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Reliability Analysis of a Shallow Foundation Considering Soil Spatial Variability

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Abstract: Random fields are widely used for describing the soil spatial variability in geotechnical engineering. The consideration of stochastic variability with spatial dependence in soil parameters is a complex problem, which renders the analysis of hundreds or thousands of random variables. Thus, it is not usually performed, and simplifications are often adopted. This article presents four different analyses to investigate the influence of the autocorrelation length on the bearing capacity of a shallow strip foundation, when random fields are explicitly considered. The studied soil property is undrained shear strength (Su) randomized with the random field theory. The Random field approach integrates the Karhunen-Loeve expansion method with Monte Carlo simulations (MCS) to develop a probabilistic analysis. The software OPTUM G2 which follows the finite element limit analysis (FELA) method was used to perform all deterministic and field analysis.

Keywords: Random field; probability of failure; spatial variability; foundation.

1 Introduction

Geotechnical engineers are usually faced with large uncertainties in situ conditions. Uncertainties are often attributed to the inherent variability of soil mass due to the intrinsic randomness of soil formation processes, the statistical uncertainty due to the limited number of site investigations and the errors in measurement devices and data processing (Phoon and Kulhawy 1999; Baecher and Christian 2003; Papaioannou and Straub 2017). Civil structures are particularly affected by such uncertainties. A correct representation of the soil inherent spatial variabilities is an important task in probabilistic analyses. The random field theory is considered the most sophisticated way to model this soil feature and has been widely applied in different geotechnical structures (Pouya et al. 2014; Mouyeaux et al. 2018; Guo et al. 2021). This article is dedicated to addressing numerical simulations to investigate the influence of random fields correlation length on the bearing capacity of a shallow strip foundation. Karhunen-Loeve expansions are used to represent the spatial random fields, and reliability analyses are carried out using Monte Carlo Simulations.

2 Problem Setting

2.1 Performance function

From the probabilistic perspective, failure modes are mathematically described by a performance function g(**X**). The random geotechnical parameters of the problem are grouped in a random variable vector $\mathbf{X}=\{X_1, X_2,\ldots, X_n\}$. In the space of random variables, the domain is divided into: $\{\boldsymbol{x}:g(\mathbf{X})\leq 0\}$ is the failure domain (D_f) and the safety domain (D_s) is represented by $\{\boldsymbol{x}:g(\mathbf{X})>0\}$. One realization of this vector is denoted by **x**. In an "*n*" dimensional hyper-space of variables, the limit state function $g(\mathbf{x})=0$ is the boundary between safe and failure domains. The probability of failure (P_f) is given by:

$$P_f = P\left[g(\boldsymbol{X})\leq 0\right] = \int_{D_f} f_{\boldsymbol{X}}(\mathbf{x})d\boldsymbol{x} \tag{1}$$

where $f_X(\mathbf{x})$ represents the joint probability density function of **X**.

The minimal distance of the limit state function $g(\mathbf{y})=0$ to the origin is the so-called reliability index (β), and the point over the limit state with minimal distance to the origin is called design point (*DP*). The limit state function is approximated by:

$$P_f \approx \Phi(-\beta) \tag{2}$$

where Φ(.) is the standard Gaussian cumulative distribution function (Siacara et al. 2020a, b, 2022a).

2.2 Random variable (*RV*) approach

A spatially variable soil property can be modelled with a single random variable **X**. The property at a specific location is not explicitly modelled and the inherent variability of the soil property within the area of interest is represented by the probability density function (PDF) of **X**, f_X. It means that the soil is assumed to be homogeneous (Guo et al. 2021). This corresponds to the classical statistical approach, which is based on modelling the variability within a population into a probabilistic distribution (Papaioannou and Straub 2017).

2.3 Random field (RF) approach

A more accurate but also more demanding approach is to model spatially variable properties at each location explicitly. In this approach, the property is modelled by a *RF* **X**(*z*), which represents a *RV* at each location *z* (Rackwitz 2000; Phoon 2008). The *RF* is usually modelled by the marginal distribution at each location F_X and the auto-correlation coefficient function ρ_X. Usually, the marginal distribution of soil parameters is modelled by a non-Gaussian distribution model.

