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The Spatial Variability of the Cone Tip Resistance of Weathered Mudstone Profiles from CPT Testing

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Abstract: Cone penetration test (CPT) data were used to explore the vertical spatial variability in weathered layers of the Charmouth Mudstone Formation (Lias Group) at a site in Oxfordshire, England. The Charmouth Mudstone Formation is a stiff, fissured Jurassic mudstone that was formed 183-199 million years ago and weathered by glacial and periglacial conditions during the past 200,000 years. Forty-six CPT profiles were considered in the variability assessment. The vertical variability of the cone tip resistance was characterized by determining the vertical scale of fluctuation analyzed within a homogenous layer via the random field theory. The results showed that the profiles of weathered mudstone had a low scale of fluctuation, which translates to high variability among the property values. The vertical scale of fluctuation of q_c for the weathered mudstone profiles varied between 0.10 m and 0.69 m, with an estimated theoretical average of 0.30 m and a coefficient of variation equal to 42%. A probability histogram of the vertical scale of fluctuation of q_c showed higher probability of occurrence between 0.11m and 0.49m. The vertical scale of fluctuation q_c did not show a distinction between the weathered mudstone layers and the underlying, unweathered material.

Keywords: Spatial variability; Weathering; Scale of fluctuation; Uncertainties; Mudstone.

1 Introduction

An understanding of the spatial variability in soil parameters is essential for the design and risk management of geotechnical structures (Pieczyńska-Kozłowska et al. 2021). The engineering properties of soils and rocks, and their inherent spatial variability, are influenced by their deposition and subsequent weathering processes (Baecher and Christian 2006). A random process model, based on a hypothesis about the site geology, can be used to describe how the soil and rock properties vary in space using a limited number of observations and in situ tests (Baecher and Christian 2006). This can be used to support the development of a conceptual ground model for a geotechnical design that accounts for extreme local variations due to fluctuations of geotechnical properties (Vanmarcke 1977). The spatial variability of geotechnical properties can be modeled as a random field characterized by a probability density function, with its descriptive statistics (mean and standard deviation) and the scale of fluctuation, θ (Vanmarcke 1977). However, a complete characterization of the spatial variability of soils for both the horizontal and the vertical scales is limited by the large number of samples required to characterize the scale of fluctuation (Zhang et al. 2021). Measurements from the cone penetration test (CPT) can be used to quantify the spatial variability of soil parameters in both the vertical and horizontal directions.

Continuous CPT profiles with measurements taken at close (e.g., 0.01m) intervals can detect small changes in the stratigraphy and vertical variability (θ_v) of soil properties. These profiles can be aggregated if assessment of horizontal variability is desired. In this study, the vertical spatial variability, θ_v, of the cone tip resistance (q_c) of weathered layers of the Charmouth Mudstone Formation at a site in Oxfordshire, England, was determined using Cone penetration test (CPT) data. The Charmouth Mudstone Formation is a stiff, fissured Jurassic mudstone classified as a medium to high plasticity clay soil in its weathered state (Hobbs et al. 2012; Briggs et al. 2022). A total of forty-six CPT profiles were considered for the assessment. The vertical variability of q_c was characterized by determining the autocorrelation function of the q_c detrended residuals for each profile and fitting a theoretical single exponential Markov correlation model. The best fit line coefficients and overall mean were determined from the theoretical values of θ_v. The coefficient of variation of θ_v was used to measure the dispersion of the estimates. The results were compared with measurements in other stiff, clays. They can be combined with a horizontal variability assessment to generate random fields for weathered Charmouth Mudstone and to inform the design of engineering structures.

2 Materials and methods

2.1 Study site

A large site investigation for the High Speed Two (HS2) railway was undertaken in the Charmouth Mudstone Formation (Lias Group) along an 11-12 km length near Banbury, England. The Charmouth Mudstone Formation is a Jurassic mudstone that was formed 183-199 million years ago and has been subjected to glacial, periglacial, and contemporary weathering (Foster et al. 1999). The site investigation included more than 568 individual borehole and trial pit excavations and 89 CPT profiles. Most of the CPT profiles were conducted within weathered layers of the Charmouth Mudstone, and many encountered refusal on contact with the unweathered Charmouth Mudstone at approximately 14 m below ground level (mbgl). Standpipe piezometer data (not shown) indicated a groundwater level of 0.5-1 mbgl across the site. The weathering profile was recorded in the borehole logs according to the code of practice for ground investigations BS 5930:2015+A1:2020 'Approach 4' for weak rocks (Classes A-E). Classes E, D, and C correspond to reworked, de-structured, and distinctly weathered clay. Class B corresponds to partly weathered, and Class A corresponds to unweathered or intact mudstone. For this study, the soil profile was characterized as either 'weathered' (Classes B-E) or 'unweathered' (Class A) groups. CPTs were performed with a 20.5-tonne track-truck mounted CPT unit (UK3) equipped with a 17-tonne capacity hydraulic ramset. This used an electric penetrometer conforming to the requirements of the International Standard ISO 22476-1:2012. The measurements included the cone tip resistance (q_c), friction sleeve resistance (f_s), and dynamic pore water pressure (u_2), sampled at a 10 mm resolution (also known as CPT$_u$). The locations of the CPT profiles are shown in Figure 1.

