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Probabilistic Slope Stability Analysis with Spatial Soil Variability Using Improved Multiple Kriging Metamodels

Lei-Lei Liu[1,*] and Shi-Ya Huang[2]

[1]Associate Professor, Key Laboratory of Metallogenic Prediction of Nonferrous Metals and Geological Environment Monitoring, Ministry of Education, School of Geosciences and Info-Physics, Central South University, Changsha 410083, P. R. China.
E-mail: csulll@foxmail.com
[2]PhD student, Key Laboratory of Metallogenic Prediction of Nonferrous Metals and Geological Environment Monitoring, Ministry of Education, School of Geosciences and Info-Physics, Central South University, Changsha 410083, P. R. China.
E-mail: huang_shiya@csu.edu.cn

Abstract: Recently, a multiple Kriging metamodel method has been proposed to simultaneously evaluate slope reliability and assess risk of slope failure within the framework of limit equilibrium method. However, the inherent spatial variability (ISV) of soil properties was not considered because of the “curse of dimensionality”. This study develops an improved multiple Kriging (IMK) method for efficient slope reliability analysis and risk assessment considering ISV of properties by a four-stage dimension reduction method. The proposed method proceeds with the simulation of random fields of soil properties with a small number of independent variables by using Karhunen-Loève expansion method. Then, representative slip surfaces (RSSs) of a slope are identified to reduce the number of Kriging models to be built, which is followed by using sliced inverse regression (SIR) to further reduce the number of variables. Thereafter, a Kriging metamodel is established for each RSS with the reduced random variables, while using a new sequential sampling strategy to actively learn and update the MK models to further improve the efficiency. A slope example is studied to illustrate the accuracy and efficiency of the IMK method for slope reliability analysis and risk assessment. The influence of the size of the training samples on the accuracy of the proposed model is discussed. The results show that the proposed IMK method performs accurately and effectively on slope reliability analysis and risk assessment considering ISV of soil properties.

Keywords: Slope reliability analysis; risk assessment; spatial variability; Kriging; active learning; representative slip surface; sliced inverse regression; Karhunen-Loève expansion.

1 Introduction

It is advantageous to use Kriging metamodel for slope reliability analysis (Zhang et al., 2011; Liu and Cheng, 2018) because it can be performed efficiently. The traditional single Kriging model is, however, unable to accurately assess the risk of slope failure. To this end, a novel method of multiple Kriging (MK) metamodels, which is constructed by establishing a unique Kriging metamodel for each potential slip surface (PSS) of a slope stability analysis model, has been recently proposed by the authors to solve this problem (Huang et al., 2021).

Although effective, the MK method cannot be directly applied to probabilistic slope stability analysis with considering the inherent spatial variability (ISV) of soil properties for the following three reasons. First, the construction of the Kriging metamodel may suffer from the curse of dimensionality because of the discretized large number of correlated random variables for representing ISV. Second, too much computational time is spent on building Kriging models for numerous trivial slip surfaces with high correlations, which is a kind of waste. Third, the design of experiments for calibrating MK metamodels is based on trials and errors, which is low in both efficiency and accuracy.

In view of the above three issues, a four-stage dimension reduction method is developed for slope reliability analysis and risk assessment considering ISV within the framework of the limit equilibrium method, which is abbreviated as IMK hereafter. The IMK method is a further improvement of the MK method, which is realized by integrating the Karhunen-Loève expansion (KLE), sliced inverse regression (SIR), representative slip surface (RSS) identification and active learning with the MK method. To explicate the proposed IMK method, this paper is organized as follows. Section 2 provides the introduction of the proposed IMK method. The proposed method is then illustrated and validated by a three-layered cohesive soil slope in Section 3, and finally comes the main conclusions of the study.

2 The IMK method

Conventionally, the employment of the Kriging metamodel to approximate the LEM slope stability model has two purposes: (1) improving the computational efficiency; (2) improving the accuracy of the metamodel. For the first purpose, the efficiency can be improved from two aspects. One is to reduce the dimension of input variables.

