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Random Finite Element Analysis for Reliability-Based Assessment of Cement-Treated Soil Columns

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Abstract: The paper presents the reliability-based assessment for cement-treated soil columns based on the analysis results of the finite element method with random field theory, random finite element method (RFEM). The analysis method in which the statistical uncertainty is considered in RFEM is adopted to assure the quality of the cement-treated soil column. In the analysis method, the realizations of the statistical uncertainty are estimated using a Bayesian inference method and the random fields of strength are generated with the realizations involving the statistical uncertainty. In this study, the mean μ_{qu}, the standard deviation σ_{qu} and the autocorrelation distance θ_{qu} of the unconfined compressive strength q_u are estimated using the data acquired in practical projects. The estimated statistical parameters of q_u are adopted to generate the random fields of strength for a cement-treated soil column model. In the RFEM analysis, the compression behavior of a cement-treated soil column is simulated to calculate the overall strength Q_u of a cement-treated soil column accounting for the spatial variability. The analysis results provide the probability characteristic of Q_u of the column. Based on the empirical accumulative distribution of Q_u, the reliability-based assessment is performed by calculating the probability of failure for the design strength.

Keywords: Cement-treated soil; Statistical uncertainty; Spatial variability; Random finite element method.

1 Introduction

The strength of cement-treated soil column by deep mixing method varies spatially owing to variabilities of in-situ soil properties, amount of injected cement slurry and mixing effectiveness. The critical issue of the deep mixing method is to rationally evaluate the influence of the spatial strength variability on the performance of the cement-treated soil ground. In quality assurance procedures of this method (e.g., CDIT 2002), the mean μ_{qu} and standard deviation σ_{qu} of unconfined compressive strength q_u of core samples retrieved from the constructed column are adopted to assure the quality of the improved ground. Since μ_{qu} and σ_{qu} of core strength are the sample statistical parameters, the statistical uncertainty emerges when estimating the population mean and standard deviation. Moreover, the spatial correlation of q_u affects the overall behavior of cement-treated soil columns.

The finite element method (FEM) with random field theory, random finite element method (RFEM), is a powerful tool for evaluating the overall behavior of cement-treated soil ground. RFEM has been performed to investigate the behavior of the cement-treated soil ground in past studies (e.g., Namikawa and Koseki 2013). The RFEM analysis results provide the probability characteristics of the performance of the ground. Based on the probability characteristics, a reliability-based assessment is possible in the quality assurance procedures. In the RFEM analysis, the random field of the strength is generated with the statistical parameters, i.e., μ_{qu}, σ_{qu} and autocorrelation distance θ_{qu}. When the statistical parameters are estimated from the core strength, the statistical uncertainty should be accounted for in the analysis.

The paper presents the reliability-based assessment for the cement-treated soil column based on the RFEM analysis results. The author (Namikawa 2021) has proposed the analysis method in which the statistical uncertainty is considered in RFEM. In the analysis method, the realizations of the statistical uncertainty are estimated using a Bayesian inference method, and the random fields of strength are generated with the statistical parameters involving the statistical uncertainty. In this study, μ_{qu}, σ_{qu} and θ_{qu} are estimated using the data acquired in practical projects. The estimated statistical parameters of q_u are adopted to generate the random fields of strength for a cement-treated soil column model. In the RFEM analysis, the compression behavior of a cement-treated soil column is simulated to calculate the overall strength Q_u of the column considering the spatial variability. The analysis results provide the probability characteristic of Q_u of the column. Based on the empirical accumulative distribution of Q_u, the reliability-based assessment is performed by calculating the probability of failure for the design strength.

2 Approach

Namikawa (2021) has proposed the RFEM analysis method in which statistical uncertainty can be considered. This analysis framework is illustrated in Figure 1. In this method, the realizations of μ_{qu}, σ_{qu} and θ_{qu} are estimated using a Bayesian inference method and the random fields of strength are generated with the realizations involving the statistical uncertainty. The RFEM analysis is performed for the generated realizations of the random fields. In this study, the probability characteristic of Q_u is estimated using this analysis method. The outline of the analysis method is given in this section.

