

# Shallow foundation settlement using a hardening soil model for spatially variable soil

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**Abstract:** In the study, a probabilistic analysis of foundation settlement is performed for spatially variable soil. The impact of elasticity modulus spatial variability is examined on the resulting foundation settlement probability distribution characteristics. For characterising random field that is used for describing the elasticity modulus variability structure, fluctuation scales are used. Both, isotropic and anisotropic correlation structures were assumed in the analysed examples, in the case of anisotropic correlation structure greater horizontal fluctuation scales are assumed in comparison with the vertical ones. The random finite element method (RFEM) was used in the study in combination with an advanced material model (Hardening Soil Model). The study propose a method that may improve our understanding of the behaviour of more advanced soil models than the Coulomb Mohr model in probabilistic applications. This is important direction because so far, RFEM has been used sporadically with advanced soil models such as Hardening Soil model. Together with elasticity modulus, also the shear strength spatial variability on settlement was investigated and discussed in the study. The proposed approach connects commercial software (ZSoil) with the authors own procedures implemented in MATLAB in a framework of Monte Carlo method to obtain settlement values and their spatial distribution.

Keywords: RFEM; random field; spatial variability; Monte Carlo.

## 1 Introduction

In geotechnical engineering, foundation settlement was traditionally evaluated using a deterministic empirical approach Chen and Yu (2014), and Shahnazari et al. (2014). It is also made possible by the use of software employing advanced material models Truty (2018). Hence, probabilistic analysis considering spatial variability of soil properties is necessary and helpful to fully understand foundation settlement. Currently, many scholars have considered the spatial variability of soil properties when studying geotechnical engineering. As a result, settlements calculated using different methods have been studied

fields and the finite element method in a Monte Carlo framework. It estimates the mean and standard deviation of the calculated outputs.

The hardening soil (HS) model, formulated by Schanz et al. (2019), or the HS model with small-strain stiffness Benz (2007), is one of the most advanced constitutive models for soils. Therefore, according to a study Kawa et al. (2021), the combination of the RFEM method for modeling soil properties using random fields with the hardening soil model appears to be an adequate approach to wall deflection, wall settlements, and maximum bending moment along the wall.

The main objective of this work is to study the distribution of probabilistic foundation settlement using the hardening soil model on two-layered and spatially variable soil to be performed in combination with ZSoil (User Manual 2018) and MATLAB Jianye and Sanjay (2013), software using the Monte Carlo method. In particular, the influence of the spatially variable unloading-reloading stiffness and also the influence of shear strength spatial variability on settlement was investigated and discussed.

## 1 Problem Statement

### 1.1 Finite element model

As an illustrative example, this study examines a rigid rough strip footing with width  $B = 1$  m subjected to a uniformly applied vertical load  $P = 200$  kN/m. The  $8\text{m} \times 6\text{m}$  plane strain rectangular domain is modelled by the FE mesh. Each FE is a 4-noded element of size  $= 0.17\text{ m} \times 0.17\text{ m}$ . In total, there are  $48 \times 36 = 1,728$  elements. The nodes along the vertical boundary are constrained against horizontal displacement (roller). Whereas the nodes on the bottom boundary are fixed (hinge).

The stiffness of the soil layer is defined by unloading-reloading stiffness ( $E_{ur}$ ), and also the shear strength parameters are defined by cohesion ( $C'$ ), and effective friction angle ( $\varphi$ ), who in the probabilistic analyses were assumed to be a lognormally distributed random field with inherent mean ( $\mu$ ) and coefficient of variation (COV). The influence of spatially variable unloading-reloading stiffness and the influence of shear strength on settlement are modeled by random fields on both layers. Accordingly, the coefficient of variation 0.1 was applied to both soil layers and their effects were investigated.

In this study, a commercially available FEM code ZSoil is adopted to analyze a two-dimensional foundation plane strain model. Accordingly, the study was developed for the strip footing linear elastic constitutive model, and, furthermore, the stress-strain behaviour of the layered soil is modelled using the constitutive hardening small strain (HSS) relationship that considers the small strain effect. For both the strip footing and the soil for finite element modelling, the assumed parameters are presented in Tab. 1 and Tab. 2.