The *RF* can then be expressed as a function of an underlying Gaussian field *U*(*z*) with zero mean and unit standard deviation, through application of the following marginal transformation:

$$\mathbf{X}(z) = F_X^{-1}\left[\Phi(U(z))\right] \tag{3}$$

where F_X^{-1} is the inverse of the marginal CDF of **X**(*z*) and Φ(.) is the standard normal CDF. The transformation of Eq. (3) implies that the joint distribution of the *RVs* for any collection of points in the spatial domain is described by a Gaussian copula, also known as the Nataf distribution (Der Kiureghian and Liu 1986). In general, the specification of the correlation structure of *U*(*z*) in terms of the one of **X**(*z*) involves solving an integral equation (Papaioannou and Straub 2017).

In order to numerically represent the continuous *RF* **X**(*z*), it is necessary to discretize it with a finite set of *RVs* gathered in a vector **X**. Several methods have been proposed for the discretization of RFs (see (Sudret and Der Kiureghian 2000) for a comprehensive review). Depending on the correlation structure of the *RF* and the discretisation method used, the number of *RVs* in **X** can be considerable. A method that is optimal in terms of the average mean-square error in the discretization, and hence can lead to efficient *RF* representation with relatively small number of *RVs*, is the Karhunen–Loève expansion (e.g. (Ghanem and Spanos 1991; Betz et al. 2014)).

2.4 Karhunen–Loève expansion

Let **X**(*z*, ω) be a random field, where *z* ∈ D defines the physical space and ω ∈ Ω defines the probability space. The correlation structure of a random field is modeled by the covariance function, denoted by C_X (**s**, **t**), where **s**, **t** ∈ D, are bounded, symmetric and positively defined.

Several random field generation methods are available (see e.g. (Fenton and Griffiths 2008)). In OPTUM G2 (OptumCE 2009), the Karhunen-Loeve expansion method is used. This method is convenient as it provides analytical solutions for the exponential covariance function. Using Mercer's theorem (OptumCE 2009), the covariance function can be decomposed according to

$$C_{\mathbf{X}}(\mathbf{s},\mathbf{t}) = \sum_{i=1}^{\infty} \lambda_i f_i(\mathbf{s}) f_i(\mathbf{t}) \tag{4}$$

where λ_i and f_i are, respectively, the eigenvalues and eigenfunction of the $C_{\mathbf{X}}$. Since the above sum has to be truncated to a finite number of terms, a significant concern is that the simulated variance will be reduced. In order to control this reduction, the eigenvalues are sorted in descending order and the number of terms, *n*, is decided on by the eigenvalues having decayed sufficiently to satisfy the condition.

3 Application problem and boundary conditions

In this section, random field analysis is applied to a common foundation engineering example: bearing capacity of a shallow foundation. The geometry, equation of performance function, random variables, and problem explanation are given as follows.

The finite element limit analysis (FELA) method was used to perform all deterministic and field analysis. The FELA method embraces the advantages of finite element model and the classic limit analysis methods, and is particularly suitable for the analysis and design of geotechnical stability problems (Sloan 2013; Ji et al. 2021). The best size of the model has been examined and tested to ensure that the failure zone does not exceed the boundary of the model. The surface ground can move freely and the bottom is fixed in all directions. The Tresca criterion was used in the analysis and the random field information is presented in Figure 1 and Table 1. The undrained shear strength (*Su*) is modelled as a Random Field. Additional deterministic parameters of the soil are Modulus of Elasticity (E_u)=30.0 [MPa], Unit weight (γ)= 10.0 [kN/m^3], Earth Pressure Coefficient (k_o)= 0.5. The

Unit weight of the rigid strip footing (γ_c) is 23 [kN/m3]. A constant load q of 1.0 [kN/m^2] is applied above of the foundation. The width and high of the numerical model are shown in Figure 1.

The OptumG2 (OptumCE 2009) provides three options for FELA of the shallow foundation stability, namely, the upper bound (UB), the lower bound (LB), and the mixed principle (Mixed). In this case, the UB and LB are used to analyze the footing bearing capacity. An automatic adaptive mesh refinement is used in this model, and greater power dissipation intensity, the denser the mesh as is shown in Figure 2. In OptumG2, the Karhunen-Loeve expansion method is used. The P_f and β were found using MCS, with 1000 realizations.

Table 1. Probabilistic parameters of the soil property.