When the dimension of the input variables is large, the number of training samples required to construct the IMK metamodels is large accordingly, of which the computational cost sometimes even exceeds that for the direct MCS. Therefore, the proposed IMK method adopts the KLE to achieve the first dimensionality reduction and then performs the second dimension reduction based on SIR (Li et al., 2019). Another is to reduce the number of IMK metamodels. When the number of PSSs is numerous, the computational resources spent on building the metamodels are also abundant. Thus, this paper adopts the method proposed by Jiang et al. (2015) to identify RSSs, and then the IMK method only establishes the Kriging metamodels for the RSSs. For the second purpose, the IMK method proposes a new active learning method to sequentially update the Kriging metamodels by a significant domain and a new learning function *H*, and then the accuracy of the Kriging metamodels can be improved quickly and accurately.

To sum up, the overall process of applying the IMK method to probabilistic slope stability analysis consists of four stages: (1) discrete the random field by KLE to simulate the ISV; (2) identify the RSSs; (3) reduce the dimension of the input variables through SIR; and (4) establish the initial IMK metamodels for the RSSs based on the input variables after dimensionality reductions and further update the prediction results through active learning for the initial kriging metamodels. Each of the four stages is described in the following subsections.

2.1 Karhunen–Loève expansion

Due to weathering and sedimentation, soil parameters (e.g., cohesion c and friction angle ϕ) in nature present ISV. The ISV can be simulated by a random field, and the construction of a random field requires the known point statistics and correlation structures of soil parameters. The correlation structures are usually represented by an autocorrelation function (ACF), which can be spectrally decomposed by KLE. For a stationary lognormal random field $H(\boldsymbol{x})$, it can be expressed as:

$$H(\boldsymbol{x}) = \exp[\mu_N + \sum_{i=1}^{m} \sigma_N \sqrt{\lambda_i} \varpi_i(\boldsymbol{x}) \xi_i] \tag{1}$$

where $\boldsymbol{x}$ is the space coordinate; μ_N and σ_N are the mean and standard deviation of soil parameters, respectively; $\boldsymbol{\xi} = [\xi_1, \xi_2, ..., \xi_m]$ is a set of standard independent random variables, and *m* is the number of truncation terms depending on the precision required for the discretization of the random field (Phoon et al., 2002); λ_i and $\varpi_i(\boldsymbol{x})$ are the eigenvalues and eigenfunctions of the ACF, respectively, which can be obtained by solving the homogeneous Fredholm integral equation of the second kind. Noted that the squared exponential ACF is adopted in this paper.

2.2 Representative slip surface identification

In this subsection, A simple but effective method proposed by Jiang et al. (2015) is employed to identify the RSSs in this study. According to Jiang et al. (2015), since the realizations of random fields are different, the failure modes of the slope system may also be different, suggesting that the critical slip surface (CSS) may be different from realization to realization of the random fields. However, the distribution of all possible failure modes may not be as large as that of all PSSs because of the ISV. Therefore, the RSSs can be selected easily by performing the deterministic slope stability analysis (e.g., the Bishop's simplified method) for a number of N_S realizations of random fields to obtain N_{RSS} CSSs. It is worth noting that $N_{RSS}<N_S$ because the failure modes of the slope system are generally correlated.

2.3 Sliced inverse regression

After the above two stages, the random field is represented by *m*-dimension standard independent random variables $\boldsymbol{\xi}$, and N_{RSS} RSSs are identified. However, directly establishing a Kriging metamodel for the input variables after KLE may still suffer from the curse of dimensionality and the modeling time may be large. Therefore, SIR is used to further reduce the dimension of the input variables.

SIR is a commonly used dimensionality reduction method of a nonparametric model. It maps input variables from high-dimensional space to low-dimensional space through a sufficient dimension reduction subspace. Different from PCA (Principal Component Analysis) that only uses the input variable information to reduce the dimension, SIR fully considers the relationship between multiple input variables $\boldsymbol{\xi}$ and a single output variable (e.g., FS). It can be seen as an improvement of PCA in the context of regression, exploring the regression of input variables to output variables.

Therefore, for R_S sets of standardized samples $[\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, ..., \boldsymbol{\xi}_{R_S}]^{\mathrm{T}}$, the implementation steps of SIR are as follows: (1) sort the samples according to the corresponding output variables; (2) divide the samples into *s* slices as evenly as possible; (3) compute the sample mean of each slice; (4) perform the weighted PCA on the sample mean to obtain the eigenvalues and eigenvectors of the weighted covariance matrix; (5) identify the first *d* largest eigenvalues, and the corresponding normalized eigenvectors are the dimensionality reduction direction. More details about SIR can be seen in Li et al. (2019).

