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Probabilistic Investigation of Infinite Undrained Slopes with Seismic Loading and Linearly Increasing Mean Strength

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Abstract: The infinite model is often used, for an initial assessment of the mechanics and stability of shallow landslides. Spatial variability is a dominant feature of natural soils, with a trend of increasing strength with depth. In this paper, an algorithm to generate 1D non-stationary random fields of undrained strength is proposed, and used to analyze the influence of spatially variable undrained strength on infinite slope reliability, with a linearly increasing mean trend. Both static and pseudo-seismic loading is considered. A critical slope angle leading to a minimum reliability index is identified, and shown to be related to the horizontal seismic coefficient.

Keywords: Infinite slope; Failure mechanism; Non-stationary random field; Spatial variability; Probability of failure.

1 Introduction

In the realm of slope stability analysis, the infinite slope procedure is relatively simple to use, as it leads to an analytical solution for the factor of safety (e.g., Duncan et al. 2014). The infinite slope model can assess the mechanics of shallow landslides, and analyze long slope stability. The traditional criterion that defines the slope stability is the factor of safety (FS), which is calculated using constant soil properties based on characteristic values. However, soil properties such as shear strength vary 'from point to point' (Terzaghi 1948), suggesting a statistical approach is needed for characterizing spatial variability of soil properties in the assessment of slope stability.

Griffiths et al. (2011b) firstly proposed a random field method to perform probabilistic stability analyses of infinite slopes, in which shear strengths were generated by random field theory. This early random field analyses considered infinite slopes with stationary random properties, i.e. the mean and standard deviation of shear strength parameters were constant with depth. However, soil strength may display an increasing trend with depth (e.g., Phoon and Kulhawy 1999), and this non-stationary character of soil properties has received attention to predict the reliability of infinite slopes; for example, Li et al. (2014) adopted the Karhunen-Loeve (KL) expansion to discretize the random fields of soil strengths which increase linearly with depth to study the reliability of infinite slopes; and Zhu et al. (2019) used the limit equilibrium method combined with 1D non-stationary random fields discretized by the KL expansion to conduct reliability analysis of infinite clay slopes under static and pseudo-seismic conditions. This study will extend the work of Griffiths et al. (2011b), focusing on the reliability analysis of infinite clay slopes with non-stationary random fields of undrained strength by the random field method.

Consider a typical column of an infinite clay slope with linearly increasing undrained strength as shown in Figure 1. The factor of safety (e.g., Zhu et al. 2019) can be given by

$$\mathrm{FS} = \min\left\{\frac{c_{\mathrm{u0}} + \rho z}{(\sin\beta + k_{\mathrm{h}}\cos\beta)\gamma z\cos\beta}\right\},\ 0 < z \le H \tag{1}$$

where c_{u0} is the undrained strength at the top of the infinite slope; ρ is the gradient of strength increasing with depth z ; γ is the saturated unit weight; β is the slope angle; k_{h} is the horizontal seismic coefficient; H is the depth of the soil above bedrock. For simplicity, only the undrained strength (c_{u}) is modelled as a random variable, while other parameters are treated as deterministic throughout this study.

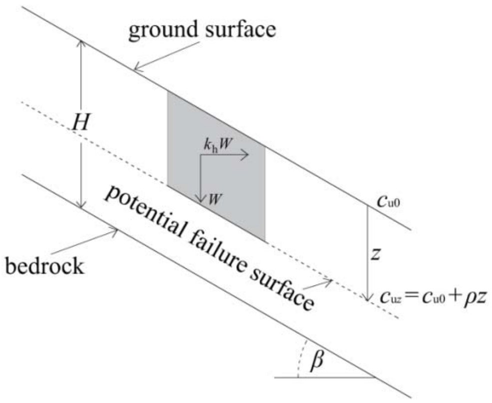

Figure 1. A model for the infinite undrained slope with linearly increasing strength.