Case of study	A	B	C	D
Material	Undrained shear strength – Su [kPa]			
Statistical moments				
Distribution	Lognormal (LN)			
Mean - μ	z=0 with 10 [kPa]			
	z=-6 with 28 [kPa]			
Coefficient of variation – *COV* (%)	30			
Autocorrelation length (m)				
Horizontal (L_x)	50.0	1.0	1.0	50.0
Vertical (L_z)	1.0	1.0	50.0	50.0
Modelling approach	Random fields			

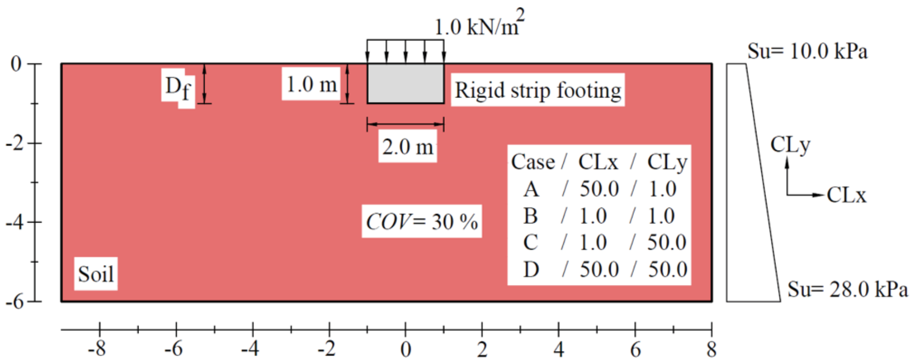

Figure 1. Model used in the analysis.

4 Shallow foundation results

Initially, a deterministic analysis with mean values of Figure 1 and Table 1 is carried out. The UB gives a bearing capacity (q_u)= 94.27 [kN/m^2] and the LB present q_u= 91,83 [kN/m^2]. The mean value between UB and LB is q_u= 93,05 [kN/m^2], and the best estimate for collapse load is q_u= 93,05 ±1.3%. The difference of results between UB and LB could be closer using a smaller mesh (independently of the strategy used), but it would be more computationally demanding. In whatever case, the mean value of q_u is the same. In this study, the adaptative mesh is applied and presented in Figure 2.

The results of the UB for the total displacements U and collapse solution with intensity of plastic multiplier are presented in Figure 2 a and c, respectively. In this case, the results of displacements *U* are only a representation. The load displacement curve needs to be performed using the classic Multiplier Elastoplastic analysis which is a combination of the Limit Analysis and Elastoplastic analysis types, this will not be our case of analysis.

The vertical collapse extent of the failure mechanism is approximately 0.7 of the base foundation (B). These initial analysis helps to see if the influence of the vertical autocorrelation length (CL_y). The case A, B, C and D are studied modifying the horizontal and vertical autocorrelation length. The Figure 3 a, b and c represent a random field simulation of Su for the case A, B and C, respectively. The random field variation of the Su property in the collapse surface produce different surfaces for every MCS simulation. The limit state function describing failure of the foundation is represented by the Eq. (5). The geometry and values are defined in Table 1 and Figure 1. Although Su is variable in the deep, this equation helps in this study.

$$g(\mathbf{X}) = q_u - q \tag{5}$$

The q is assumed as 61.4 [kPa]. The relationship between both values represents the factor of safety FS= q_u/61.4. In the deterministic case, the mean value FS= 1.51 is assumed.

