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Uncertainty Propagation Assessment in CPTu-Based Lithological Modeling Using Stochastic Co-Simulation

Diego Di Curzio[1] and Giovanna Vessia[1]

[1]Department of Engineering and Geology (InGeo), University "G. d'Annunzio" of Chieti-Pescara, Via dei Vestini 31, 66013 Chieti, Italy.
E-mails: diego.dicurzio@unich.it; g.vessia@unich.it

Abstract: Reliability-based designing in geotechnical engineering represents a modern approach to estimating the probability of failure due to spatial variability of soil and rock properties as well as the errors and uncertainties related to measured parameters and calculated designing variables. As spatial data are often used as inputs in models of spatially distributed variables, such as CPTu-based litho-mechanical models, the propagation of spatial uncertainty in model predictions becomes a critical issue even in engineering design. This is commonly known as model uncertainty and several different methods to handling it has been proposed in the literature. This paper proposes an efficient geostatistical simulation approach, based on the use of the Sequential Gaussian Co-Simulation, for uncertainty propagation assessment in the calculation of a Soil Behavior Index from raw CPTu profiles (i.e., q_c, f_s, u_2). These measurements have been collected in a portion of the Bologna district (Italy) by the Emilia Romagna Region.
A Linear Model of Coregionalization was fitted to the matrix of experimental direct and cross-variograms of the input data, and one thousand realizations that honor the experimental data were provided. These realizations were used to calculate the corresponding values of the litho-mechanical subsoil index. As a result, the uncertainty estimates of subsoil lithology can be efficiently used within geotechnical designing of several structures so avoiding assumptions on probability distribution of natural materials that may in turn show site-specific hydro-mechanical behavior, which needs to be investigated in detail.

Keywords: Uncertainty propagation; 3D litho-mechanical model; CPTu profiles; Sequential Gaussian Co-Simulation.

1 General Aspects

To perform a geotechnical reliable design, spatial variability of soils and rocks and uncertainties related to the Engineering Geological Model (EGM) adopted cannot be disregarded. According to Phoon and Tang (2019) site understanding *refers to how well the ground measured mechanical properties are known while* model understanding *refers to the degree of confidence that a designer has in the subsoil model used to predict the geotechnical resistance*. Thus, whenever an EGM is conceived it cannot be simply a deterministic and expert judgment-based model, but it is characterized by uncertainties known as model uncertainty. This latter involves two aspects viz.: (1) the bias in the mathematical expression that transforms the measured parameters into design ones and (2) the uncertainty associated with the variability of the soil and rock parameters in the prediction equations. The need for the assessment of model uncertainty has been emphasized and considered in the current revision process of the Eurocodes (Lesny, 2017). A review paper by Lee and Chen (2009) identifies five categories of uncertainty propagation methods (UP): 1) the simulation; 2) the local expansion; 3) the most probable point; 4) the functional expansion; 5) the numerical integration. Hereinafter, the first category method has been used named Sequential Gaussian Co-Simulation method (SGCS). It has been applied to propagate the uncertainty in the calculation of a Soil Behavior Index from raw CPTu profiles (i.e., q_c, f_s, u_2) through a Linear Model of Coregionalization. This latter was fitted to the matrix of experimental direct and cross-variograms of the input data, and one thousand realizations that honor the experimental data were provided. These realizations gave the derived distribution of I_{SBT} values which was used to calculate the mean and the standard deviation maps of I_{SBT}. The study area is the Po plain in the East part of the Bologna district (Italy).

2 Study area

The selected study area (Figure 1) is a 900-square-kilometer-wide portion of the eastern Bologna district, located in the southern part of the Po plain (Italy). From the geological standpoint, the Po plain is a tectonic depression, filled by hundreds-of-meter-thick continental and/or marine-transitional deposits (Amorosi and Farina 1995). Within the study area, there are alluvial deposits made up of undifferentiated fine silty-sandy deposits (i.e. flooding plain), characterized by coarser (i.e. alluvial fans and paleo-channels) and finer (i.e. lacustrine lenses) geological bodies (ISPRA 2009a, 2009b). From a lithological point of view, these inclusions consist of sandy, gravelly, and silty-clayey soils. On one hand, sandy-gravelly alluvial fans are prevalent nearby the Apennine reliefs, in the south; on the other hand, sandy paleo-channels become predominant moving northward. Conversely, fine lacustrine lenses can be found all over the study area. It is worth pointing out that all the types of inclusion have shape, size, and depth that can be predicted through direct investigations, within the whole subsoil volume.