**Table 1.** Strip footing parameters assumed in finite element modelling.

Parameter	Notation	Layer 1	Layer 2	Unit
Saturated unit weight	$\gamma$	19.4	19.8	kN/m <sup>3</sup>
Dry unit weight	$\gamma$	16	17	kN/m <sup>3</sup>
Unl/ relo stiffness	$E_{ur}^{ref}$	45,000	90,000	kN/m <sup>2</sup>
Reference stress	$\sigma_{ref}$	100	100	kN/m <sup>2</sup>
Poisson ratio	$V_{ur}$	0.2	0.2	-
Exponent for power law	$m$	0.62	0.54	-
Lower bound stiffness	$\sigma_L$	10	10	kN/m <sup>2</sup>
Demand secant reference	$E_{50}^{ref}$	15,000	30,000	kN/m <sup>2</sup>
Friction angle	$\phi$	31.1	34	deg
Dilatancy angle	$\psi$	1.1	4	deg
Cohesion	$C'$	5	2	kN/m <sup>2</sup>
Oedometric modulus	$E_{oed}$	15,000	30,000	kN/m <sup>2</sup>
Reference vertical stress	$\sigma_{oed}^{ref}$	200	227	kN/m <sup>2</sup>
Ko coefficient	$K_0^{NC}$	0.5	0.5	-
OCR	-	1	1	-

**Table 2.** Soil parameters adopted in finite element modelling (Brinkgreve et al.2010).

Parameter	Notation	Value	Unit
Unit weight	$\gamma$	25	kN/m <sup>3</sup>
Young modulus	$E$	30,000,000	kN/m <sup>2</sup>
Poisson ratio	$\nu$	0.3	-

Note: The upper layer is 2 m thick and the lower layer is 4 m thick, with an angle of dilatancy ( $\psi = \phi - 30^\circ$ ) for both.

## 2 Modelling Soil Variability as a Random Field

Random fields were used for modelling soil parameters. The correlation structure of a field, defined by its correlation function, plays an important role in modelling the spatial variability of soil properties. In this study, the Markov correlation function Fenton and Griffiths (2008) shown in Eq. (1) was used.

$$\rho(\tau_x, \tau_y) = \exp\left(\frac{-2|\tau_x|}{\delta_x} + \frac{-2|\tau_y|}{\delta_y}\right) \quad (1)$$

where  $(\tau_x, \tau_y) = (x_2 - x_1, y_2 - y_1)$ . Parameters  $\delta_x$  and  $\delta_y$  denote the horizontal and vertical scale of fluctuations (SoF), respectively;  $x, y$  denote the horizontal and vertical coordinate difference of any two points of concern, respectively.

Many studies have shown, the horizontal fluctuation scale is usually much larger than the vertical fluctuation scale Keaveny et al. (1989) and Chorghini

and  $\delta_x=\delta_y=10$  m and the anisotropic correlation structure,  $\delta_x=5$  m,  $\delta_y=0.5$  m,  $\delta_x=10$  m,  $\delta_y=1$  m and  $\delta_x=20$  m,  $\delta_y=2$  m.

### 3 Generating FE Models with Random Parameters

In this study, eight scenarios of the fluctuation scales previously mentioned were analyzed. For each of these scenarios, 1000 Monte Carlo simulations (MCSs) were carried out with different realizations of the unloading-reloading stiffness ( $E_{ur}$ ) and shear strength parameters i.e., cohesion ( $C'$ ), and effective friction angle ( $\phi$ ) random field. These parameters that are randomly distributed are not correlated. Also, both the top and bottom layers are not correlated to each other.