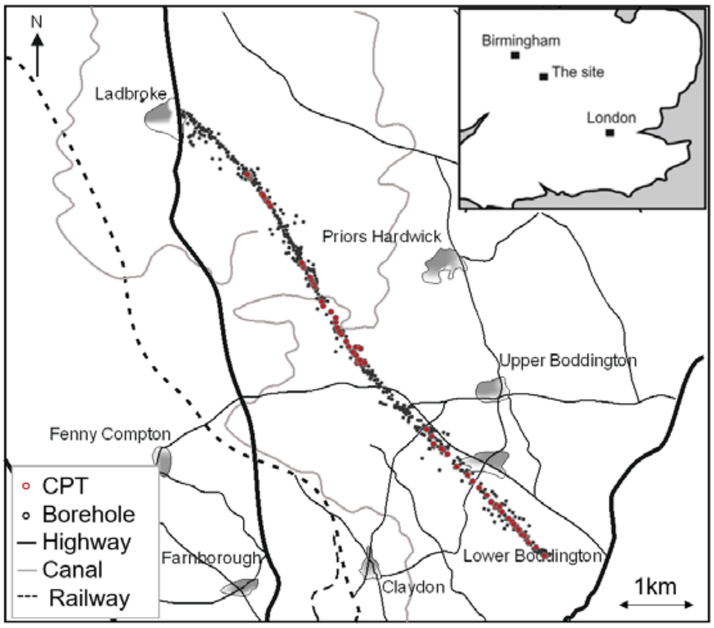


Figure 1. Location of the CPT tests, trial pit, and borehole excavations in the Charmouth Mudstone Formation along an 11-12 km length of ground investigation 12 km north of Banbury, England.

Soil classification data from the ground investigation showed greater variation in material properties near the ground surface (up to 5 mbgl) than at greater depth (Briggs et al. 2022). Classification data from excavated samples plotted above the A-line on a Casagrande Chart, indicating a clay/silt of medium to high plasticity and compressibility; this is in agreement with results from elsewhere in the Charmouth Mudstone Formation outcrop (Hobbs et al. 2012). The plastic limit, liquid limit, and moisture content of samples decreased slightly with depth.

2.2 Cone penetration test (CPT) and pre-processing

During the CPT, the soil resistance along with the friction sleeve (including pore water pressure for CPTu) are measured continuously while pushing a standardized cone into the ground at a constant penetration rate. Empirical correlations can be used to interpret parameters such as strength, permeability, and stiffness from CPT data.

Some correlations use the uncorrected cone resistance, q_c, or the sleeve friction f_s, while others use the corrected cone resistance, q_t. The sleeve friction, f_s, is often considered to be unreliable, e.g., due to sleeve wear (Kenarsari et al. 2013; Zuidberg 1982). Therefore, in this study, the raw q_c measurements were selected as the input signal to determine the vertical scale of fluctuation of the cone tip resistance. Spatial variability analyses require the data to be homogenous and stationary (Baecher and Christian 2003). Of the eighty-nine CPT profiles, sixty-seven were of at least 6 m in length and therefore selected for pre-processing. These profiles reduced to forty-six viable profiles after the homogeneity criteria (59 CPT profiles were eligible) and stationarity criteria (forty-six CPT profiles were eligible using a linear trend line) had been evaluated. Soil homogeneity is required because the correlation structure of the soil properties is soil type dependent (Bouayad 2017). Uniform units were determined based on the coefficient of variation of soil behavior type index (I_c) given by Robertson and Wride (1998). Profiles were considered homogeneous when COV of $I_c < 10\%$ (Tian and Sheng 2020; Uzielli et al. 2007). Stationarity is a statistical assumption that considers data as independent of the location at a site (Baecher and Christian 2003). For the CPT data, the deterministic trend should be removed to access the stationary data. Trends are low-order polynomial functions no higher than a quadratic, obtained by using the ordinary least square method (Uzielli et al. 2007; Jaksa et al. 1999). In this study, a linear trend was removed from the data as a pre-processing step, and a threshold of coefficient of determination R^2 of 0.25 was used. A coefficient of determination larger than 0.25, which in the case of a simple regression with a constant term is the square of the correlation coefficient ($r= 0.5$), is set as a threshold for considering cases where the linear trend with depth is at least moderate (Asuero et al. 2006). The profiles were then grouped by depth to calculate the average vertical scale of fluctuation for the weathered mudstone (6 mbgl). To compare the weathered and unweathered materials, a limited number of deeper CPTs were available. CPT profiles from depths greater than 12 mbgl were selected and divided into two layers of equivalent length (weathered and unweathered) for a one-to-one comparison with their corresponding unweathered profiles. Figure 2 shows a representative example of the measured data from CPT signal processing (showing results for the weathered layers).