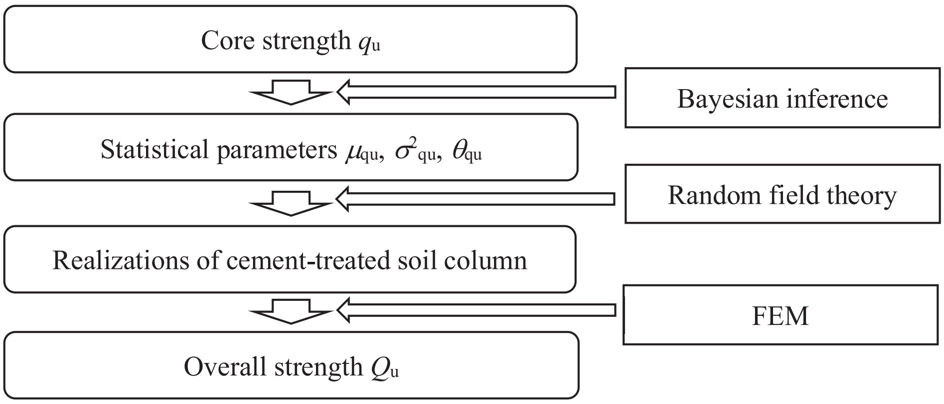


Figure 1. Analysis framework.

2.1 Bayesian inference of statistical parameters

2.1.1 Probability distribution of q_u

In this study, q_u is assumed to follow the multivariate normal distributions as follows:

$$p(\mathbf{q_u}|\mu_{qu},\sigma_{qu}^2,\theta_{qu}) = \frac{1}{\sqrt{(2\pi)^m(\sigma_{qu}^2)^m|\mathbf{C}|}}\exp\left\{-\frac{1}{2\sigma_{qu}^2}(\mathbf{q_u}-\boldsymbol{\mu}_{\mathbf{qu}})^T\mathbf{C}^{-1}(\mathbf{q_u}-\boldsymbol{\mu}_{\mathbf{qu}})\right\}$$

$$\mathbf{q_u} = \begin{bmatrix} q_u(\mathbf{r_1}) \\ \vdots \\ q_u(\mathbf{r_n}) \end{bmatrix}, \quad \boldsymbol{\mu}_{\mathbf{qu}} = \begin{bmatrix} \mu_{qu} \\ \vdots \\ \mu_{qu} \end{bmatrix}, \quad \mathbf{C} = \rho_{qu}(\mathbf{d}) = \exp\left(-\frac{|\mathbf{r_i}-\mathbf{r_j}|}{\theta_{qu}}\right) \tag{1}$$

where σ^2_{qu} is variance of q_u, m denotes the number of q_u, $\mathbf{C}$ denotes the correlation matrix, and $\mathbf{r_i}$ denotes the space vector at a point i. An exponential type autocorrelation function is assumed for the spatial variability of q_u.

2.1.2 Posterior probability distribution of statistical parameters

In Bayesian inference, the posterior probability distribution of the estimated values is defined as a product of the prior distributions and the sampling distribution of observed data. The joint probability distribution $p(\mu_{qu}, \sigma^2_{qu}, \theta_{qu}|\mathbf{q_u})$ after observing q_u values is described as follows:

$$p(\mu_{qu},\sigma_{qu}^2,\theta_{qu}|\mathbf{q}_u) \propto p(\mathbf{q}_u|\mu_{qu},\sigma_{qu}^2,\theta_{qu})p(\mu_{qu})p(\sigma_{qu}^2)p(\theta_{qu}) \tag{2}$$

where $p(\mathbf{q_u}|\mu_{qu}, \sigma^2_{qu}, \theta_{qu})$ is the likelihood function, and $p(\mu_{qu})$, $p(\sigma^2_{qu})$, and $p(\theta_{qu})$ denote the prior distributions of μ_{qu}, σ^2_{qu}, and θ_{qu}. The statistical parameters of the population can be estimated from $p(\mu_{qu}, \sigma^2_{qu}, \theta_{qu}|\mathbf{q_u})$.