2.4 Active learning Kriging method

Based on the training samples $[\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, ..., \boldsymbol{\eta}_{R_S}]^{\mathrm{T}}$ after SIR dimensionality reduction, an initial IMK metamodel is first established for each RSS. At this time, the accuracy of the initial IMK metamodels may not be high due to dimensionality reduction. Therefore, a new sequential sampling strategy (Huang et al., 2022) is adopted to actively learn and update the initial IMK metamodels.

This new sampling strategy mainly consists of two parts: identification of significant domain and determination of additional training samples (ATSs). For the slope series system with multiple failure modes, the significant domain is the safe domain. However, the safe domain cannot be known in advance unless performing the slope stability analysis. Thus, the prediction $\hat{g}(\boldsymbol{\eta})$ using the Kriging metamodel is adopted as the substitute for the true-response value $g(\boldsymbol{\eta})$ to estimate the significant domain.

Before the identification of the significant domain, a large number of MCS samples are randomly generated as the initial sample pool Ω_{ini}, also known as the initial samples, which are substituted into the initial Kriging metamodels to obtain the corresponding Kriging predictions $\hat{g}(\boldsymbol{\eta})$. Based on the Kriging predictions, the significant domain can be defined as:

$$S_{sys} = \{\boldsymbol{\eta} \in \Omega_{ini} \left| \cap_i^{N_{RSS}} g_i(\boldsymbol{\eta}) \geq 0\right.\} \approx \left\{\boldsymbol{\eta} \in \Omega_{ini} \left| \cap_i^{N_{RSS}} \mu_{\hat{g}}^i(\boldsymbol{\eta}) \geq \delta\sigma_{\hat{g}}^i(\boldsymbol{\eta})\right.\right\} = \left\{\boldsymbol{\eta} \in \Omega_{ini} \left| \cap_i^{N_{RSS}} 1 - \Phi\left(\frac{0 - \mu_{\hat{g}}^i(\boldsymbol{\eta})}{\sigma_{\hat{g}}^i(\boldsymbol{\eta})}\right) \geq \Phi(\delta)\right.\right\} \quad (2)$$

where $\boldsymbol{\eta}$ is an input random variable vector after SIR dimensionality reduction, $\boldsymbol{\eta} = [\eta_1, \eta_2, ..., \eta_d]$; N_{RSS} is the number of the failure modes, also known as the number of RSSs herein; $g_i(\boldsymbol{\eta}) = FS_i - 1$ is the true-response value of the limit state function (LSF) of the ith RSS; FS_i is the factor of safety (FS) of the ith RSS.; $\mu_{\hat{g}}^i(\boldsymbol{\eta})$ is the mean of the predicted value $\hat{g}_i(\boldsymbol{\eta})$ by using the ith current Kriging metamodel; $\sigma_{\hat{g}}^i(\boldsymbol{\eta})$ is the corresponding variance of $\mu_{\hat{g}}^i(\boldsymbol{\eta})$; $1 - \Phi\left(\frac{0 - \mu_{\hat{g}}^i(\boldsymbol{\eta})}{\sigma_{\hat{g}}^i(\boldsymbol{\eta})}\right)$ represents the probability that $\hat{g}_i(\boldsymbol{\eta})$ is positive; and $\Phi(\delta)$ is determined as 80%. As such, the samples, which are located in S_{sys}, compose a candidate sample pool Ω_{can} from which the ATSs will be selected. Then, a new learning function H is used to determine the ATS and the new selection strategy based on the H function is as follows:

$$H = \left|\Phi\left(\frac{0 - \mu_{\hat{g}}^i(\boldsymbol{\eta})}{\sigma_{\hat{g}}^i(\boldsymbol{\eta})}\right) - 50\%\right| \quad (3)$$