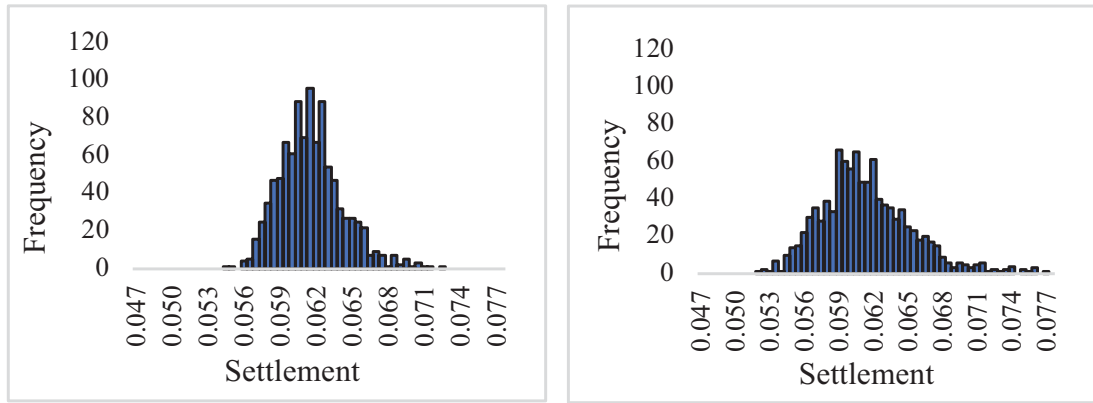
### 4 Results of Probabilistic Analysis Simulations

The vertical deformation (settlement) obtained for all MCSs was collected and the results of the analysis are presented in Tab.3.

**Table3.** Probabilistic values obtained for all scenarios.

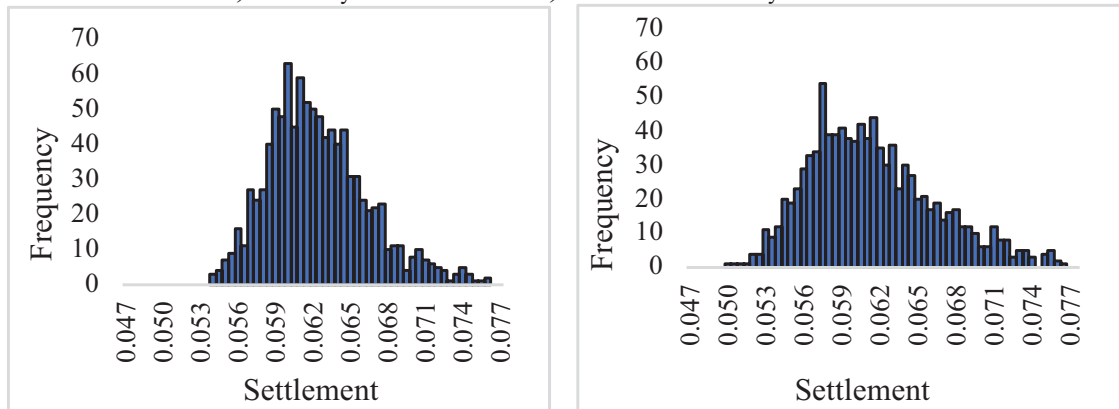
Scale of fluctuation (SoF)	Vertical deformation (settlement)					
	Max (m)	Min (m)	Mean (m)	Median (m)	S. dev (m)	COV %
$\delta_x = \delta_y = 0.5$ m	0.0721	0.0545	0.0613	0.0612	0.002624	4.28
$\delta_x = \delta_y = 1$ m	0.0804	0.0537	0.0622	0.0616	0.004076	6.55
$\delta_x = \delta_y = 2$ m	0.1183	0.0511	0.0627	0.0617	0.006077	9.69
$\delta_x = \delta_y = 5$ m	0.1144	0.0480	0.0629	0.0613	0.008760	13.93
$\delta_x = \delta_y = 10$ m	0.1175	0.0476	0.0625	0.0606	0.009236	14.77
$\delta_x = 5$ m, $\delta_y = 0.5$ m	0.0808	0.0515	0.0612	0.0607	0.004068	6.65
$\delta_x = 10$ m, $\delta_y = 1$ m	0.1347	0.0500	0.0616	0.0606	0.006167	10.01
$\delta_x = 20$ m, $\delta_y = 2$ m	0.1169	0.0473	0.0620	0.0602	0.008254	13.31

The results for the scale of fluctuation (SoF) for both isotropic and anisotropic correlation structures are shown below in Figs. 1, 2, and 3, respectively. As can be seen in the figures below, the results obtained from the histograms show that the distribution is not symmetrical. This means that the mean is greater than the median and the distribution is right-skewed. According to the results, the mean settlement values obtained for the non-random case are 0.0587m, but the mean settlement values obtained for the random cases are noticeably higher than for the non-random case.



