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# Slope Reliability Analysis Based on Deep Learning of Digital Images of Conditional Random Fields Using CNN

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**Abstract:**Considering the site investigation data and the spatial variability of soil strength, a deep learning model of conditional random field (CRF) characteristics is proposed for reliability analysis of slope stability. The random fields of soil slope are discretized by Karhunen-Loeve Expansion(KLE) method and constrained by the Cone Penetration Tests (CPTs) using the kriging interpolation method. The discretization results are converted into digital images. Then, a Convolutional Neural Network (CNN) surrogate model is established to approach the implicit relationship between the images and responses of the performance function. Based on the surrogate model, the reliability index of the slope is calculated. Finally, the effectiveness of proposed method is demonstrated by a single layer saturated clay slope under undrained condition. The result shows that the proposed method can reduce the spatial uncertainty, and significantly reduce the computational cost of slope reliability analysis.

Keywords: spatial variability; slope reliability analysis; Convolutional Neural Networks; digital image; surrogate model; conditional random field.

## 1 Introduction

The mechanical properties of soil vary spatially because of the different physical and chemical effects in the geological history. In the reliability analysis of geotechnical engineering, it is pointed out that the spatial variability of soil parameters is important for reliability index calculation (Griffiths and Fenton 2004). Therefore, random field theory is adopted in the slope stability model to characterize the variability of soil parameters.Griffiths and Fenton (2004) investigated the effects of spatial variation of the undrained cohesion on the slope system reliability using a random finite element method (RFEM). Ji et al. (2012) proposed two 2-D random field discretization methodsknown as interpolated autocorrelations and autocorrelated slices for slope reliability analysis.

Most of the studies in the slope reliability analysisfocus mainly on the unconditional random field (URF) simulation. Site investigation data, albeit limited, generally exist in an engineering project, which can be considered as constraints in the random field simulations. Conditional random field (CRF) can be used to parameterize soil properties known at these selected locations. Li (2016) performed 3D slope stability analysis based on CRF and investigated the optimization for the layout of Cone Penetration Test (CPT) through sampling data. Liu(2017) adopted CRF modeling with five-point in-situ test data and investigated theperformance of CRF simulation according to sampling distance and autocorrelation distance. Huang (2017) conducted slope probabilistic analysis by CRF and RFEM, the effects on reliability indices are also investigated.

The objectives of this paper are to presentthe application of CNN surrogate model in slope stability analysis that account forthe known data from CPT tests and efficiently evaluate the reliability of a slope based on the proposed method. The study on the effects of different layouts of CPT tests on the CRF simulation is carried out. To achieve these objectives, the simulations of the unconditional and conditional random fields are respectively introduced. Then the surrogate model of probabilistic analysis utilized in this study is described. The implementation procedure for the proposed conditional probabilistic analysis of slope stability is then introduced in detail. The stability of a hypothetical saturated clay slope under undrained condition is evaluated as an example to illustrate the proposed method.

## 2 Methodology

### 2.1 Simulation of unconditional random field

Random field theory has been extensively applied to characterized soil properties in the slope stability analysis. In order to consider the spatial variability of soil parameters, it is necessary to clarify the mean, standard deviation and autocorrelation function of soil parameters. The 2-D exponential autocorrelation function is utilized:

$$\rho\left(\tau_h,\tau_v\right)=exp\left(-\tau_h/L_h-\tau_v/L_v\right) \tag{1}$$

where $\tau_h$ and $\tau_v$ denote lag distance in the horizontal and vertical directions, respectively. $L_h$ and $L_v$ denote the horizontal and vertical autocorrelation distances of a random field, respectively.