2 Static infinite slope analysis

2.1 Deterministic stability analysis of infinite undrained slopes with linearly increasing strength

For the static case ($k_h = 0$), the Eq. (1) can be simplified to

$$\mathrm{FS} = \min\left\{\frac{c_{u0} + \rho z}{\gamma z \sin\beta \cos\beta}\right\} = \min\left\{\frac{c_{u0}}{\gamma z \sin\beta \cos\beta} + \frac{\rho}{\gamma \sin\beta \cos\beta}\right\}, 0 < z \le H \tag{2}$$

It can be seen from Eq. (2) that, as the variables (c_{u0}, ρ, γ, β) are constant, the minimum value of the factor of safety (corresponding to the smallest value of the factor of safety for all slices) will occur at the base of the infinite slope ($z = H$). Therefore, for an infinite undrained slope with linearly increasing strength, the critical failure plane will be at the full depth of the slope, with the factor of safety given by

$$\mathrm{FS} = \frac{c_{u0} + \rho H}{\gamma H \sin\beta \cos\beta} \tag{3}$$

By observing Eq. (3), the critical slope angle (β_{min}) which leads to the minimum factor of safety will be at 45°, as noted by Griffiths et al. (2011a).

2.2 Probabilistic analysis of infinite undrained slopes with linearly increasing mean strength

For the problem definition shown in Figure 1, the mean undrained strength can be given by

$$\mu_{c_{uz}} = \mu_{c_{u0}} + \rho z \tag{4}$$

where $\mu_{c_{uz}}$ is the mean undrained strength at depth z and $\mu_{c_{u0}}$ is the mean undrained strength at the top of the infinite slope. In this study the non-stationary random field with the properties by Eq. (4) will be simulated using the following steps (e.g., Zhu et al. 2017):

Step 1: Adopt the local averaging subdivision method to generate an initial 1D stationary lognormally distributed random field according to the parameters at the top of the infinite slope, i.e. the mean ($\mu_{c_{u0}}$), standard deviation ($\sigma_{c_{u0}}$) and spatial correlation length (θ). These initial values (c_{0i}, $i = 1, 2, \cdots, 100$) are assigned to all slices (the column is split into 100 equal slices), over depth, of the infinite slope. The selection of 100 slices is a reasonable compromise (Griffiths et al. 2011b).

Step 2: Slice values are then adjusted to take account into other depths ($z > 0$) by using the scaling factor

$$c_{zi} = c_{0i} \frac{\mu_{c_{u0}} + \rho z}{\mu_{c_{u0}}} \tag{5}$$

where c_{zi} is the undrained strength for the i^{th} slice and z is sampled at the centroid of each slice.

After the generation of the non-stationary random field and assignment to each slice, the factor of safety for the i^{th} slice at depth z is given as

$$\mathrm{FS}_{zi} = \frac{c_{zi}}{\gamma z \sin\beta \cos\beta} \tag{6}$$

The column of an infinite clay slope consists of 100 slices, each slice would return a different factor of safety, and the smallest one is the "correct" value. In the previous section, the deterministic FS always occurred at the base of the infinite slope ($z = H$), however, this is not necessarily the case in a random soil, where the minimum c_{zi}/z may occur elsewhere (Griffiths et al. 2011b).

In a Monte-Carlo framework, each repetition by the random field method involves a generation of a non-stationary random field and a subsequent calculation giving the lowest factor of safety and the critical failure plane. In the current work, 5000 Monte-Carlo simulations were performed from which the probability of failure (p_f) was computed as the proportion of the 5000 simulations where $\mathrm{FS} < 1$.

To investigate the effect of non-stationary property of undrained strength on the reliability of the infinite slope, for the static case, an example slope with $\mu_{c_{\mathrm{u}0}} = 25$ kPa, $H = 2.5$ m, $\gamma = 20$ kN/m^3 and $\beta = 30°$ is firstly considered with four different values of the gradient of mean strength ($\rho = 0$, 0.2, 0.4, 0.6 kN/m^3). It may be noted that the case of $\rho = 0$ and $v_{c_\mathrm{u}} = 0.1$ involves a stationary random field, that was also the test slope considered by Griffiths et al. (2011b).