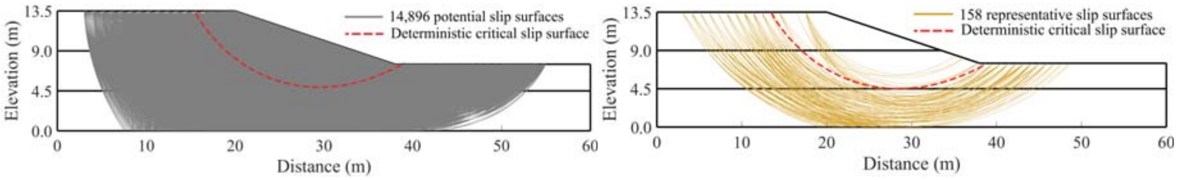
where $\Phi\left(\frac{0 - \mu_{\hat{g}}^i(\boldsymbol{\eta})}{\sigma_{\hat{g}}^i(\boldsymbol{\eta})}\right)$ is the probability that $\hat{g}_i(\boldsymbol{\eta})$ is negative, and the ATS can be determined as:

$$(k^*, \boldsymbol{\eta}^*) = \arg \min_{1 \leq k \leq N_{RSS}, \boldsymbol{\eta} \in \Omega_{can}} (H_k(\boldsymbol{\eta})) \quad (4)$$

where $\boldsymbol{\eta}^*$ is the ATS; and k^* is the index of the Kriging metamodel to be updated. The stop criterion of the H function is set as $H_{k^*}(\boldsymbol{\eta}^*) > 0.4986$. It is worth noting that the active learning is completed through iterations, and only one ATS is selected in each iteration.

3 Illustrative example

The proposed method is applied to the reliability analysis and risk assessment of a three-layered cohesive slope studied by Li et al. (2016) in this section. The cross-section of the three-layered cohesive slope is shown in Figure 1(a). The slope is 14.3 m high, and the slope ratio is 1:3. The strength parameters of the three undrained layers are considered spatially variable here and shown in Table 1, and the soil unit weight of the three layers is a constant of 18 kN/m^3. With the mean values of these parameters, a deterministic slope stability analysis of the slope is first carried out by Bishop's simplified method. A total of 14,896 reasonable arc-shaped PSSs are predefined in the model, as shown by the solid gray lines in Figure 1(a). Based on the mean values of s_{u1}, s_{u2} an d s_{u3}, the minimum FS is calculated as 1.281 and the critical deterministic slip surface is shown by the red dashed line in Figure 1(a), which are basically consistent with the calculation results of Li et al. (2016).



(a) Potential slip surfaces (b) Representative slip surfaces

Figure 1. The geometry of the slope with 14,896 potential slip surfaces and 158 representative slip surfaces.

Table 1. The strength parameters of the three undrained layers.

Layer	Parameter	Mean (kPa)	COV	Distribution	Autocorrelation distance (m)
Clay 1	s_{u1}	18	0.3	Lognormal	θ_h =20m, θ_v =2m
Clay 2	s_{u2}	20	0.2	Lognormal	
Clay 3	s_{u3}	25	0.3	Lognormal	

To simulate ISV, three random fields of s_{u1}, s_{u2} and s_{u3} are divided into 495, 939 and 1080 elements, respectively, corresponding to the three layers from up to down with a side length of 0.5 m, as shown in Figure 2. The horizontal autocorrelation distance θ_h and vertical autocorrelation distance θ_v are taken as 20 m and 2.0 m, respectively. For the squared exponential ACF, KLE with 29 truncation terms can well represent the three random fields (Phoon et al., 2002). Therefore, for this slope, the dimension of the input random variable $\boldsymbol{\xi}$ is 29.

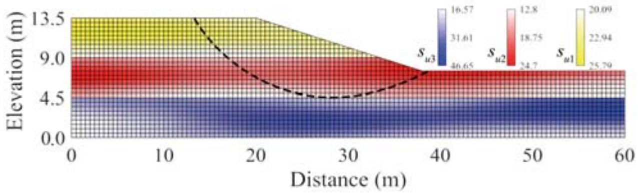


Figure 2. Typical random field realization for the three-layered cohesive slope.

3.1 Probabilistic slope stability analysis results

First, 1,000 realizations of s_{u1}, s_{u2} and s_{u3} are simulated by LHS using Eq. (5), and their corresponding CSSs are determined using the Bishop's simplified method, resulting in a total of 158 RSSs herein, as shown in Figure 1(b). It can be seen that the deterministic CSS is also included in the RSSs. Thereafter, 320 realizations of random fields with their corresponding FSs are randomly selected from the above 1000 realizations to perform the dimensionality reduction by SIR. Here, the reduced dimension *d* is set as 1, which is consistent with Li et al. (2019), and the number of slices *s* is set as 10. The parameter settings for the number of initial training samples are discussed in Section 3.2. It is worth noting that the sufficient dimension reduction subspace obtained by SIR is different for each RSS because FSs obtained by 320 random field realizations are different, thereby needing to perform 158 SIR dimensionality reductions in this example.