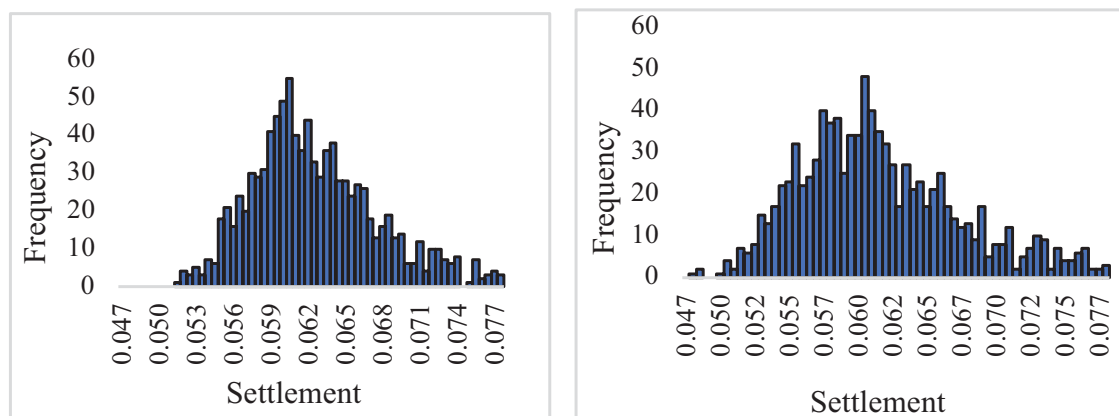
a) b)

**Figure 1.** Typical frequency of the histogram, scale of fluctuation (SoF) for a)  $\delta_x = \delta_y = 0.5$  m and b)  $\delta_x = 5$  m and  $\delta_y = 0.5$  m.



a) b)

**Figure 2.** Typical frequency of the histogram, scale of fluctuation (SoF) for a)  $\delta_x = \delta_y = 1$  m and b)  $\delta_x = 10$  m and  $\delta_y = 1$  m.



a) b)

**Figure 3.** Typical frequency of the histogram, scale of fluctuation (SoF) for (a)  $\delta_x = \delta_y = 2$  m and (b)  $\delta_x = 20$  m and  $\delta_y = 2$  m.

For comparison, the mean settlement values in the isotropic case (Fig. 1a, 2a, and 3a) are slightly higher than in the anisotropic correlation structure case (Fig. 1b, 2b, and 3b). Similarly, the median value on the anisotropic correlation structure is slightly lower. Also, according to the results, the distance between

result in the anisotropic scenario. In general, the coefficient of variation of the unloading-reloading stiffness ( $E_{ur}$ ) and shear strength parameters, i.e., cohesion ( $C'$ ), and the effective friction angle ( $\varphi$ ) applied to both soil layers contributes significantly to the results obtained from the vertical SoF analysis and the horizontal SoF analysis. Because when the coefficient of variation applied to the soil layer is different, the probabilistic vertical deformation values will change.

## 5 Concluding Remarks

- Spatial variability has an impact on settlement prediction and settlement distributions. Thus, the results show that scenarios with larger scale fluctuation in both the isotropic correlation structure and the anisotropic correlation structure have slightly larger settlement result than the smaller scale of fluctuation. However, the result of the median value on the anisotropic case is countertrend.
- The results obtained in the isotropic correlation structure and the anisotropic correlation structure show that the mean value is higher than the median value. That is, if the mean is greater than the median, the distribution is positively skewed.
- The combination of Soil and MATLAB software with Monte Carlo framework proposed in the study seems to be promising for further applications of the Hardening Soil Model in probabilistic settlement analysis. According to the results, the mean settlement values obtained for the non-random case are noticeably lower than for the random case.

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