Figure 2 shows the probability of slope failure plotted against the nondimensional spatial correlation length ($\Theta = \theta/H$) for different gradients of mean strength with $v_{c_\mathrm{u}} = 0.1$. It can be seen that the probability of slope failure p_f decreases as Θ is increased and eventually flatten out for all values of ρ. This indicates that the "single random variable" (SRV) approach (implying infinite spatial correlation length $\Theta \to \infty$) always underestimates p_f for infinite undrained slopes with linearly increasing mean strength. This is the same conclusion reached by Griffiths et al. (2011b) for a stationary soil ($\rho = 0$). For details of the infinite slope problem by the SRV approach, the reader can refer to Griffiths et al. (2011b). Also indicated in Figure 2, for a fixed value of Θ, is that p_f decreases with increasing ρ. The reason is that when other parameters are fixed, a larger value of ρ will lead to higher values of mean undrained strength across the soil layer, indicating increasing values of FS at all depths which decreases the probability of slope failure.

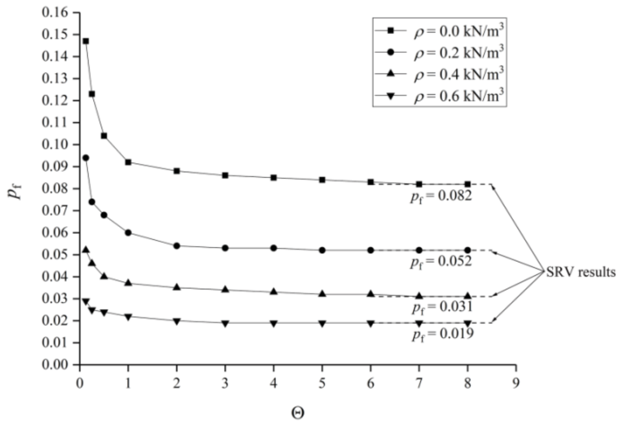

Figure 2. Probability of failure versus the nondimensional spatial correlation length for different gradients of mean strength for an infinite slope with $H = 2.5$ m, $\gamma = 20$ kN/m^3, $\beta = 30°$, $\mu_{c_{\mathrm{u}0}} = 25$ kPa and $v_{c_\mathrm{u}} = 0.1$.

2.3 Observations on the frequency of the critical depth

As stated in the previous section, the critical depth may not necessarily be at the full depth of the random soil in an infinite slope with linearly increasing mean strength, because the lowest factor of safety occurs when c_{zi}/z is at minimum. Figure 3 shows histograms of the frequency of the critical depth for the example slope for different values of ρ when $\Theta = 0.04$ and $v_{c_\mathrm{u}} = 0.1$, which clearly indicates that the critical depth is most likely to be near the base of the infinite slope for all cases. It can also be noted that the frequency of the depth of critical mechanisms appears to be essentially unaffected by the value of ρ. This is due to the fact that, in the generation of the 1D non-stationary random fields, while all depths will be assigned greater values of c_u on average for larger values of ρ, the critical depth just depends on the minimum of all values of c_{zi}/z. It should be noted that, this phenomenon may only be suitable for undrained slopes. The critical depth may occur at other depths if a cohesive-frictional slope is considered.

Figure 4 shows histograms of the frequency of the critical depth for the example slope with different values of v_{c_u} when $\Theta = 0.04$ and $\rho = 0.2$ kN/m^3. It can be observed from Figure 4 that the critical depth is still most likely to be at the base, however, the frequency of the critical depth occurring at the base decreases as the value of v_{c_u} increases. Only about 10% of simulations occurred at the base when $v_{c_u} = 0.3$, while over twice that the number occurred at the base for $v_{c_u} = 0.1$. This observation might be expected, because the increase of v_{c_u} leads to greater fluctuations of c_{zi} about the mean at all elevations, and hence a greater probability of minimum values of c_{zi}/z occurring at elevations above the base.