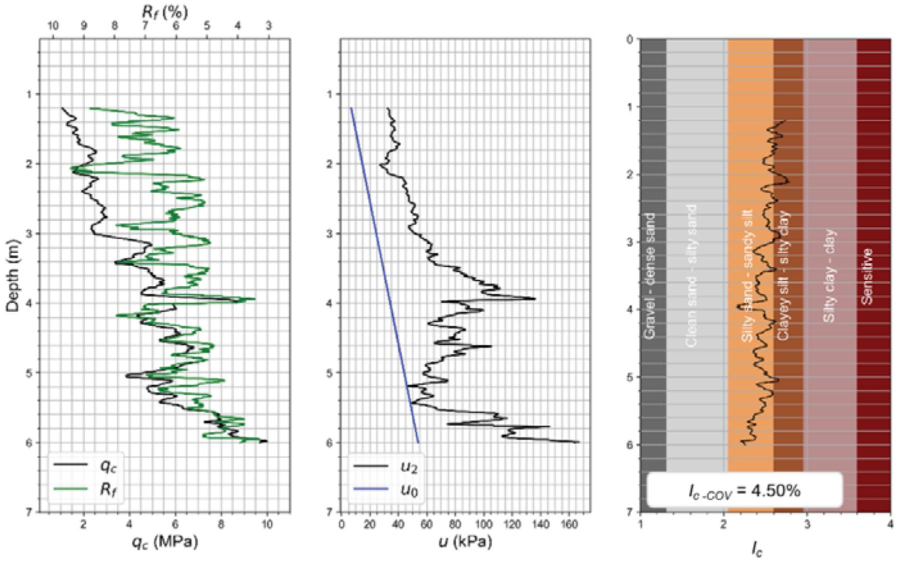


Figure 2. Typical measured CPT data from a single profile, showing the cone resistance (q_c), friction ratio ($R_f = 100.f_s / q_c$), measured and shoulder pore water pressures (u_0 and u_2, respectively), and corresponding soil behavior type index (I_c).

The soil behavior type index (I_c) was used to define the soil's behavior type according set numerical boundaries. For the analyzed data, I_c ranged from sandy silt to silty clay. I_c was calculated as follows (Robertson and Wride 1998):

$$I_c = \sqrt{(3.47 - \log{(Q_t)^2} + (1.22 + \log(F_r))^2} \tag{1}$$

where

$$Q_t = \frac{q_t - \sigma_{vo}}{\sigma'_{vo}} \tag{2}$$

$$F_r = \frac{f_s}{q_t - \sigma_{vo}} . 100\ (\%) \tag{3}$$

q_t is the corrected cone tip resistance (q_t is determined from the uncorrected cone resistance, tip net area ratio, and shoulder pore pressure, respectively as $q_t = q_c + [1 - a_n]\ u_2$), f_s is the sleeve friction, σ_{vo} and σ'_{vo} are the in-situ vertical total and effective stress, respectively, and Q_t and F_r are the normalized values of tip resistance, and sleeve friction.

