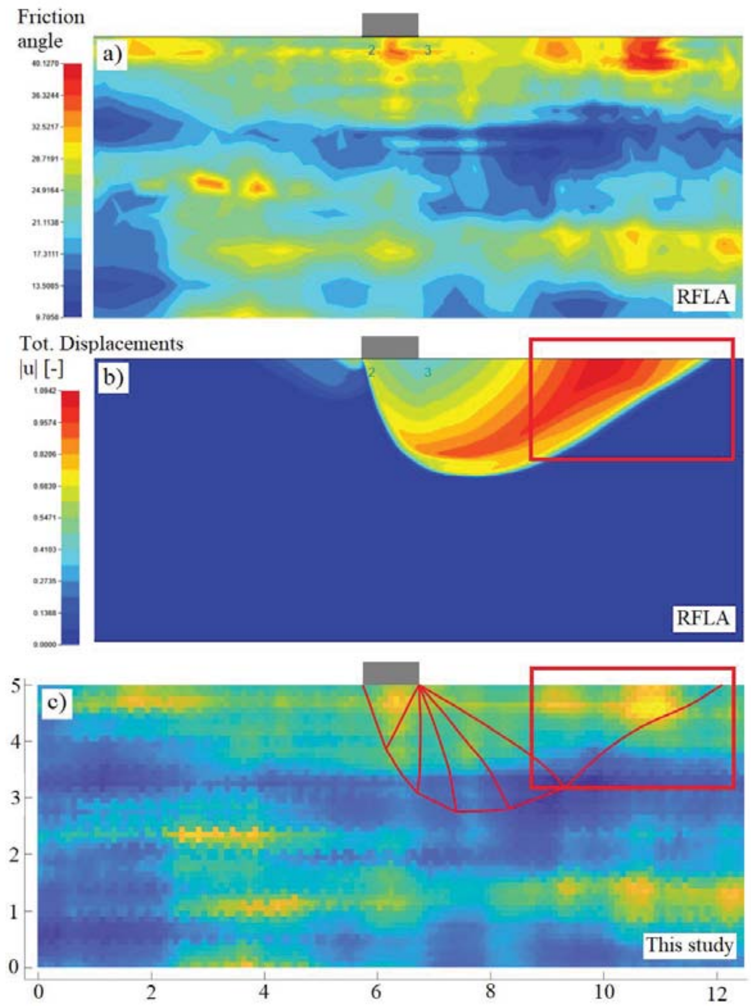

Figure 5. Exemplary velocity hodographs for double-sided (a) and one-sided (b) failure mechanism. Note that the indexes of velocity jump vectors denotes the rigid block according to Fig.2.

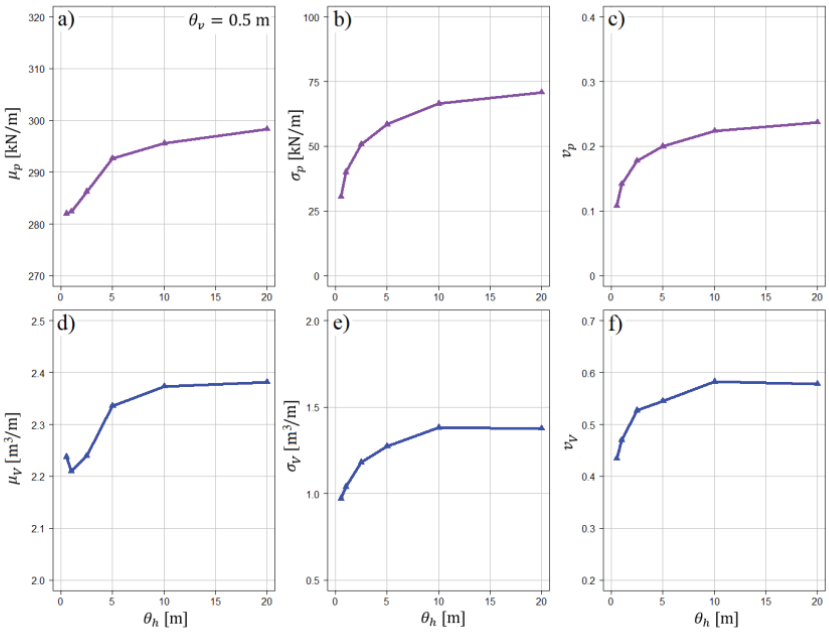

Figure 6. Bearing capacity mean values (a), standard deviations (b), and coefficients of variation (c) with corresponding volume mean values (d), standard deviations (e), and coefficients of variation (f).

fluctuation is demonstrated (the assumed vertical fluctuation scale is 0.5 m). The results are obtained for 1000 random field realizations. It is shown that both the bearing capacity standard deviation and the coefficient of variations increased with the increase of horizontal scale of fluctuation; however, for values greater than 10 m the stabilization is observed. Similar observation can be made for volumes of the failure mechanisms, with one difference that for mean values a local minimum is observed. To investigate if this is a manifestation of some phenomena or just numerical accuracy, further analysis is needed. The objective of this study is to demonstrate the applicability of the broken line method in considering bearing capacity of spatially variable soils. The formulation is shown to be efficient and reliable in terms of accuracy and ability to adjust to weaker failure zones in spatially variable soil. The flexibility of the failure mechanism is greatly improved compared to classical failure mechanism approaches.

6 Conclusions

In the study, the successful application of the method to consider spatially variable soil in shallow foundations bearing capacity has been demonstrated. The described method is capable of identifying the weakest path in the spatially variable soil within a competitive efficiency with respect to finite element-based approaches. Two failure mechanisms have been analyzed, that is the double-sided failure mechanism and the one-sided failure mechanism. The proposed idea can be used for other geotechnical applications and extended to scenarios that considered a foundation near a slope or considering cracks in cohesive soils. The method is also an interesting alternative to using classical kinematical failure mechanisms for spatially variable soils.

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