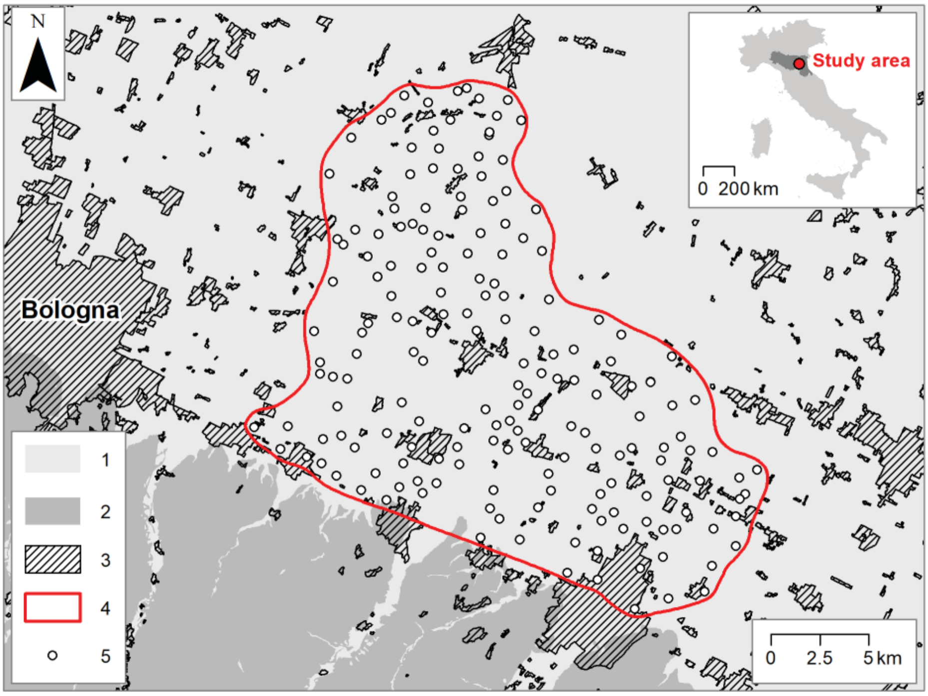

Figure 1. Map showing the location and main geological features of the study area as well as the CPTus' distribution within the selected domain. In legend: 1) alluvial deposits; 2) bedrock; 3) urban areas; 4) geostatistical domain; 5) CPTus' locations.

3 Material and methods

3.1 Dataset

The dataset used in this research consists of 182 CPTus performed across a 900-square-kilometer-wide area and collected in a comprehensive database by the Regional Office for Territorial Protection and Development of the Emilia-Romagna region (http://geoportale.regione.emilia-romagna.it/it), subsequently made available by Di Curzio and Vessia (2021).

3.2 Sequential Gaussian Co-Simulation

To get a deeper insight into the propagation of uncertainty while using transformation equations, the Sequential Gaussian Co-Simulation (SGCS) method has been selected (Gooverts, 1997; Webster and Oliver, 2007; Chilès and Delfiner, 2012). It derives from the general Sequential Gaussian Simulation approach (SGS). This advanced geostatistical technique is one of the most straightforward and used among Conditional Simulation methods (Delbari et al., 2009; Emery and Peláez, 2011; Nussbaumer et al., 2018), which are the simulation approaches that honor measured data. Unlike kriging methods, stochastic simulation techniques are devoted to assessing spatial uncertainty (Castrignanò and Buttafuoco, 2004). Furthermore, since these methods can preserve the spatial variability, which is instead smoothed in kriging methods, stochastic simulation approaches can be also used to obtain optimized maps of estimates (ASTM International, 2018).
SGS is based on the multi-gaussian assumption and the conditionally simulated values ($z_c^*(\mathbf{x_0})$) at each node of the grid is obtained conditioning results with the Kriging estimator ($z^*(\mathbf{x_0})$), as follows:

$$z_c^*(\mathbf{x_0}) = z^*(\mathbf{x_0}) + [s^*(\mathbf{x_0}) - z_s^*(\mathbf{x_0})] \tag{1}$$

where, $s^*(\mathbf{x_0})$ is the simulated field calculated with the same variogram model as that of experimental data, while $z_s^*(\mathbf{x_0})$ is the kriging estimates obtained by considering the simulated values at the sampling points. In SGS, this process is repeated several times by random seeds, which correspond to different paths through the data. As a result, several equiprobable representations of the spatial distribution of the considered variable can be obtained, namely, realizations, providing a statistical distribution for each node of the grid, instead of an estimated value and the corresponding error (i.e., as in kriging methods).