2.1.3 Markov chain Monte Carlo method

An Markov chain Monte Carlo (MCMC) method is adopted to draw the realization values of μ_{qu}, σ^2_{qu}, and θ_{qu} from the joint probability distribution. In the MCMC simulation, the values of μ_{qu}, σ^2_{qu}, and θ_{qu} are sequentially sampled from the conditional posterior distributions described as follows:

$$\begin{aligned} p(\mu_{qu}|\sigma_{qu}^2,\theta_{qu},\mathbf{q}_u) &\propto p(\mathbf{q}_u|\mu_{qu},\sigma_{qu}^2,\theta_{qu})p(\mu_{qu}) \\ p(\sigma_{qu}^2|\theta_{qu},\mu_{qu},\mathbf{q}_u) &\propto p(\mathbf{q}_u|\mu_{qu},\sigma_{qu}^2,\theta_{qu})p(\sigma_{qu}^2) \\ p(\theta_{qu}|\mu_{qu},\sigma_{qu}^2,\mathbf{q}_u) &\propto p(\mathbf{q}_u|\mu_{qu},\sigma_{qu}^2,\theta_{qu})p(\theta_{qu}) \end{aligned} \tag{3}$$

In Eq. (3), the likelihood function $p(\mathbf{q_u}|\mu_{qu}, \sigma^2_{qu}, \theta_{qu})$ is calculated from Eq. (1) in which the previous realization values of each parameter are substituted. It is assumed that $p(\mu_{qu})$ follows a normal distribution and $p(\sigma^2_{qu})$ follows an inverse gamma distribution. Then, these prior distributions become the natural conjugate distributions. Thus, the Gibbs sampling can be used to draw the realization values of μ_{qu} and σ_{qu} in the MCMC simulation. It is difficult to select the natural conjugate distribution for $p(\theta_{qu})$. The truncated normal distribution is selected for $p(\theta_{qu})$ and the Metropolis-Hastings algorithm is adopted to draw the realization values of θ_{qu}. The MCMC framework used here has been described in detail in another publication (Namikawa 2019).

2.2 Random finite element method

The realizations of the random field of q_u are generated by the covariance matrix decomposition method. In this method, the random field in the presence of spatial autocorrelation is generated from a production of a lower triangle of **C** and a standard normal random variable vector. Normally, the random fields are calculated with the constant values of μ_{qu}, σ_{qu}, and θ_{qu}. In this study, since the statistical uncertainty is considered in the RFEM analysis, the realizations of the random field are generated with the μ_{qu}, σ_{qu}, and θ_{qu} values calculated in the MCMC simulation. Thus, the μ_{qu}, σ_{qu}, and θ_{qu} values vary for each realization of the random field of q_u. The random variables in the random field are assigned to elements in the FEM analysis.

A three-dimensional FEM analysis is performed to calculate the unconfined compressive strength Q_u of the full-scale cement-treated soil column. The FEM software DIANA is used in the analysis. A cement-treated soil column of 1 m in diameter and 2 m in height is modeled as shown in Figure 2. A mesh consists of eight-node isoparametric elements. Most of the elements are cubic with a side length of 100 mm. The boundary conditions are smooth at the top and bottom of the model. A uniform displacement is applied at the top surface in the vertical direction during the loading process.