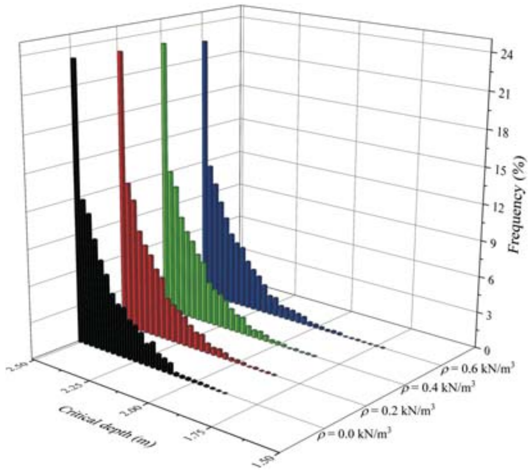

Figure 3. Histograms of frequency of the critical depth by the random field method for different values of ρ for an infinite slope with $H = 2.5$ m, $\gamma = 20$ kN/m^3, $\beta = 30°$, $\mu_{c_{u0}} = 25$ kPa, $v_{c_u} = 0.1$ and $\Theta = 0.04$.

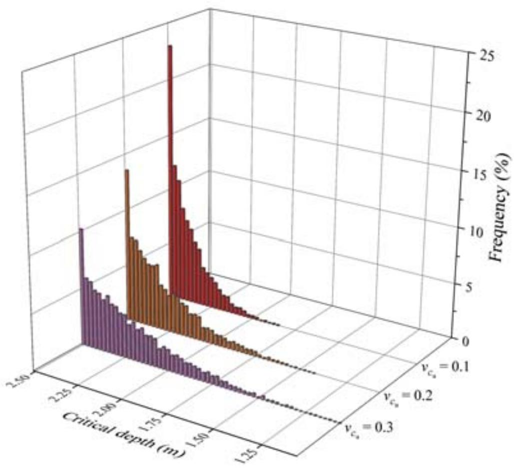

Figure 4. Histograms of frequency of the critical depth by the random field method with different values of v_{c_u} for an infinite slope with $H = 2.5$ m, $\gamma = 20$ kN/m^3, $\beta = 30°$, $\mu_{c_{u0}} = 25$ kPa, $\rho = 0.2$ kN/m^3 and $\Theta = 0.04$.

3 Pseudo-seismic infinite slope analysis

3.1 Deterministic pseudo-seismic stability analysis of an infinite undrained slope with linearly increasing strength

For pseudo-seismic case, the Eq. (1) can be further transformed into

$$\mathrm{FS} = \min\left\{\frac{c_{u0}}{\gamma z(\sin\beta\cos\beta + k_h\cos^2\beta)} + \frac{\rho}{\gamma(\sin\beta\cos\beta + k_h\cos^2\beta)}\right\},\ 0 < z \le H \tag{7}$$

As with the static case, when the parameters (c_{u0}, ρ, γ, β, k_h) are held constant, the lowest value of FS will also occur at the base. Thus, for the infinite undrained slope with linearly increasing strength under pseudo-seismic loading, the factor of safety is given by

$$\mathrm{FS}=\frac{c_{u0}+\rho H}{\gamma H(\sin\beta\cos\beta+k_h\cos^2\beta)} \tag{8}$$