The Karhunen-Loeve Expansion(KLE) method is a random field discrete method based on the spectral decomposition of a bounded, symmetric and positive definite autocorrelation function(Ghanem 2003). For the truncated KLE, a stationary lognormal random field $H(x)$ is expressed as

$$H(\boldsymbol{x})=exp\left(\mu_{ln\,X}+\sigma_{ln\,X}\sum_{i=1}^{M}\sqrt{\lambda_i}\varphi_i(\boldsymbol{x})\xi_i\right) \tag{2}$$

where $\mu_{lnX}$ and $\sigma_{lnX}$ denote the mean and standard deviation of the lognormal variable, respectively; $x$ is the location coordinate of the discrete random field; $M$ is the number of truncated terms; $\lambda_i$ and $\varphi_i(\cdot)$ are the eigenvalue and eigenfunction of the autocorrelation function, respectively. $\xi_i$ is a set of uncorrelated random variables.

### 2.2 Conditional random field

Conditioning methods include the geostatistics approach, the Hoffman method and the Bayesian method (Jiang 2022). Kriging based geostatistics approach is adopted in this study. A conditional random field, which preserves the known values at the measurement locations, can be formed from three different fields (Journel 1978; Gordon and Griffiths 2008):

$$Z_c(x)=Z_k(x)+(Z_u(x)-Z_{uk}(x)) \tag{3}$$

where $x$ denotes a location in the 2-D domain of interest; $Z_c(x)$ is the conditional random field; $Z_k(x)$ is the kriged field based on measured values at sampling locations; $Z_u(x)$ is an unconditional random field; $Z_{uk}(x)$ is the kriged field based on URF simulation values at the same positions.

The kriged fields are obtained by krigingmethod, which is an interpolation technique originally developed in mining engineering. The technique is a "Best Linear Unbiased Estimation" (BLUE) interpolator, which gives the best estimation with respect to the variance in the expected error between the estimation and generic field. The kriged interpolation of a point, $Z_k(x_0)$, based on the known (conditioning) points $Z(x_i)$, is given by (Cressie, 1990)

$$Z_k(x_0)=\sum_{i=1}^{n}\beta_i Z(x_i) \tag{4}$$

where $\beta_i$ are the weights for the linear combination of the known points $Z(x_i)$ with $\sum_{i=1}^{n}\beta_i=1$( in order to ensure that the estimator is unbiased). The weights can be determined by minimizing the variance ($\sigma^2$) of the kriging error, which is given as

$$\sigma^2=\begin{pmatrix}\lambda_1\\ \vdots\\ \lambda_n\\ \mu\end{pmatrix}\begin{pmatrix}\gamma(x_1-x_0)\\ \vdots\\ \gamma(x_n-x_0)\\ 1\end{pmatrix} \tag{5}$$

where $\gamma(x_n-x_0)$ denotes the variogram between location $x_n$ and $x_0$. To minimize the variance, the Lagrange method is used, and $\mu$ is the Lagrange parameter(Wackernagel 2003). Note that the estimation variance is independent of the values at the known points $Z(x_i)$.

The kriging method ensures that the values at the known points remain unchanged. In-situ tests provide solid evidence that the shear strength parameters of soil can be well estimated (Motaghedi 2014, Ma 2016). Hence, it is assumed that the cohesion and friction angle can be obtained by CPT tests and the transformation is not further discussed in this paper.

### 2.3 Surrogate model

#### *2.3.1 Convolutional Neural Networks (CNNs)*

CNN is a kind of neural network specially designed to process data with grid-like topology structure (e.g. a picture)(LeCun 2015). The major components of a CNN are the input layer, convolutional layer, pooling layer, activation layer, dropout layer, fully connected layer and output layer. The architecture of a CNN refers to the configuration of these layers.

The traditional input of a CNN model is colour digital image, which is made of pixels. Each pixel has three channels that correspond to the three primary colours, i.e. red, green and blue. Similarly, the input of the surrogate model are channels representing the soil properties, each type of soil property is described by a random field.If more than one property is characterized, the same number of channels are used as input. To map the position accurately, the inputs of the current study are grayscale images converted from random fields. After convolution, pooling, activation and other operations, a regression model of high-level features and outputs is established, which could be used to obtain the slope factor of safety from digital image. A detailed description about CNN layers can be found in Cha (2017).