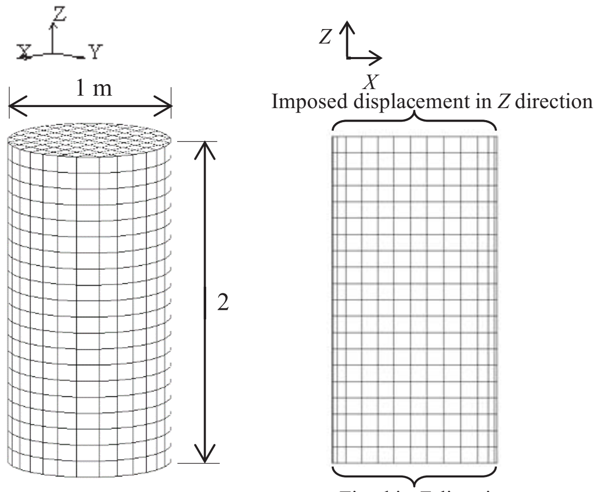


Figure 2. Finite element mesh.

Table 1. Material parameters for cement-treated soil with q_u = 2 MPa.

Parameter	Stochastic or Deterministic	Value	Parameter	Stochastic or Deterministic	Value
Elastic modulus E	Stoch.	3520 MPa	Hardening parameter e_y	Deter.	0.0002
Poisson's ratio ν	Deter.	0.167	Fracture energy G_f	Stoch.	10.6 N/m
Friction angle ϕ	Deter.	30 degree	Softening parameter e_r	Deter.	0.4
Cohesion c	Stoch.	0.577 MPa	Dilatancy coefficient D_c	Deter.	-0.4
Tensile strength T_f	Stoch.	0.448 MPa	Localization size t_{s0}	Deter.	0.6
Hardening parameter α	Deter.	1.05	Characteristic length l_c	Deter.	100

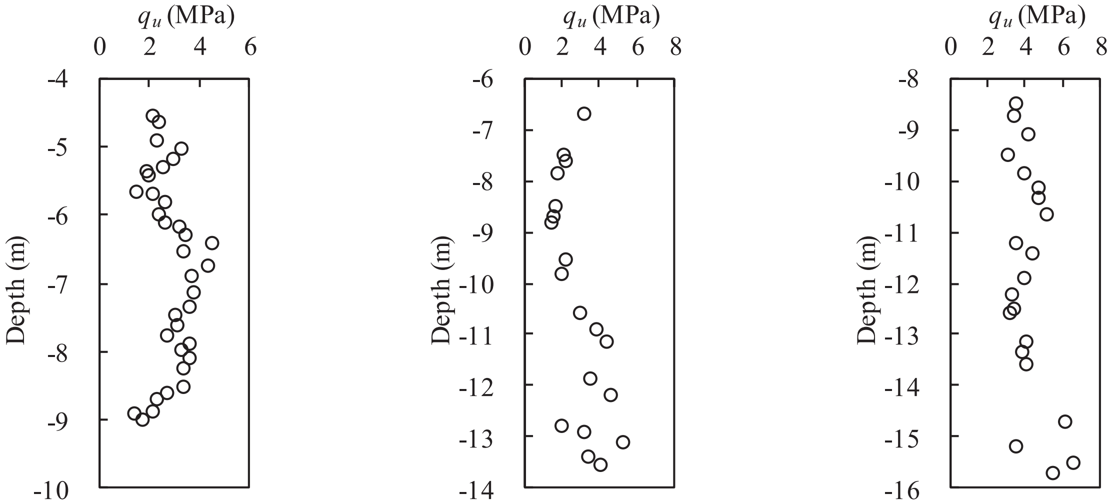
Namikawa and Mihira (2007) have proposed an elasto-plastic model for the mechanical behavior of cement-treated soils. This model that can appropriately describe the compressive and tensile failure behavior of cement-treated soils is adopted in the FEM analysis. The material parameter values of the elasto-plastic model are listed in Table 1. These values for q_u = 2 MPa are determined based on past studies (e.g., Namikawa and Mihira 2007). In the RFEM analysis, the q_u values for each element are calculated from the assigned random variables. Thus, the material parameter values should be determined from the q_u values assigned for each element. The elastic modulus E, cohesion c, tensile strength T_f, and fracture energy G_f are assumed to be stochastic parameters and vary with q_u proportionally. The other parameters, friction angle ϕ, Poisson's ratio ν, hardening parameter α, and e_y, softening parameter e_r, dilatancy coefficient D_c, localization size t_{s0}, and characteristic length l_c, are assumed to be constant. The determination of the material parameters has been described in detail in other publications (Namikawa and Koseki 2013; Namikawa 2021).