In the previous section, Eq. (3) results in a critical slope angle ($\beta_{\min}$) which is 45° for the static case. For the pseudo-seismic case, the critical slope angle can be obtained analytically by finding the maximum value of the function

$$f(\beta,\ k_h)=\sin\beta\cos\beta+k_h\cos^2\beta \tag{9}$$

By differentiation of Eq. (9) and setting the result to zero, the critical slope angle will be

$$\beta_{\min}=\frac{1}{2}\arctan\frac{1}{k_h} \tag{10}$$

To investigate the critical slope angle phenomenon, for the pseudo-seismic case, an example infinite slope is used here with the properties $H=2.5$ m, $\gamma=20$ kN/m³, $c_{u0}=25$ kPa and $\rho=0.2$ kN/m³. Figure 5 shows analytical solutions from Eq. (8) for the example slope with different horizontal seismic coefficients over a range of slope angles. It can be seen clearly from Figure 5 that as k_h increases, the critical slope angle $\beta_{\min}$ decreases. Also indicated in Figure 5, is that FS decreases with increasing k_h, especially for lower values of slope angle.

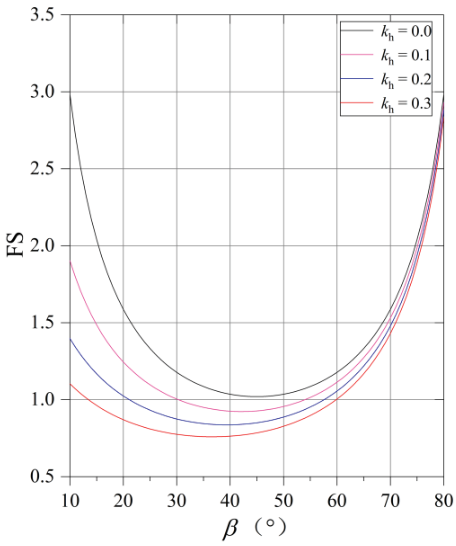

Figure 5. Analytical solutions from Eq. (8) with different horizontal seismic coefficients over a range of slope angles for an infinite slope with $H=2.5$ m, $\gamma=20$ kN/m³, $c_{u0}=25$ kPa and $\rho=0.2$ kN/m³.

3.2 Probabilistic pseudo-seismic analysis of infinite undrained slopes with linearly increasing mean strength

Figure 6 shows the probability of failure plotted against the nondimensional spatial correlation length with different horizontal seismic coefficients for the example slope with $H = 2.5$ m, $\gamma = 20$ kN/m³, $\beta=30°$, $\mu_{c_{u0}}=25$ kPa, $\rho=0.2$ kN/m³ and $v_{c_u}=0.1$. It can be seen from Figure 6 that, for a fixed value of Θ, the probability of failure increases rapidly when k_h increases from 0 to 0.15, indicating that pseudo-seismic loading has a significant influence on the reliability of infinite undrained slopes with linearly increasing mean strength in a random field case.

Figure 7 shows the probability of failure plotted against slope angle for the example slope with $H=2.5$ m, $\gamma=20$ kN/m³, $\mu_{c_{u0}}=25$ kPa, $\rho=0.2$ kN/m³, $v_{c_u}=0.1$, $\Theta=4$, $k_h=0.0$ and 0.1. It can be seen that, for static case shown in Figure 7a, the probability of failure reaches a maximum when $\beta=45.0°$. For pseudo-static case shown in Figure 7b, when $\beta=42.1°$, which is about the value given by Eq. (10) for $k_h=0.1$, the highest p_f is observed. This phenomenon can also be predicted by deterministic analysis where the factor of safety reaches a minimum for the $k_h=0.0$ and 0.1 cases shown in Figure 5.

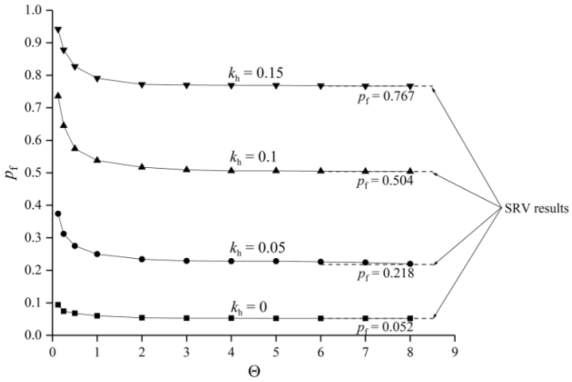

Figure 6. Probability of failure versus the nondimensional spatial correlation length for different horizontal seismic coefficients for an infinite slope with $H = 2.5$ m, $\gamma = 20$ kN/m³, $\beta = 30°$, $\mu_{c_{u0}} = 25$ kPa, $\rho = 0.2$ kN/m³ and $v_{c_u} = 0.1$.