*2.3.2 Training of CNNs*

The CNN training is a process of parameter optimization, which consists of hyperparameter optimization and weight optimization. The hyperparameters (e.g. the network architecture and training options)in the current studyare optimized by trial-and-error method and Bayesian optimization method. Most of the hyperparameters such as network structure, filter size and loss function are determined by trial-and-error method and some important hyperparameters such as initial learning rate are automatically determined by Bayesian optimization. The optimization aims at minimizing the loss function of the validation set, 30% of the data will be used for validation to improve the accuracy during hyperparameter optimization. The loss function used in this paper is Mean Absolute Error (MAE), which is given by

$$Loss = \frac{1}{N}\sum_{n=1}^{N}\left(\frac{1}{R}\sum_{i=1}^{R}\left|Y_{ni}-T_{ni}\right|\right) \tag{6}$$

where $N$ is the number of samples in the mini-batch;$R$ is the number of responses;$Y$ and $T$ are the predicted value and the true value, respectively.

Weights and biases are the parameters to be optimized (learned) in the network training. The Adam algorithm is employed to update the network parameters by minimizing the same loss function (Kingma2014). For the purpose of preventing overfitting, the dropout layer, the $L_2$ Regularization and 5-fold cross-validation are applied in the training process. The MATLAB CNN Toolboxis used for implementation in the present study.

### 2.4 Probabilistic analysis of a slope based on CNN

After the CNN surrogate model is established, simulation methods can be used to calculate the reliability index. Latin hypercube sampling (LHS)is employed to generate samples on account of its sampling efficiency. Taking slope stability analysis as an example, the performance function is given by

$$g\left(\boldsymbol{X}\right)=F_s\left(\boldsymbol{X}\right)-1 \tag{7}$$

where $X$ denotes a random variable of soil property. The random field images are preprocessed and fed into the CNN surrogate model for the factor of safety (*FS*) calculation. The probability of failure ($P_f$) is given by

$$P_f = P\left(g\left(\boldsymbol{x}\right)<0\right) \approx \frac{n'\left(F_s\left(\boldsymbol{x}\right)<1\right)}{n} \tag{8}$$

where $n$ is the number of total simulations. The corresponding reliability index is given by

$$\beta \approx -\Phi^{-1}\left(P_f\right) \tag{9}$$

where $\Phi^{-1}$is the inverse function of the standard normal cumulative distribution function. The implementation procedure is as follow:

**Step 1**: Performing preliminary survey to obtain the CPT data and statistics of the variable properties. CRF samples are generated according to the known data;

**Step 2**: Constructing the numerical model of the slope to calculate the associated *FS* against slope failure;

**Step 3**: The $N$ pairs of random fields and their associated *FS* values are then divided into a training set and a validation set. The architectured CNN is set up, followed by training and validation datasets training;

**Step 4**: After sufficient training and proper validation, the random field numerical model is replaced by the trained CNN to predict the *FS* values associated with the CRFs generated using LHS. These results are then used to evaluate the $P_f$ of the slope.

The numerical analysis is carried out on FLAC3D 6.0 software. The other work in the present study is implemented on the MATLAB platform.

## 3 Illustrative Example

### 3.1 Basic model

For illustration, this section applies the proposed CNN-based conditional probabilistic analysis approach to evaluate the probabilistic response of a hypothetical saturated clay slope under undrained conditions. The slope has also been successively studied by Cho (2010) in the literature. As seen from Figure 1, the slope has a height of 5 m and a slope angle of 26.6°, and the slope consists of a single soil layer with a unit weight of 20 kN/m$^3$. The mean value of cohesion($c$)is23 kPa (friction angle $\varphi = 0$). The elastic modulus $E$ is 100 MPa, the Poisson's ratio is 0.3. Based on the mean values of $c$, the deterministic slope stability model is preliminarily established using finite difference method (FDM), which provides a similar $FS$ value (1.332) to the values (1.322) calculated by Cho (2010) with the same method. The corresponding slip surface is also shown in Figure 1.