As in this work we dealt with a multivariate case, the Kriging estimator in Eq. (1) with the Co-Kriging one, in fact moves from SGS to SGCS. In this case, the simulation relies on the fitting of a Linear Model of Coregionalization (LMC) of the considered variables (i.e., q_c, f_s and u_2), below represented in matrix notation (Wackernagel, 2003; Castrignanò et al., 2015; Di Curzio et al., 2019, Vessia et al., 2020a):

$$\boldsymbol{\Gamma}(\mathbf{h}) = \sum_{u=1}^{N_S} \mathbf{B}^u g^u(\mathbf{h}) \tag{2}$$

where, $\mathbf{B}^u$ is the Coregionalization matrix of the LMC coefficients, which is symmetric and positive, $\boldsymbol{\Gamma}(\mathbf{h})$ is the n x n matrix with direct variograms (i.e., diagonal elements) and cross-variogram (i.e., non-diagonal elements) modeled as a linear combination of N_S basic variogram functions, $\mathbf{h}$ is the lag, $g^u(\mathbf{h})$ is the spatial structure, and u is the spatial scale.

It is worth pointing out that, since a Gaussian distribution is required, non-Gaussian data transformation or standardization is needed. In this study, raw measurements of q_c, f_s, and u_2 have been transformed through the Gaussian Anamorphosis function (Chilès and Delfiner, 2012). This function converts a Gaussian-distributed variable (Y) into a new variable ($Z = \Phi(Y)$), whatever its statistical distribution, by fitting a Hermite polynomial expansion $H_i(Y)$:

$$\Phi(Y) = \sum \Psi_i H_i(Y) \tag{3}$$

where, Ψ_i are the coefficients of the Hermite polynomials.

Once defined the Gaussian Anamorphosis function, the transformation from a non-Gaussian variable into a standardized one (i.e., the one required to use SGCS) is performed by inverting the function, as follows:

$$Y = \Phi^{-1}(Z) \tag{4}$$

The selected estimation domain has a cell size equal to 500x500x0.5 m. All the geostatistical analyses have been performed using Isatis 2018, whose results have then been visualized through Isatis.neo (www.geovariances.com/en/software/isatis-neo-geostatistics-software/).

3.3 Lithotype classification

Lithotype classes within the studied subsoil have been defined through the normalized Soil Behavior Type index (I_{SBTn}) defined by Robertson (2009), which allowed following the spatial distribution of the classes listed in Table 1 for all the 1000 realizations of q_c, f_s, and u_2, estimated through the SGCS. This allowed obtaining an improved estimation of lithotypes within the considered subsoil volume as well as the statistical distribution of I_{SBTn} values at each cell of the grid, to be compared to the q_c, f_s, and u_2 ones to assess the uncertainty propagation.

Table 1. Soil Behavior Type classes as defined by Robertson (2009), with the corresponding I_{SBTn} values.

Soil Behavior Type	I_{SBTn}	Class
Gravelly sand to dense sand	< 1.31	SBT2
Sands – clean sand to silty sand	1.31-2.05	SBT3
Sand mixtures – silty sand to sandy silt	2.05-2.60	SBT4
Silt mixtures – clayey silt to silty clay	2.60-2.95	SBT5
Clays – silty clay to clay	2.96-3.60	SBT6
Organic soils – clay	> 3.60	SBT7

I_{SBTn} values have been calculated through the following equation:

$$I_{SBTn} = \left[\left(3.47 - \log(Q_{tn})\right)^2 + (\log F_R + 1.22)^2\right]^{0.5} \tag{5}$$

Where, $Q_{tn} = \left(\frac{q_t - \sigma_{v0}}{P_a}\right) \cdot \left(\frac{P_a}{\sigma'_{v0}}\right)^n$ is the normalized tip resistance, $F_R = \frac{f_s}{q_t - \sigma_{v0}} \cdot 100\%$ is the friction ratio, $q_t = q_c - u_2(1 - a)$, coefficient a equal to 0.8 (i.e., average value), and $n = 0.381 \cdot I_c + 0.05 \cdot \left(\frac{\sigma'_{v0}}{P_a}\right) - 0.15$ and I_c is the soil behaviour type index according to Robertson (1998). These are the terms inside the expressions, whereas σ'_{v0} and σ_{v0} are the effective and total lithostatic stresses at each depth, respectively.

3 Results and discussion

The nested directional LMC described in Table 2, consisting of a combination of scale-dependent variabilities, and the SGCS approach allowed to reconstruct 1000 three-dimensional realizations of the raw measurements.