Based on the sufficient dimension reduction subspace, 29-dimension input variables are mapped to the 1-dimension input variable, and 158 initial IMK metamodels are established for RSSs. Meanwhile, 10,000 MCS random field samples are randomly generated as the initial sample pool. The initial samples are substituted into the initial IMK metamodels to obtain the Kriging predictions. Then, the active learning described in subsection 3.4 is performed to improve the accuracy of the IMK metamodels. Finally, the active learning stops after 72 iterations, and the high-precision IMK metamodels have been constructed.

To verify the accuracy of the proposed method, another 1,000 test random field samples are generated by LHS and substituted into the established high-precision IMK metamodel. Figure 3(a) compares the minimum FSs predicted by the IMK metamodels with those calculated by Bishop's simplified method. It can be seen that the two methods have high consistency, suggesting that the proposed method still maintains high accuracy after reducing the number of slip surfaces and SIR dimensionality reduction.

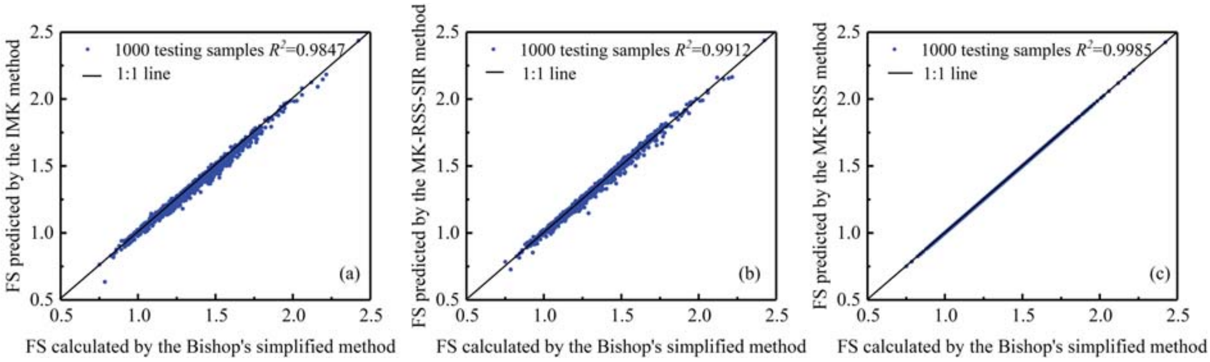


Figure 3. Comparison of the minimum FSs obtained by three methods and Bishop's simplified method.

In addition, to further analyze the accuracy of the IMK method, another 10,000 MCS random field samples are substituted into the high-precision IMK metamodels to obtain the means, standard deviations and associated probability distribution function (PDF) of the predicted minimum FSs. Meanwhile, THE BISHOP'S SIMPLIFIED METHOD is also adopted to calculate the minimum FS and obtain its probability distribution characteristics for comparison. Figures 4(a) and 4(b) show the probability distribution characteristics of the minimum FS calculated by THE BISHOP'S SIMPLIFIED METHOD and those predicted by the IMK method, respectively. From the figures, the PDFs fitted by the two methods are log-normal distributions, and the means (1.323 vs. 1.315) and standard deviations (1.185 vs. 1.181) are also consistent. This consistency demonstrates the high accuracy of the proposed method again.

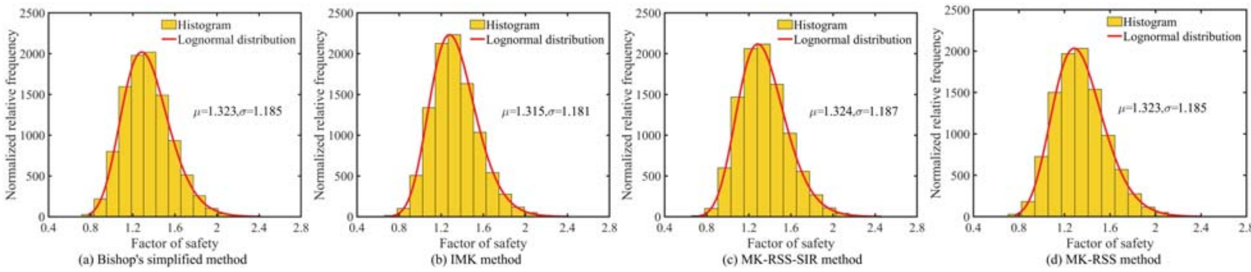


Figure 4. Histograms and fitted PDFs of the minimum FSs.

