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Field Measurements-Based Probabilistic Stability Analysis of an Earth Dam

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Abstract: Soil property uncertainties are unavoidable for the design/evaluation of earth dams, which are large structures made of naturally variable materials (i.e., soils/rocks). This paper evaluates the stability of an existing earth dam under the consideration of such uncertainties by means of probabilistic analysis. The measurements collected during the dam construction or from laboratory tests are exploited to model the relevant soil properties by random variables or random fields. One objective of the current study lies in showing a probabilistic analysis procedure for practical engineering including soil variability modelling and reliability analysis. The considered dam problem is high dimensional with over 2000 input variables due to the discretization of random fields and the low autocorrelations distances. Five reliability methods, including sampling methods and surrogate-based techniques, are compared, aiming to investigate their accuracy and efficiency when dealing with such challenging problems. The presented results are expected to be useful for engineers to probabilistically analyze earth dams; some recommendations about method selection and soil spatial modelling for the reliability analysis using random fields are also available.

Keywords: Earth dam; Reliability analysis; Spatial variability modelling; Monte Carlo Simulation

1 Introduction

Over the last few years, rationally considering soil/rock uncertainties and investigating their effects on the safety of geo-structures are being increasingly recommended in design codes. For example, the French national codes of hydraulic works, modified in 2015, requested the owners of large dams to incorporate probabilistic-based risk analyses into their safety evaluation reports. However, performing the above-mentioned tasks properly and efficiently is still challenging in practice due to the limited number of in-situ measurements and expensive computation effort. It is usually not easy to model reasonably soil spatial variabilities and couple them with deterministic models. Additionally, over thousands of simulations are needed in most probabilistic analyses, leading to heavy computational burdens.

For earth dams, the probabilistic analyses presented in the literature can be classified into three groups focusing on different failure modes: flood overtopping, piping and slope instability (Foster et al., 2000). The last group concerns the topic of this paper and the relevant contributions can be found in Chen and Chang (2011) and Siacara et al., (2020). However, most of the existing studies either worked on hypothetical cases with virtual data or used traditional reliability analysis methods, such as Monte Carlo Simulation (MCS), which are generally inefficient in terms of computation time.

This paper is dedicated to presenting a reliability analysis of an existing earth dam, in which real measurements are considered and several reliability methods (from basic to advanced) are employed. The data collected during the dam construction and from laboratory tests are exploited to estimate the necessary parameters of modelling relevant soil properties as random variables (RV) and/or random fields (RF). Such probabilistic modelling, together with the developed deterministic model, is firstly integrated into the MCS framework to estimate the failure probability (*Pf*) of the studied dam. Then, four other methods including Subset Simulation (SS), Moment Method (MM) and two metamodel-based techniques are evaluated by comparing with the MCS results.

2 Case study and deterministic models

2.1 Presentation of the studied dam

The studied dam, constructed in 1998, is a 23.8 m high earth dam located in the west of France. The width is around 140 m and the length in the 3rd dimension is 170 m for the crest. Figure 1 presents the main cross section of the dam. The dam is formed by three different zones including a core (Core) constituted with sandy silts and two backfill zones (Shell-1 and Shell-2) constituted with gravelly sands. This work focuses on the dam stability analysis under steady state flow conditions with the reservoir being kept at its normal level (20m). The soils below the phreatic level are assumed to be saturated. A horizontal pseudo-static acceleration (2.4m.s^{-2}) is considered in this work.

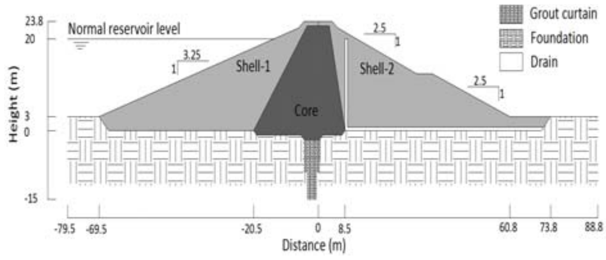


Figure 1. Main cross section of the studied dam

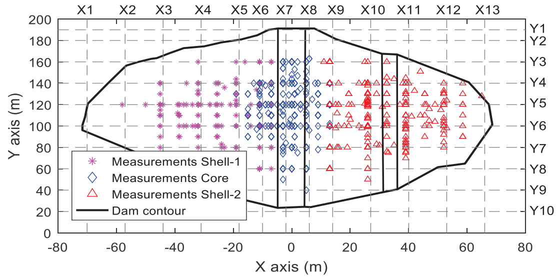


Figure 2. The γ_d measurements with the grid system

2.2 Deterministic models

Two deterministic models in two dimensions (2D) are developed in this work in order to estimate deterministically the dam safety factor (FoS) under the current calculation configuration. The first model, created in FLAC2D (finite difference code), is based on the strength reduction method (SRM) for the FoS estimation. The second model combines the limit equilibrium method (LEM), specifically Morgenstren-Price, with the genetic algorithm (GA). For more details about the models, please refer to Guo et al. (2018).