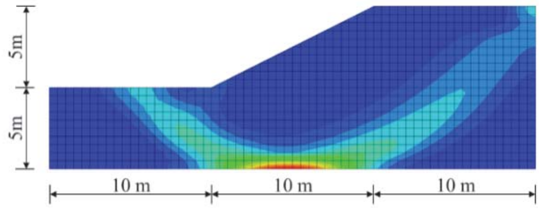



**Figure 1.**Deterministic analysis of the hypothetical slope model

Uncertainty in the undrained shear strength is considered for the whole domain, with lognormal distribution and COV of 0.3. Following Huang (2015), element size should be less than half of the autocorrelation distance, so the random field domain is first discretized into 910 elements. Then, the exponential autocorrelation functionEq. (1)is selected to characterize the spatial correlation structure, in which the horizontal and vertical autocorrelation distances are chosen as $L_h$ = 20 m and $L_v$ = 2 m, respectively (Cho 2010, Ji 2021).

For comparison purpose, probabilistic analysis based on URF simulation is carried out first. One hundred URFs are generated using KLE with 200 truncation terms. Figure 2 shows a typical realization of URFfor cohesion. Figure 3 shows its grayscale image after preprocessing, the size of the image is 20×60×1, and each grid is a pixel. The range [0,1] of the grayscale value of the pixel corresponds to the value interval of $c$ [0,50] kPa. According to the distribution of $c$, this range covers at least 99.9% of the possible values. These 100 sets of samples will be used for the establishment of the CNN surrogate model.

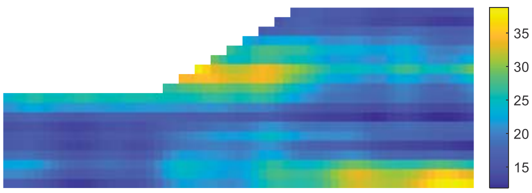




**Figure 2.**Typical realization of URF for cohesion**Figure 3.** The correspondinggray image of typical URF

### 3.2 Reliability results based on unconditional random fields

The probabilistic analysis of basic model is performed using CNN. The network architecture design of CNN is shown in Figure4, with a total of ten layers. Hyperparameters determined by Bayesian optimizer are$N_F$=16, learning rate=0.001 and dropout rate=0.24. The architecture of CNN can be replaced by any reasonable choice as long as effective features are extracted by convolutional operations (Wang 2021).

This study considers 70% of the total dataset as the training set while the remaining 30% as the validation set. Figure 5. shows the training process of CNN, which is the loss function value and RMSE of the training set and the validation set with the number of iterations. It can be seen that after the first 500 iterations, the errors on the training and validation set both drop rapidly and stabilize. After 2806 training epochs, the training stops automatically.

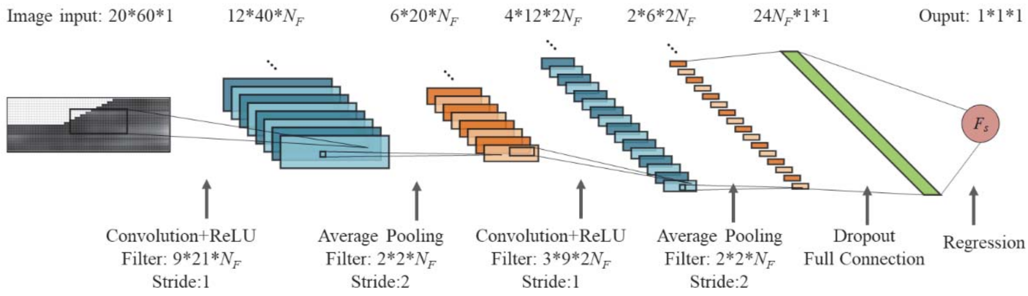
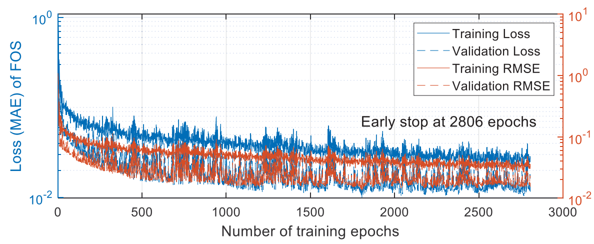