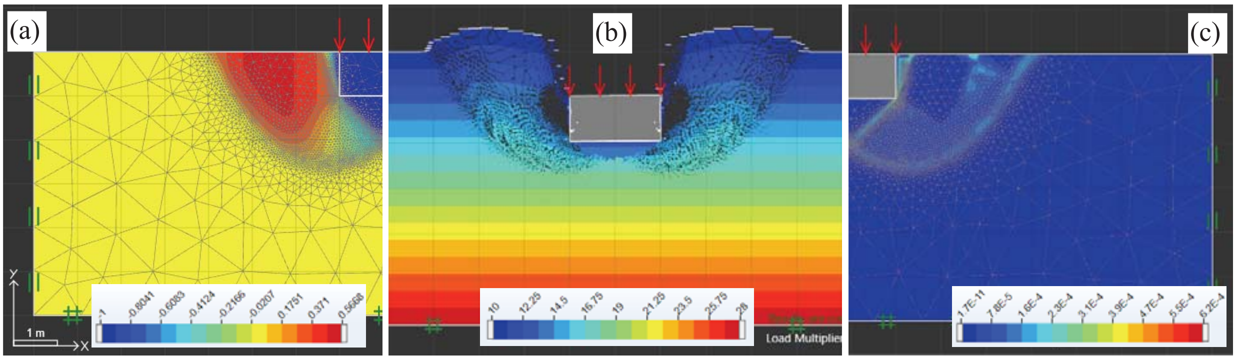

Figure 2. Results of UB for (a) Total displacements *U* [-]; (b) Undrained shear strength Su [kPa] and (c) collapse solution with intensity of plastic multiplier.

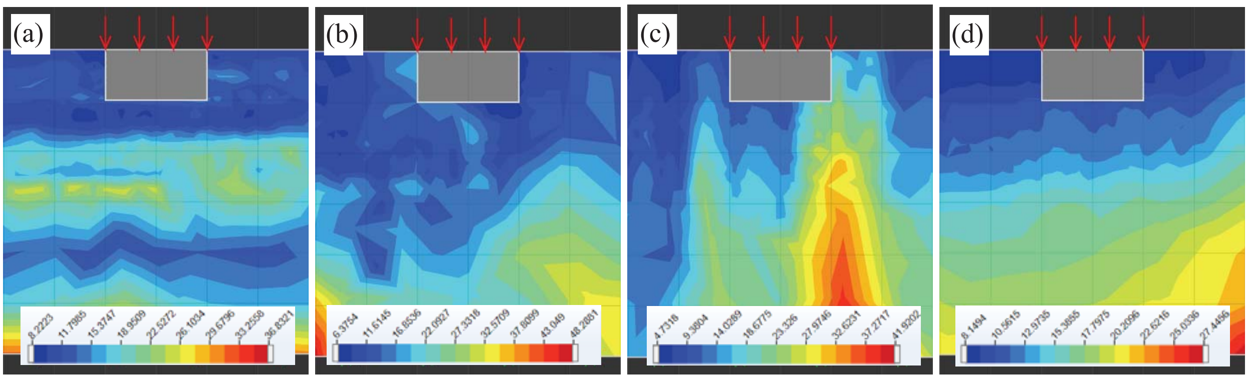

Figure 3. A random field simulation of Su for the Case: (a) A; (b) B; (c) C and (d) D.

The results reported are based on the mean value between the UB and LB for each simulation. The best fit (Figure 4 a, b, c and d) of the q_u is the Lognormal (*LN*) distribution which follows the Su distribution, as expected. The lowest probability of failure (P_f) is presented in the case B, and the highest P_f is presented in the case D as shown in Table 2 and Fig 4 b and d. This observation follows the differences in terms of autocorrelation length between both cases. In case D, P_f is larger by the large autocorrelation distance. It could be represented as a *RV* analysis. An opposite behavior is observed in case B by the lowest autocorrelation distance. The mean factors of safety (μ_{FS}) are very similar to the deterministic *FS*, the closer value is when the autocorrelation length is highest.

Table 2. Reliability results of the three analyses.

Analysis	Failure probability		Statistical moments of *FS*	
	P_f (x10^{-2})	*COV* [%]	Mean	Standard deviation
A	8.5	20.38	1.42	0.34
B	1.1	30.20	1.35	0.18
C	4.7	30.59	1.39	0.25
D	14.5	8.95	1.50	0.50