Table 2. Features of the Linear Model of Coregionalization related to the Gaussian transformed variables

Variables	Horizontal LMC structures	Range (m)
gq_c, gf_s, gu_2	Spherical	10
	Spherical	1200
	Spherical	12000
Variables	**Vertical LMC structures**	**Range (m)**
gq_c, gf_s, gu_2	Spherical	2
	Spherical	6
	Spherical	12
	K-bessel	> 100

This large amount of data, in turn, has been used as input variables of Eq. (5), providing the same number of realizations of the selected output variable (i.e., I_{SBTn}). Besides the optimized 3D models (i.e., mean values of 1000 realizations) of both measured (i.e., q_c, f_s, and u_2) and calculated (i.e., I_{SBTn}) variables (Figure 2), which depict the geo-lithological features of the considered Po plain portion, it was possible to investigate how the uncertainty of measured variables propagates at each point of the selected model domain. Histograms in Figure 3, corresponding to two points characterized by two different SBT classes, show that I_{SBTn} values of coarser lithotypes (P2), which are generally described by very high and variable q_c, f_s and u_2 values, seem to be more uncertain than the finer lithotypes' ones (P1). The intrinsic variability of the fan-shaped geological body at P2 location can account for this evidence. In addition, the location and density of actual measurement might also play a significant role in uncertainty propagation.

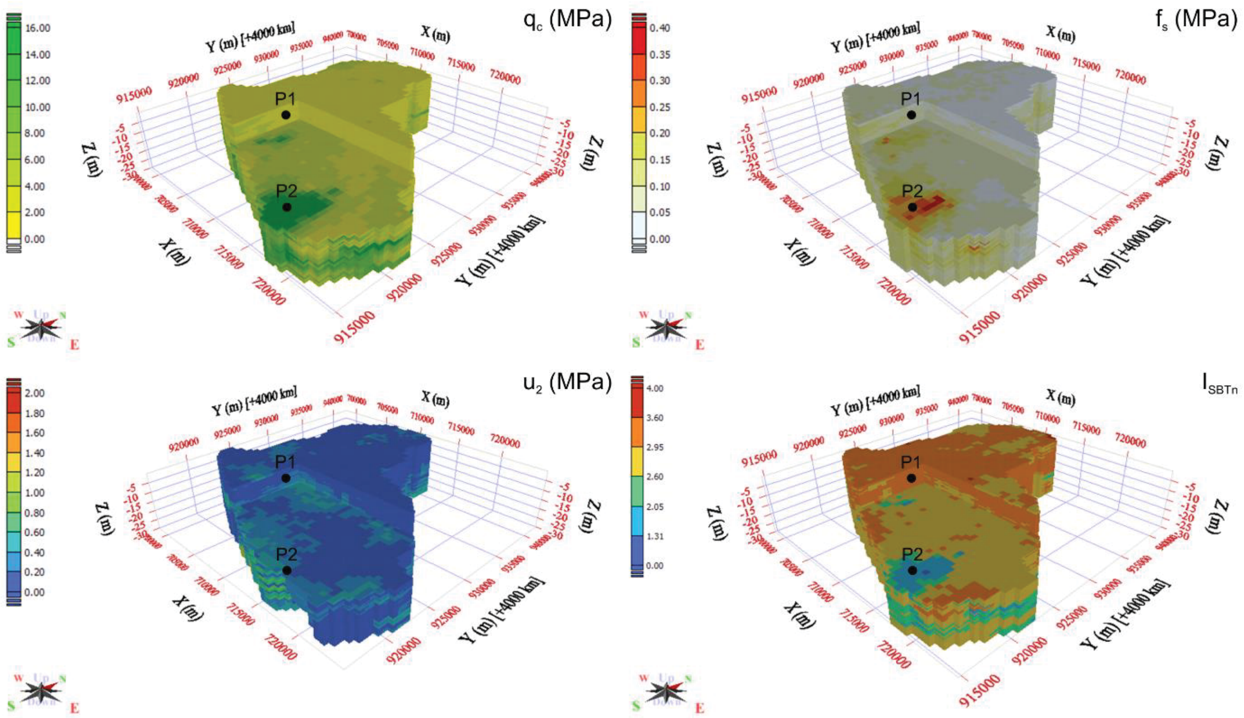

Figure 2. Optimized 3D models of q_c, f_s, u_2, and I_{SBTn}, represented by the mean values of the 1000 realizations.