Based on the high-precision IMK metamodels, MCS with 100,000 random field samples is efficiently performed to estimate the failure probability P_f and failure risk R. Herein, the risk assessment adopts the method proposed by Huang et al. (2013). The failure probability and failure risk are calculated as 0.0450 and 11.694, which are consistent with the results (P_f for 0.0487 and R for 12.022) calculated by direct MCS with 10,000 samples. Such agreement thus validates the accuracy of the proposed method.

3.2 Comparison and discussion

To illustrate the computational efficiency of the IMK method, this section further selects the following two schemes for comparison:

(1) MK-RSS-SIR method: the active learning is not conducted compared with the IMK method;

(2) MK-RSS method: the active learning and SIR dimensionality reduction are not conducted compared with the IMK method.

Table 2. Results obtained by different methods for the three-layered undrained slope.

Method	P_f	COV_{P_f}	R	COV_R	Number of training samples	Absolute calculation time (h)	Source
MCS (10,000)	4.87%	4.40%	12.02	4.57%	–	889.0	This study
IMK	4.50%	1.46%	10.98	1.50%	320+72	3.8	This study
MK-RSS-SIR	4.67%	1.43%	11.54	1.48%	435	8.3	This study
MK-RSS	4.65%	1.43%	11.50	1.49%	290	18.5	This study
LHS (1,000)	5.70%	12.86%	–	–	–	–	Li et al. (2016)

The number and dimensions of training samples are decisive factors in determining the time required to build the Kriging metamodel. According to the rule of thumb, it is sufficient to calibrate a Kriging metamodel when the number of training samples is 10-15 times the number of random variables. Therefore, based on the above three methods, 290 random field samples are initially selected to build the Kriging metamodels. However, only the results predicted by the MK-RSS method are accurate, as shown in Figure 3(c) and Figure 4(d). The

IMK method and MK-RSS-SIR method require more training samples. Finally, for the IMK method, when the number of initial training samples is 320, the predicted results are accurate, as shown in Figure 3(a) and Figure 4(b); while for the MK-RSS-SIR method, accurate results are not obtained until the number of training samples is 435, as shown in Figure 3(b) and Figure 4(c).

The probabilistic stability analysis results obtained by different methods and the absolute calculation time on a desktop with 16 GB RAM and one Intel Core i7 CPU clocked at 3.2 GHz required by different methods are summarized in Table 2. From the table, it can be observed that although the accuracy of the proposed method is slightly lower than that of the MK-RSS method, its computation time is much smaller than that of the MK-RSS method. Therefore, considering the computational accuracy and computational efficiency, this loss of accuracy is acceptable. While compared with the MK-RSS-SIR method, the computational efficiency of the proposed method is also higher. In general, the three methods can accurately replace the slope stability analysis model for slope reliability analysis and risk assessment considering ISV. However, the proposed method greatly improves the computational efficiency under the premise of ensuring accuracy.

4 Conclusion

In this paper, an improved multiple Kriging (IMK) method has been proposed for efficient slope reliability analysis and risk assessment considering ISV of soil properties. The IMK method firstly combines the KLE and SIR to reduce the dimension of input variables to improve the computational efficiency. Then, RSSs are selected to reduce the number of Kriging models to further improve the computational efficiency. Finally, , the calculation efficiency on the premise of ensuring the calculation accuracy is achieved by actively adding additional training samples around the system LSF to compensate for the accuracy loss caused by previous dimension reductions.

A three-layered cohesive slope is adopted to illustrate the effectiveness of the IMK method. The results show that the proposed method can accurately and effectively substitute the slope stability analysis model for probabilistic analysis. In addition, the advantages and disadvantages of the proposed method, MK-RSS-SIR method and MK-RSS method are compared from the perspective of accuracy and efficiency. It is shown that, although the calculation accuracy of the proposed method is slightly lower than that of the MK-RSS method, its computational efficiency is the highest. Weighing efficiency and accuracy, the loss of accuracy is acceptable.

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