3 Analysis of Overall Strength

3.1 Core strength data in practical project

The core strength data obtained in two practical projects, Project 1 (Babasaki et al. 1996) and Project 2 (Namikawa et al. 2007), were used in the assessment. In these projects, cement-treated soil columns were constructed by the wet mechanical deep mixing method. The in-situ soils are clays in these Projects.

The core strength is shown in Figure 3. This figure shows the distributions of q_u of the core samples retrieved from one column in Project 1 and two columns in Project 2. The statistical sample size n and the sample values of μ_{qu}, σ_{qu}, and θ_{qu}, are shown in this figure. The θ_{qu} values were calculated using the maximum likelihood method. The determination of the θ_{qu} values has been described in detail in other publications (Namikawa and Koseki 2013).



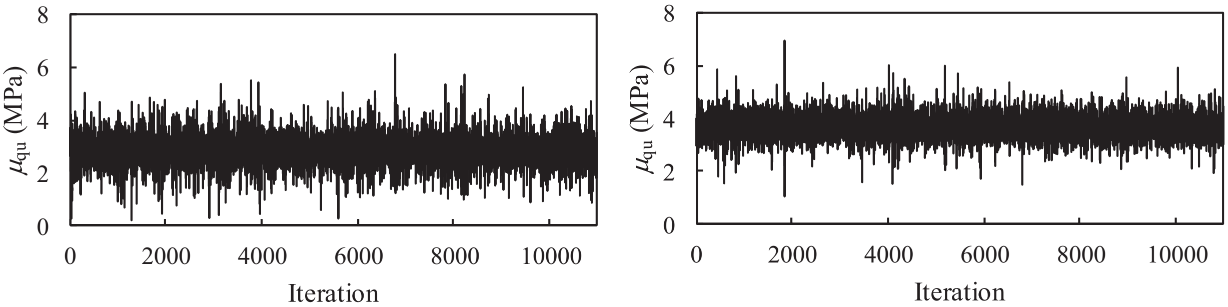
Sample values
μ_{qu} = 2.87 MPa
σ_{qu} = 0.772 MPa
θ_{qu} = 0.3 m
(a) Project 1, n = 34

Sample values
μ_{qu} = 3.59 MPa
σ_{qu} = 1.23 MPa
θ_{qu} = 0.6 m
(b) Project 2, n = 40

Figure 3. Distribution of core strength.

3.2 Bayesian inference for statistical parameter

The popular values of μ_{qu}, σ_{qu}, and θ_{qu}, were evaluated using the Bayesian inference method mentioned in the previous section. The MCMC simulation was performed to calculate the realizations of the μ_{qu}, σ_{qu}, and θ_{qu} values. Eleven thousand realizations of μ_{qu} generated in the MCMC simulation are shown in Figure 4. The variability of the generated μ_{qu} values for Project 1 is larger than that for Project 2. The variability of the inferred parameter values depends on n and the correlation between the data values. In Project 1, the core samples were retrieved from one column and the sampling interval was small, resulting in an equivalent number of independent observations (Cressie 1993) becoming small. Conversely, the core samples were retrieved from two columns and the equivalent number of independent observations becomes large in Project 2. Figure 4 indicates that the core sampling design affects the statistical uncertainty significantly. The coefficient of variation of the μ_{qu} realizations is 0.182 for Project 1, and that is 0.111 for Project 2.



(a) Project 1

(b) Project 2

Figure 4. Realizations of μ_{qu}.