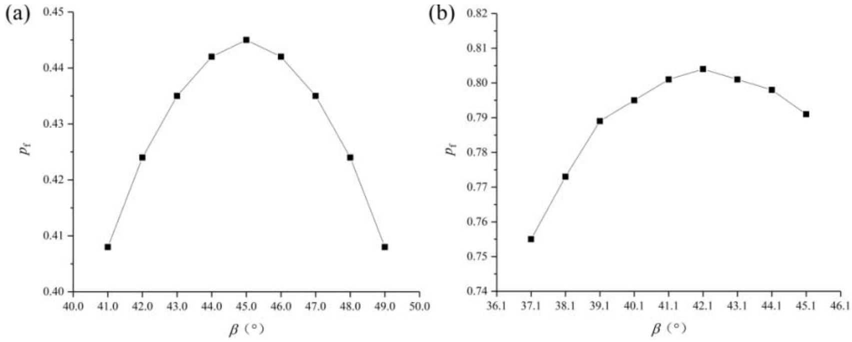

Figure 7. Probability of failure versus slope angle for an infinite slope with $H = 2.5$ m, $\gamma = 20$ kN/m³, $\mu_{c_{u0}} = 25$ kPa, $\rho = 0.2$ kN/m³, $v_{c_u} = 0.1$ and $\Theta = 4$. (a) $k_h = 0.0$, (b) $k_h = 0.1$.

4 Concluding remarks

This study investigates the reliability of infinite slopes with linearly increasing mean undrained strength under both static and pseudo-seismic loadings. The probability of infinite slope failure decreases with increasing gradient of mean strength. As the coefficient of variation of strength is increased, the frequency with which the critical depth occurs at the base is decreased. For static case, the critical slope angle is 45°, and it decreases from 45° as the horizontal seismic coefficient is increased.

Acknowledgments

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References

Duncan, J.M., Wright, S.G., and Brandon, T.L. (2014). Soil strength and slope stability, *2nd edn, John Wiley & Sons, N.J., Hoboken*.

Griffiths, D.V., Huang, J., and deWolfe, G.F. (2011a). Numerical and analytical observations on long and infinite slopes. *International Journal for Numerical and Analytical Methods in Geomechanics*, 35, 569-585.

Griffiths, D.V., Huang, J., and Fenton, G.A. (2011b). Probabilistic infinite slope analysis. *Computers and Geotechnics*, 38, 577-584.

Li, D., Qi, X., Phoon, K.K., Zhang, L.M., and Zhou, C. (2014). Effect of spatially variable shear strength parameters with linearly increasing mean trend on reliability of infinite slopes. *Structural Safety*, 49, 45-55.

Phoon, K.K. and Kulhawy, F.H. (1999). Characterization of geotechnical variability. *Canadian Geotechnical Journal*, 36 (4), 612-624.

Terzaghi, K. (1948). Foreword to the inaugural volume of Géotechnique. *Géotechnique*, 1 (1), 3-5.

Zhu, B., Pei, H., and Yang, Q. (2019). Reliability analysis of submarine slope considering the spatial variability of the sediment strength using random fields. *Applied Ocean Research*, 86, 340-350.

Zhu, D., Griffiths, D.V., Huang, J., and Fenton, G.A. (2017). Probabilistic stability analyses of undrained slopes with linearly increasing mean strength. *Géotechnique*, 67 (8), 733-746.