2.3 Method to determine the vertical scale of fluctuation θ_v

The engineering properties of soils and rocks, and their inherent spatial variability, are influenced by their deposition and subsequent weathering processes (Baecher and Christian 2006). The spatial variability of geotechnical properties can be modeled as a random field characterized by a probability density function, with its descriptive statistics (mean and standard deviation) and the scale of fluctuation, θ. The scale of fluctuation describes the distance over which values of soil properties are correlated (Vanmarcke 1977). Large scales of fluctuation represent low variability among the property values. Spatial variability studies use random field theory and time series analysis or geostatistics. In this study, the vertical scale of fluctuation of the cone tip resistance, q_c, was determined by fitting a theoretical correlation model to the correlation structure (ρ) estimated (from the CPT data) following Vanmarcke 1977; Campanella et al. 1987; DeGroot and Baecher 1993; Fenton 1999; Baecher and Christian 2003; Wackernagel 2003; Uzielli et al. 2005; Fenton and Griffiths 2008; and Lloret-Cabot et al. 2014. First, the q_c profiles were detrended to determine the correlation structure, $\rho_{\tau j}$ of the residuals (with zero mean) for different lag distances (also called separation distance) between CPT profile observations ($\tau j = j\Delta z$), $j = 1, 2, \ldots, N/4$, following (Box and Jenkins 1970), and Δz is sampling interval. This estimated correlation function can be written as:

$$\rho_{\tau_j} = \frac{\sum_{i=1}^{N-j} w_i w_{i+j}}{\sum_{i=1}^{N-j} w_i^2} \tag{4}$$

where w is the fluctuating component of the inherent variability of a depth-dependent geotechnical property, and N is the total number of data points. Second, the theoretical correlation model was fitted to $\rho_{\tau j}$. There are several types of autocorrelation models that have been used for geotechnical applications. Some of those include the Gaussian, triangular, single and cosine exponential, and Markov correlation models (Lloret-Cabot et al. 2014; Fenton and Griffiths 2008). For simplicity in this study, the theoretical correlation model was assumed to have a single exponential Markov function form:

$$\rho_\tau = exp\left\{\frac{-2|\tau|}{\theta_v}\right\} \tag{5}$$

where ρ_τ is the theoretical correlation model, and θ is the scale of fluctuation. The coefficient from the best fit line represents the value of θ_v. Figure 3 shows the processing stages undertaken for each CPT profile.

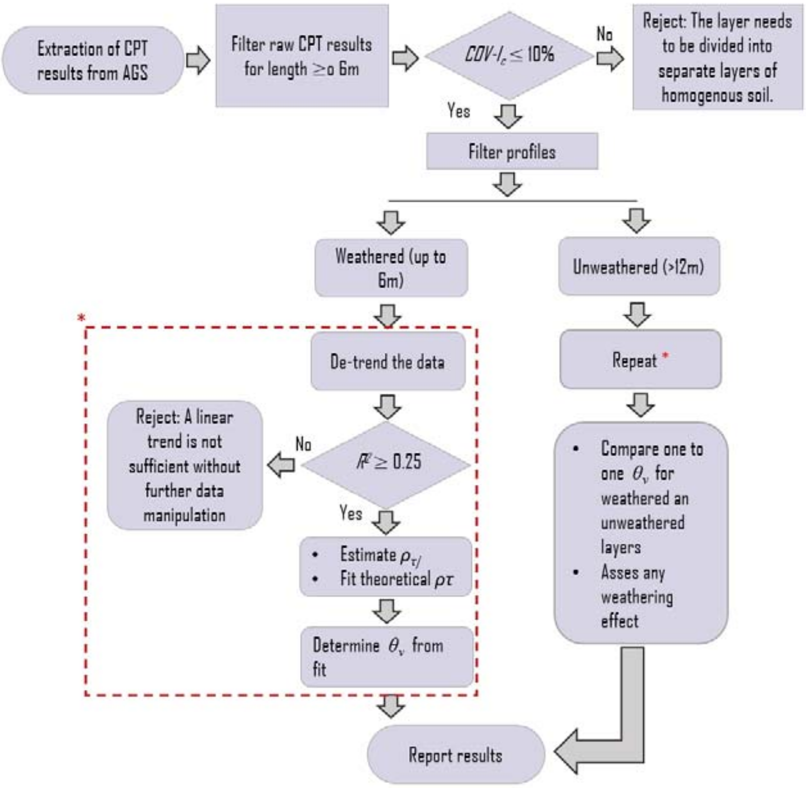


Figure 3. The processing stages for each CPT profile.

3 Results and discussions

The mean qc of the forty-six profiles was 3.6 MPa with a mean coefficient of variation of 38.4%. However, to determine the vertical scale of fluctuation, each profile was analyzed separately (see independent R2 in Table 1), and the mean vertical scale of fluctuation of all analyses was given. A typical CPT profile is presented to illustrate the methodology. Figure 4(a) shows a CPT profile with a linear trend that exhibits a strong correlation with the qc data having a coefficient of determination, R2 of 0.83. This shows that the CPT profile data are non-stationary and that the trend should be removed. Figure 4(b) shows the detrended qc (or residuals) profile. A linear trend was removed from all forty-six selected CPTs, with R2 ranging between 0.27 and 0.86.