**Figure 4.**Architecture of the CNN

After the CNN surrogate model is established, a total of 2000 random field grayscale images were generated and fed into the CNN model to obtain the corresponding response (*FS*). The $P_f$and reliability index of the slope are calculated according to Eq. (8-9). The predicted *FS* and $P_f$ by CNN are respectively 1.236 and 0.107, which is similar to 1.240 and 0.104 by FDM. The same strategy is used for CRF probabilistic analysis.

In the current study, each random field FDManalysis takes approximately 130 seconds on a single CPU computer with a 6-core Intel (R) Core(TM) i5-8400@2.80 GHz and a RAM of 8GB. The training of the CNN takes approximately 10 mins, while the time required for the trained CNN to make predictions for the 2000 samples is approximately 20 s. Compared with the LHS simulation method that requires a large number of FDM analyses, CNN greatly improves the computational efficiency.



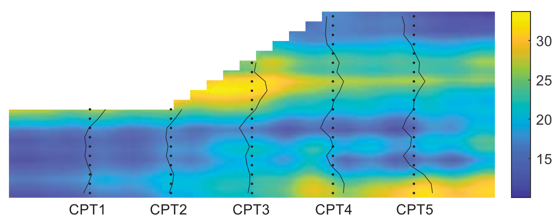




**Figure 5.**Typical training progress of CNN**Figure6.**Typical realization of CRF

### 3.3 Reliability results based on conditional random fields

The typical realization of Figure 2 is chosen as the baseline model in the CRF simulation, which representing the ‘actual’ in-situ variability. The ‘measured’ cohesion data is extracted from the baseline model using CPTs in the form of a regular grid. As shown in Figure 6, five CPT measurement locations in the *x* direction referred to as CPT1-5 (at $x$ = 5, 10, 15, 20, 25m) are available, comprising 10, 10, 15, 20, 20 data points at 0.5 m spacing in the vertical direction. The cohesion sampled from CPTs does not represent the exact cohesion distribution in the vertical direction because of the limited number of samplings. The CRF generated from CPT data by kriging based geostatistics approach is also shown in Figure 6.

The FDM analysis is performed to obtain the ‘accurate’ *FS* of the baseline model, which is 1.152.To obtain the $P_f$ of the baseline model accounting for known data, three sampling scheme are adopted in this paper: 1) sampling at the location of CPT3; 2) sampling at 3 locations of CPT2-4; 3) sampling at 5 locationsof CPT1-5. For each sampling scheme, one hundred CRFs are generated and fed into the CNN model for training while 2000 CRFs are generated for prediction. The stability analysis of FDM is performed on 2000 samples to calculate ‘real’ $P_f$, which is 0.1445, 0.036 and 0 for scheme 1-3, respectively. To demonstrate the consistency of the *FS* between CNN prediction and FDM, another 100 independent CRFs are generated and analyzed by CNN and FDM respectively. The results are shown in Figure7. For comparison purpose, we shrink the size of training set to 50 and prediction set remains unchanged. The results of *FS* and $P_f$are shown in Table 1.