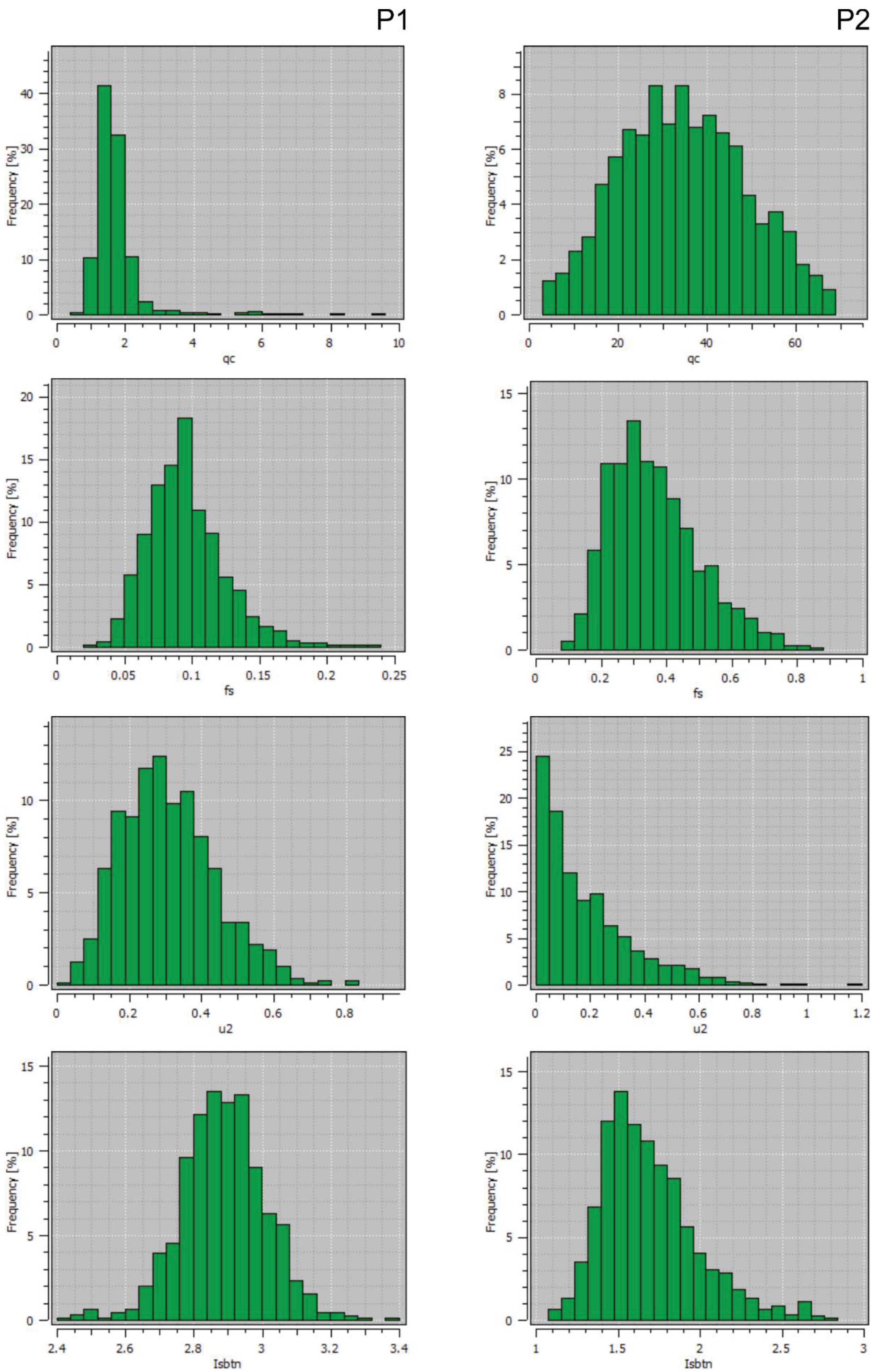

Figure 3. Histograms comparing the statistical distribution of equiprobable values of input variables (i.e., q_c, f_s, and u_2) with the output ones (i.e., I_{SBTn}), both for the silts mixtures at 5-meter depth (P1) and the sandy gravelly deposits at 10-meter depth (P2) (Figure 2).

4 Conclusions

Applying the Sequential Gaussian Co-Simulation to CPTu data through fitting a Linear Model of Coregionalization allowed thoroughly assessing the uncertainty propagation when empiric equations are used to derive variables of interest from raw data (measured). In detail, 1000 realizations of q_c, f_s, and u_2, obtained through SGCS, have been used to calculate the same number of realizations of I_{SBTn}, providing an optimized 3D model of lithotypes' distribution as well as a quantification of the uncertainty associated with the transformation expression. The same methodological approach can be used to quantitatively assess the uncertainty propagation for other critical design variables, which are generally obtained using empirical equations and raw measured data.

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