3.3 RFEM analysis results

The RFEM analysis was performed for the random fields with consideration for the statistical uncertainty. When considering the statistical uncertainty, the values of μ_{qu}, σ_{qu}, and θ_{qu} drew by the MCMC method were adopted for generating the random field of q_u. 200 realizations of the random field of q_u were generated for each case in the RFEM analysis. The number of realizations were determined considering between the reliability of the simulation results and the calculation cost. Thus, 200 realizations of μ_{qu}, σ_{qu}, and θ_{qu} were adopted for generating the random field. 200 values of μ_{qu}, σ_{qu}, and θ_{qu} were selected from the realizations generated in the MCMC simulation. The initial 1000 values were discarded in the selection to avoid the influence of the initial values.

The RFEM analysis without the statistical uncertainty was also performed for comparison. In that analysis, the values of μ_{qu}, σ_{qu}, and θ_{qu} were held constant at the sample statistical values shown in Figure 3 when generating the random field for the FEM analysis. 200 realizations of the random field of q_u were generated for each case.

The Q_u values calculated by the RFEM analysis are shown in Figure 5. The mean μ_{Qu} and standard deviation σ_{Qu} of Q_u are shown in this figure. The statistical uncertainty does not affect the mean of Q_u significantly. The variability of Q_u estimated with the statistical uncertainty is much larger than that without the statistical uncertainty in each case. This indicates that the statistical uncertainty significantly affects the variability of Q_u.

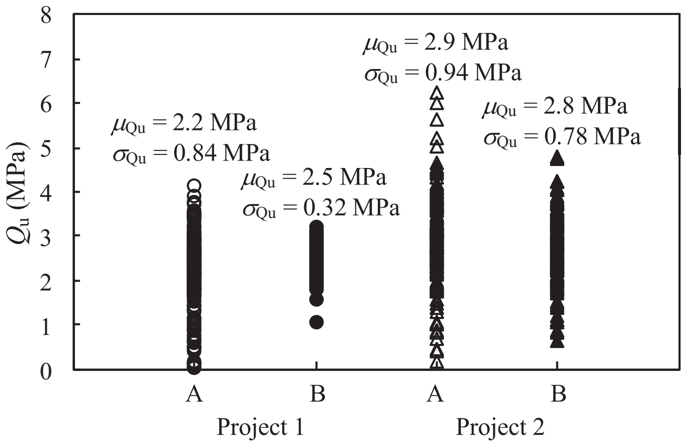


Figure 5. Overall strength of a cement-treated soil column Q_u; A denotes Q_u with statistical uncertainty and B denotes Q_u without statistical uncertainty.

4 Reliability-based Assessment for Overall Strength of Cement-treated Soil Column

The design strength q_{uck} is normally determined from the mean μ_{qud} and standard deviation σ_{qud} of q_u in the design process (CDIT 2002). q_{uck} is defined as

$$q_{\mathrm{uck}} = \mu_{\mathrm{qud}} - K\sigma_{\mathrm{qud}} \tag{4}$$

where K is the coefficient determined from a defect rate of the core strength. Normally, K is set to be 1.3 for a 90% defect rate of the core strength. The allowable compressive strength σ_{ca} is defined as

$$\sigma_{\mathrm{ca}} = q_{\mathrm{uck}}/F_{\mathrm{s}} \tag{5}$$

where F_s is the safety factor. Normally, F_s is set to be 3 for a static condition.

On the basis of the RFEM results, the probability of failure is evaluated for the design and allowable strength in each project. The μ_{qud} values are 1.49 MPa in Project 1 and 2.35 MPa in Project 2. When the coefficient of variation is set to be 0.3, the q_{uck} values are 0.909 MPa in Project 1 and 1.43 MPa in Project 2. The σ_{ca} values are 0.303 MPa in Project 1 and 0.476 MPa in Project 2. The empirical cumulative distribution functions (ECDF) for the calculated Q_u value is shown in Figure 6. When the statistical uncertainty is considered, the probabilities of failure for q_{uck} are 10% in Project 1 and 6% in Project 2. Those for σ_{ca} are 3.5% in Project 1 and 1.5% in Project 2. The sample mean values of q_u are 2.87 MPa in Project 1 and 3.59 MPa in Project 2. Although those values satisfy the design mean values, the RFEM analysis results indicate that the probability of failure becomes more than 1% for the static condition in both projects. When not considering the statistical uncertainty, the probability of failure for σ_{ca} becomes less than 0.5 % in both projects. This indicates that if the probability of failure calculated from the core strength data is not accepted, the design probability of failure and the additional core boring are required to reduce the statistical uncertainty in the reliability-based assessment.