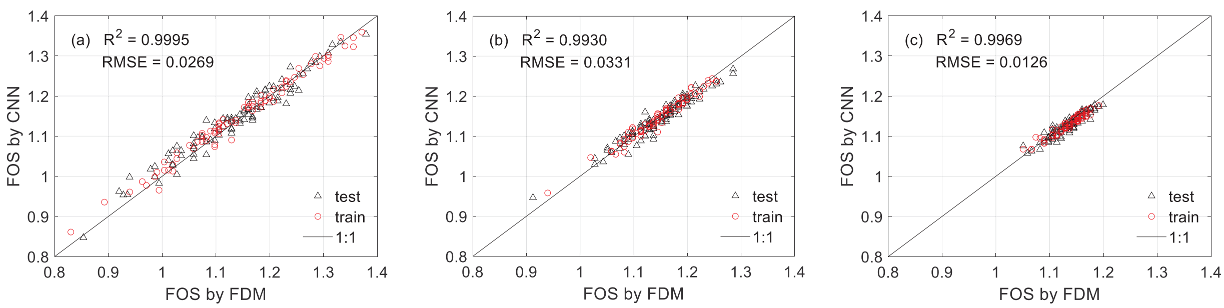



**Figure 7.** FDM versus CNN predictions of *FS* (a) 1 CPT (b) 3 CPTs (c) 5 CPTs

It can be seen from Figure7 that the predicted *FS*s of CNN are in reasonably well agreement with FDM analysis. The RMSE values are all smaller than 0.04 while the coefficients of determination($R^2$) are all greater than 0.99, thus indicating a strong correlation between the CNN and FDM predictions. As for different sampling schemes, the range of *FS*in Scheme 1 is from 0.8 to 1.4;the range of *FS* in scheme 2 is about 0.9 to 1.3; the range of *FS* in scheme 3 is about 1.0 to 1.2. The decreasing trend of the range for*FS* indicating that the more CPTs are planned, the less variability remains.

As shown in Table 1, the predicted values of *FS*are basically in the range of 1.13 to 1.14, which are similar to the baselinemodel (1.152) while the *FS* obtained by deterministic analysis ignoring CPT data is 1.332, indicating that reliable *FS* predictions can be made with less data. In addition, the standard deviation of *FS* decreases with increasing number of CPTs, indicating that the conditioning method decreased the simulation variability to a certain extent. Compared with the $P_f$ of URF analysis (0.107), $P_f$(0.138) is larger when 1 CPT is used for probabilisticanalysis; when 3 CPTs are used, the $P_f$ is smaller (0.038) and when the number of CPTs increases to 5, the $P_f$ is 0. Considering that the baseline model *FS* is greater than 1, the prediction of $P_f$ may be misleading when less data is available, but it is more reasonable when the amount of data increases. Moreover, the reliability index is acceptable when 50 samples are employed though $R^2$ of the CNN model are less convincing. In this regard, the employment of known data shows promising prospects in reducing the number of samples needed to train the model.

**Table 1.** Comparison of reliability results.

| Probabilistic approach | CPT data | Number of samples | *FS* | | $P_f$ | $\beta$ | $R^2$ |
|---|---|---|---|---|---|---|---|
| | | | Mean | Standard deviation | | | |
| LHS-FDM | 0 | 2000 | 1.240 | 0.205 | 0.104 | 1.259 | - |
| CNN-FDM | 0 | 100 | 1.236 | 0.203 | 0.107 | 1.243 | 0.9650 |
| | 1 (CPT3) | 100 | 1.138 | 0.125 | 0.138 | 1.092 | 0.9995 |
| | | 50 | 1.129 | 0.119 | 0.138 | 1.085 | 0.9983 |
| | 3 (CPT2-4) | 100 | 1.134 | 0.066 | 0.038 | 1.774 | 0.9930 |
| | | 50 | 1.137 | 0.071 | 0.045 | 1.751 | 0.7881 |
| | 5 (CPT1-5) | 100 | 1.132 | 0.024 | 0 | Inf | 0.9969 |
| | | 50 | 1.140 | 0.020 | 0 | Inf | 0.9429 |

## 4 Summary and Conclusion

This paper presents the use of CNN as a surrogate model to efficiently perform slope reliability analysis accounting for known data in spatially variable soils.Three schemes of sampling are performed on a hypothetical slope. As a result, the features of CRFs can be well learned by CNN.The CRF helps to reduce the variability in soil heterogeneity simulation, thus less samples are needed to construct the surrogate model. The reliability indices calculated accounting for known data are more accurate than that without CPT data.

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