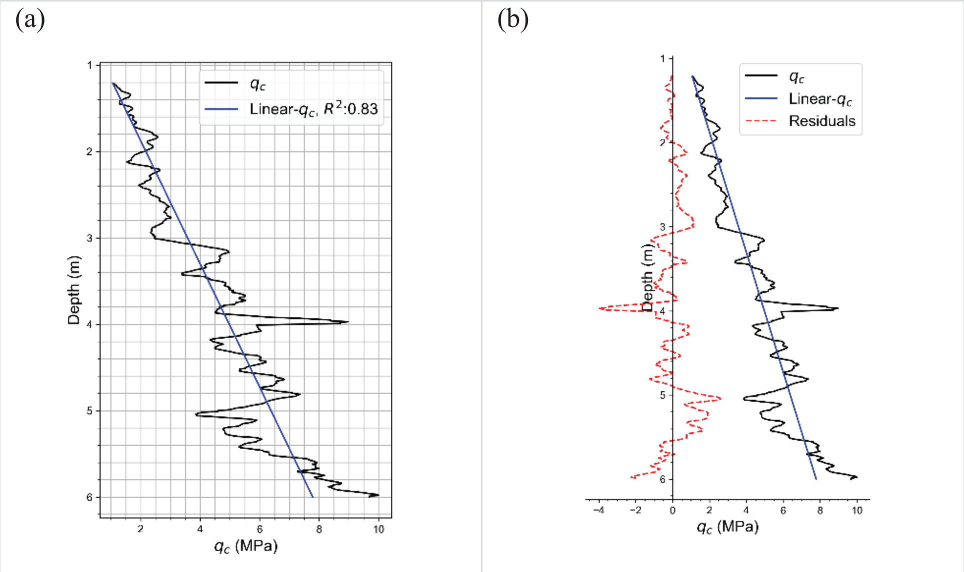


Figure 4. Non-stationary and stationary CPT data (From Figure 2) are shown as (a) a linear trend that exhibits a strong correlation with the corrected q_c data and (b) the detrended q_c (or residuals).

The results in Figure 4(b) show no pronounced trend with depth. After de-trending, the correlation function of the residuals was calculated using Equation 4. Figure 5 shows the estimated correlation function ($\rho_{\tau j}$) for each CPT profile (shown grey), the average of the estimated correlation functions for all forty-six CPT profiles (shown blue), and the theoretical correlation model (ρ_τ) fit to the average of the estimated functions for all forty-six CPT profiles (shown red). The results for all CPTs are presented in Table 1.

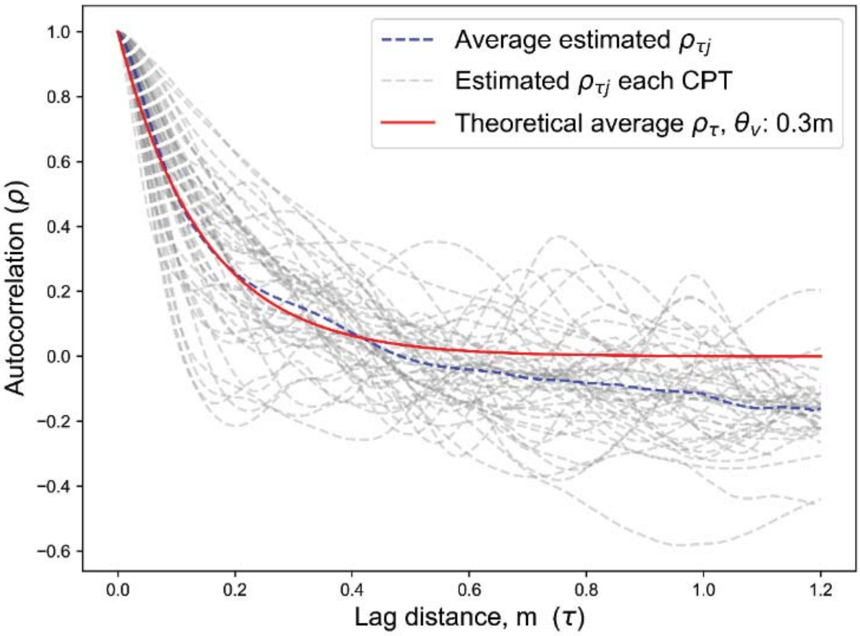


Figure 5. Estimated correlation functions for the weathered mudstone layers for all forty-six CPTs and a theoretical correlation function fit to the average of the estimated correlation functions for all tests.