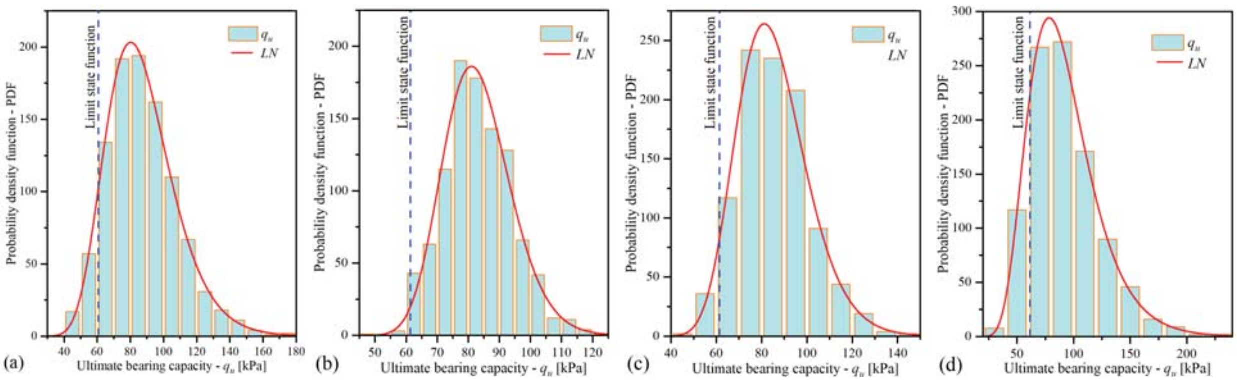

Figure 4. Random field Su result for the Case: (a) A; (b) B; (c) C and (d) D.

Convergence plots allow us to know if the number of simulations is enough. In this study, the cases C and D are presented in Figure 5 a and b, respectively. In the case D, the accuracy of the P_f can be obtained with as 600-700 runs. The case C, on the other hand, requires somewhat more runs to be determined with the same degree of accuracy of case D.

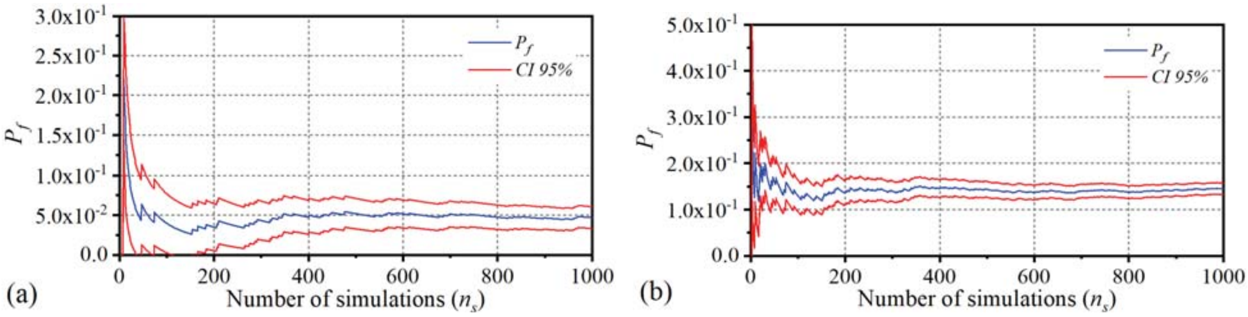

Figure 5. Convergence of MCS simulation in terms of number of samples and Failure probability (P_f) and 95% confidence interval: (a) Case C, and (b) Case D.

In reliability analysis using high-fidelity numerical models, computational cost and the method employed are important factors, which determine whether the analysis is feasible or not (Siacara et al. 2022b). In this study, a common desktop computer was used, with a processor speed of 1.8 GHz and RAM memory of 8 GB. The computational cost for deterministic analysis was less than 1 minute, and 6 hours for every case (A, B, C and D) of study using the random field analysis (RF).

5 Conclusions

This paper presented a numerical simulation approach to investigate the influence of the autocorrelation length of random field on the bearing capacity of a shallow strip foundation. The studied soil property is undrained shear strength (Su) randomized with the random field theory. As the Monte Carlo simulation (MCS) is a robust approach that can capture the inherent variability of soil properties in classic geotechnical problems, in this case, 1000 simulations are carried out to determine the statistical properties of the bearing capacity of the footing.

In OPTUM G2, the Karhunen-Loeve expansion method is used. The results reported are based on the mean value between the UB and LB for each run. This method is convenient as it provides analytical solutions for the exponential covariance function.

In this case, the mean factor of safety (μ_{FS}) on a spatially variable soil profile is always lower than the deterministic factor of safety (*FS*) obtained from a constant mean value. It also has demonstrated that an increase the degree of heterogeneity, increases this deviation.

In all cases, the best fit of the q_u is the Lognormal (*LN*) distribution which follow the Su distribution. The lowest probability of failure (P_f) is presented in the case B, and the highest P_f is presented in the case D. This observation follows the differences in terms of autocorrelation length between both cases. In the case D, P_f is large by the autocorrelation distance. In this case, the random field could be represented as a *RV*. The mean factors of safety (μ_{FS}) are very similar to the deterministic *FS*, the closer value is when the autocorrelation length is larger.

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