q_{uck} can be regarded as a characteristic value. Eurocode 7 (CEN 2004) recommends, "If statistical methods are used, the characteristic value should be derived such that the calculated probability of a worse value governing the occurrence of the limit state under consideration is not greater than 5%". Based on the single-core boring data, the quality of the cement-treated soil does not satisfy the Eurocode 7 criterion in Project 1. The additional core boring is required for satisfying the Eurocode 7 criterion in Project 1. The statistical uncertainty depends on the equivalent number of independent observations. The reliability-based assessment indicates that when the probability of failure evaluated from the RFEM analysis with the observed core strength is not accepted, the additional core boring is recommended to reduce the statistical uncertainty.

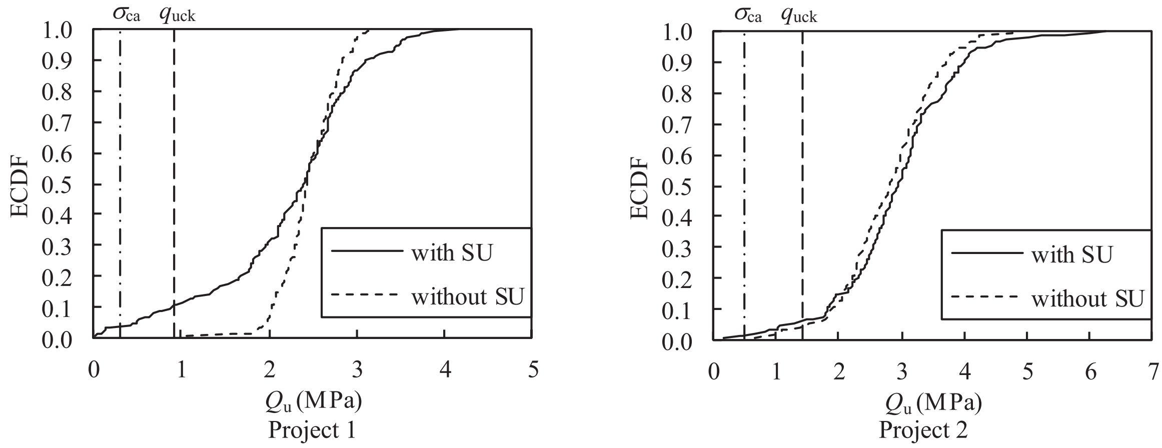


Figure 6. Empirical cumulative distribution function (ECDF) of Q_{qu}; with and without statistical uncertainty (SU).

5 Conclusions

The paper presented the reliability-based assessment for the cement-treated soil column based on the RFEM analysis results. Using the Bayesian inference method, the realizations of μ_{qu}, σ_{qu}, and θ_{qu} were estimated from the core strength acquired in practical projects. The estimated statistical parameters of q_u were adopted to generate the random fields of strength for a cement-treated soil column model. In the RFEM analysis, the compression behavior of a cement-treated soil column was simulated to calculate the overall strength Q_u of a cement-treated soil column.

The RFEM results showed that the statistical uncertainty significantly affects Q_u of a cement-treated soil column. The variability of Q_u estimated with the statistical uncertainty was much larger than that without the statistical uncertainty. Based on the calculated Q_u values, the probability of failure is evaluated for the design strength. The proposed reliability-based assessment framework provides the overall failure probability characteristic with considering the statistical uncertainty and the spatial variability.

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