As shown in Table 1, the vertical scale of fluctuation of q_c for the weathered mudstone profiles varied between 0.10 m and 0.69 m (see Table 1), with an estimated theoretical average of 0.30 m and a coefficient of variation of 42%. Published data in clays show vertical correlation distances varying between 0.13 m and 8.6 m, with stiff clays lying at the lower limit of the range and comparable with the values in this study (Jaksa et al. (1999); Li and Lee (1991)). Histograms and probability density functions for both the cone tip resistance and the vertical scale of fluctuation of q_c are shown in Figure 6. Although the vertical scale of fluctuation (θ_v) of q_c varied between 0.1m and 0.69 m, a higher probability of occurrence lies between 0.11 m and 0.49 m.

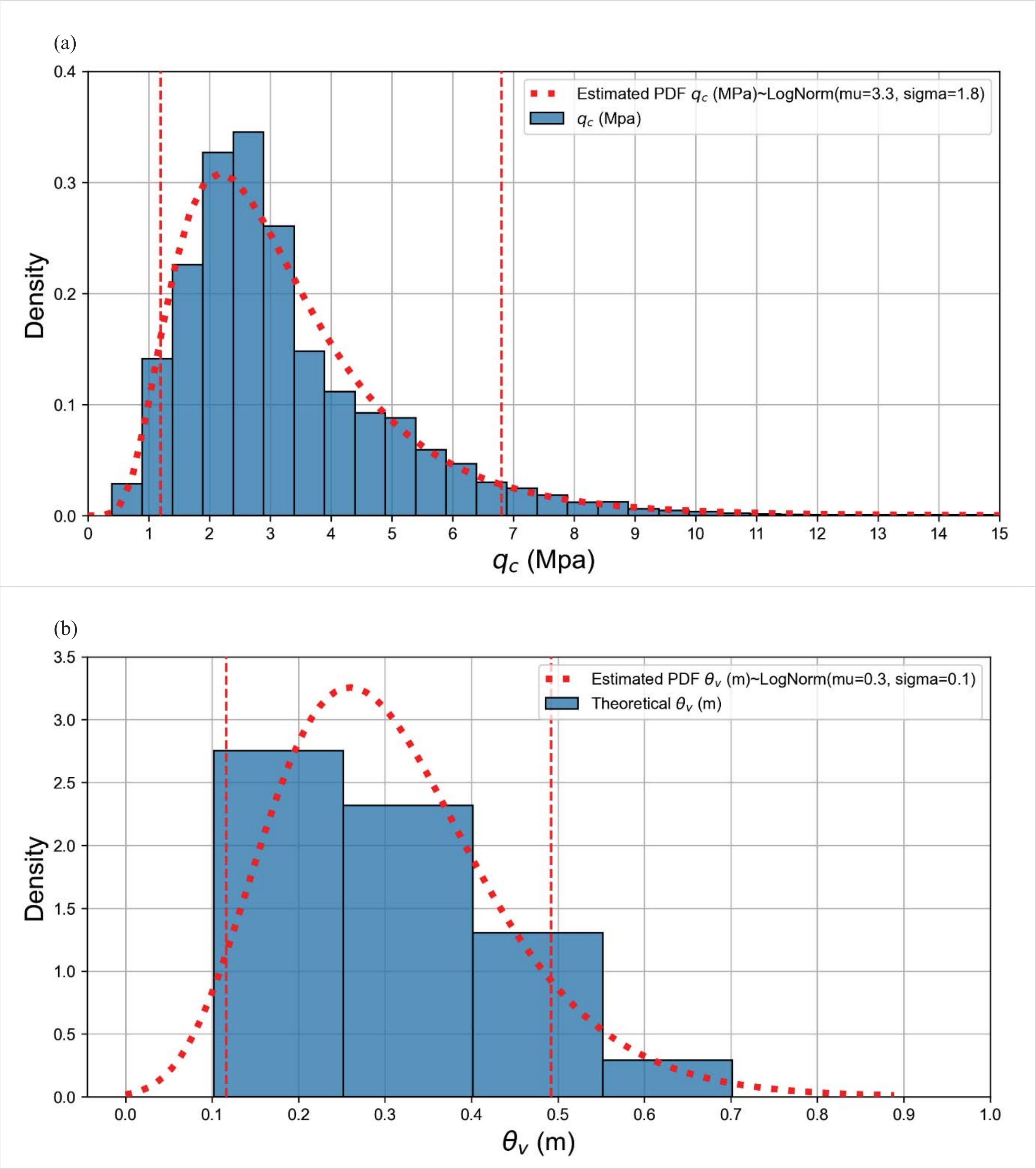


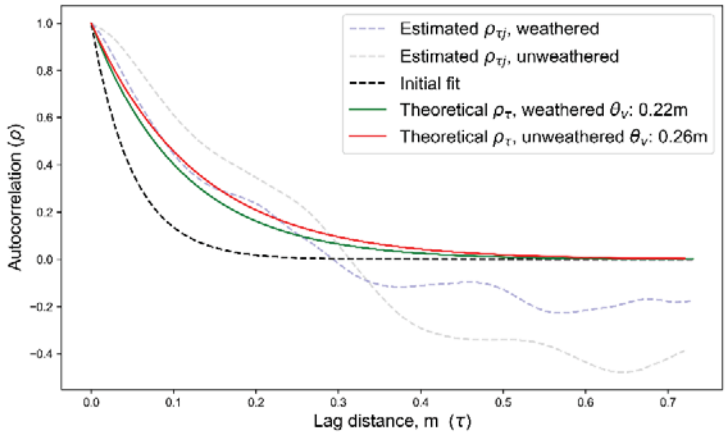
Figure 6. (a) Histogram and probability density function (PDF) of the point tip resistance q_c for the selected forty-six CPT profiles. The best-fit theoretical PDF (log-normal function) is shown in red with parameters mean of 3.3 MPa and a standard deviation of 1.8 MPa (b) Histogram and PDF of the determined vertical scale of fluctuation, θ_v of q_c for the selected forty-six CPT profiles. The best-fit theoretical PDF (log-normal function) is shown in red with parameters mean of 0.3 m and a standard deviation of 0.1 m. The 5% and 95% percentile of the histograms are shown by the dashed vertical lines.

Table 1. A summary of the linear fit (coefficient, *a*, and intercept *b*, for the given fit) and scale of fluctuation results for the theoretical correlation function for each CPT profile

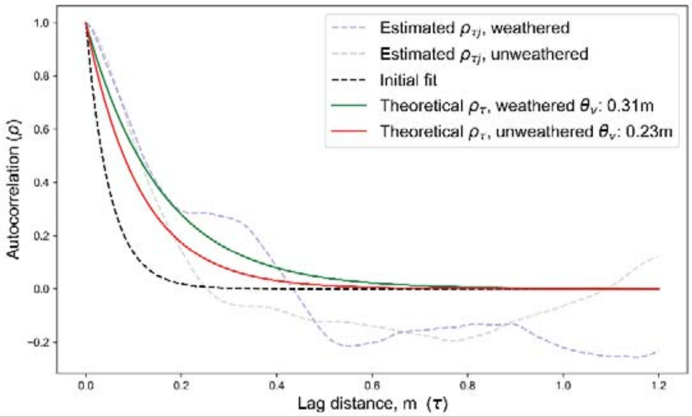
CPT test	Mean q_c (MPa)	COV q_c (%)	R^2 (a x $Depth + b$)	a	b	θ_v (m)
1	3.05	0.48	0.82	0.96	-0.43	0.40
2	2.69	0.64	0.47	0.85	-0.39	0.45
3	3.33	0.55	0.44	0.87	0.18	0.32
4	2.12	0.42	0.47	0.44	0.54	0.25
5	2.63	0.53	0.36	0.60	0.46	0.38
6	2.00	0.30	0.27	0.23	1.18	0.19
7	3.48	0.44	0.60	0.85	0.41	0.58
8	4.40	0.67	0.42	1.38	-0.57	0.10
9	4.43	0.48	0.83	1.40	-0.63	0.23
10	4.99	0.59	0.72	1.80	-1.51	0.12
11	5.03	0.65	0.71	1.99	-2.14	0.40
12	4.26	0.56	0.80	1.53	-1.25	0.31
13	4.31	0.55	0.73	1.47	-1.00	0.44
14	4.43	0.53	0.63	1.34	-0.40	0.42
15	5.64	0.51	0.36	1.24	1.17	0.12
16	4.66	0.28	0.72	0.80	1.77	0.23
17	2.13	0.44	0.72	0.57	0.08	0.47
18	3.21	0.23	0.45	0.36	1.91	0.19
19	2.53	0.36	0.49	0.46	0.86	0.48
20	3.03	0.35	0.63	0.60	0.85	0.43
21	2.60	0.33	0.86	0.57	0.55	0.11
22	2.45	0.31	0.78	0.48	0.73	0.14
23	2.19	0.37	0.83	0.54	0.25	0.34
24	2.82	0.29	0.78	0.52	0.94	0.31
25	2.36	0.33	0.74	0.49	0.61	0.36
26	2.79	0.30	0.55	0.45	1.17	0.69
27	3.03	0.50	0.77	0.95	-0.45	0.35
28	1.96	0.47	0.81	0.60	-0.22	0.24
29	3.04	0.44	0.51	0.69	0.55	0.35
30	3.02	0.49	0.52	0.76	0.26	0.36
31	3.57	0.83	0.35	1.27	-1.01	0.18
32	3.72	0.38	0.59	0.78	0.91	0.19
33	2.60	0.36	0.60	0.56	0.65	0.34
34	2.53	0.28	0.27	0.27	1.56	0.16
35	2.73	0.39	0.77	0.67	0.31	0.34
36	2.70	0.36	0.44	0.46	1.07	0.38
37	2.22	0.41	0.76	0.57	0.17	0.19
38	2.95	0.38	0.81	0.71	0.35	0.21
39	3.36	0.68	0.42	1.08	-0.54	0.16
40	4.12	0.67	0.64	1.59	-1.62	0.22
41	3.25	0.42	0.77	0.87	0.11	0.43
42	3.40	0.43	0.75	0.92	0.09	0.33
43	3.51	0.43	0.83	1.00	-0.09	0.27
44	3.42	0.45	0.76	0.98	-0.10	0.50
45	3.36	0.37	0.86	0.82	0.39	0.25
46	3.55	0.44	0.82	1.01	-0.11	0.25
mean	3.25	0.45	0.64	0.86	0.17	0.31
COV	27.02	28.4%	27.4%	-	-	42.8%
Min.	1.96	0.23	0.27	-	-	0.10
Max.	5.64	0.83	0.86	-	-	0.69

A comparison between the weathered and unweathered CPT profiles showed limited differences in the magnitude of the scale of fluctuation for the weathered and the unweathered layers. The results are shown in Figure 7. Both the weathered and unweathered layers showed low magnitudes of the vertical scale of fluctuation of q_c, indicating high vertical variability of q_c values.

(a) CPT1



(b) CPT2



(c) CPT3

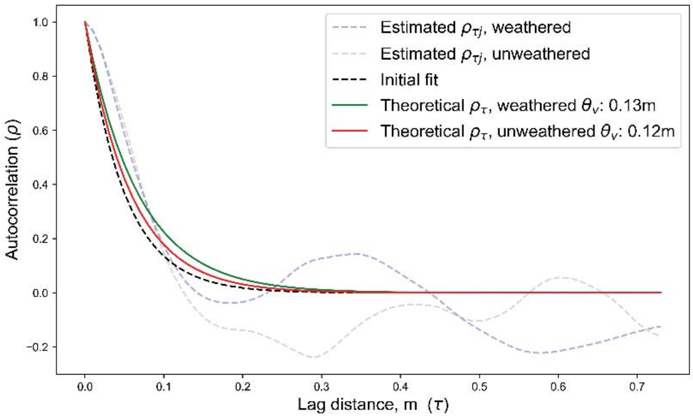


Figure 7. Estimated and theoretical values of the vertical scale of fluctuation for weathered and unweathered mudstone layers in a one-to-one comparison for three different tests (a) CPT1, (b) CPT2, and (c) CPT3.

4 Conclusions

Measurements of the cone tip resistance q_c from CPT profiles were used to characterize the variability of weathered mudstone profiles through random field theory. The vertical scale of fluctuation of the profiles and their coefficient of variation were compared with published values for corresponding materials. The results showed that the mudstone is variable throughout the vertical profile. The vertical scale of fluctuation of q_c for the weathered mudstone profiles varied between 0.10 m and 0.69 m, with an estimated theoretical average of 0.30 m (based on the Markov exponential function) and a coefficient of variation equal to 42%. There was no distinction between the vertical scale of fluctuation values in the weathered mudstone and the underlying unweathered mudstone. Low values of the scale of fluctuation indicate variability of the profile over small distances. This can inform numerical analyses using probabilistic approaches, such as reliability-based assessment of data that might form the basis of design analysis for engineering structures in weathered mudstone.

Acknowledgments

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