As each model has its own positive and negative features, it is thus worth developing more than one deterministic model, especially for a big project so that different models can be selected to address different problems. Additionally, having more than one model enables cross-comparison and validation.

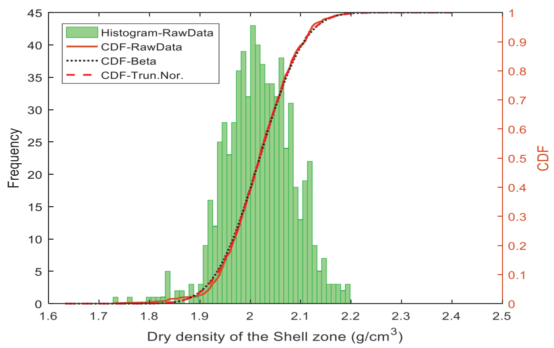
3 Soil variability modelling

The uncertainties of three soil parameters (dry density γ_d, cohesion c', and friction angle ϕ'), which are highly relevant to the slope stability analysis, are modelled by RVs or RFs.

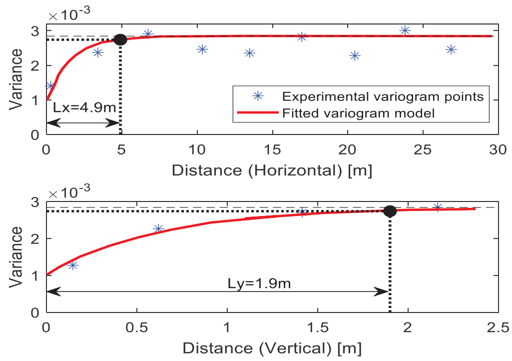
3.1 Uncertainty modelling of γ_d

During the dam's construction, measurements of the γ_d after compaction were collected in-situ using a gamma-densimeter within a grid monitoring system (Figure 2). This leads to a large number of geo-localized γ_d data: 381 measurements for the Core zone, 248 for the Shell-1 zone and 272 for the Shell-2 zone.

By fitting the measured data with an assumed probabilistic distribution, the γ_d variability can be described by means of RVs. The Beta distribution is employed to fit the data given that such a distribution can avoid generating unreasonable values by defining the upper and lower bounds. As an illustration, Figure 3.a presents the histogram and cumulative density function (CDF) of the γ_d measurements in the Shell zone, together with the fitted CDF curve. Then, the spatial correlations (autocorrelation distance in the theory of RF) among these distributed data are estimated for both horizontal and vertical directions. This is achieved by performing a variogram analysis on the geo-localized γ_d measurements. Taking the Shell-2 zone as an example, an experimental semi-variogram is firstly obtained by using all the γ_d measurements of this zone. Then, the autocorrelation distances can be estimated by fitting a mathematical model (exponential model in this work (Cami et al., 2020)) to the experimental semi-variogram as shown in Figure 3.b.



(a)



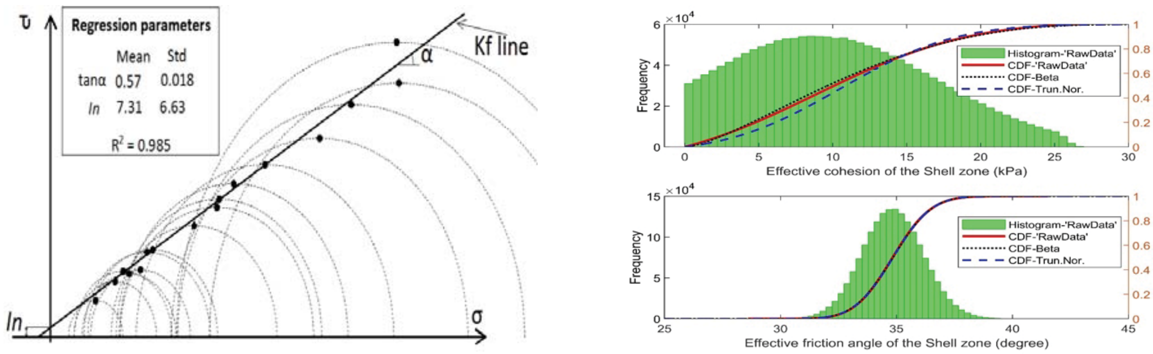
(b)

Figure 3. (a) Histogram and CDF of the γ_d data (Shell); (b) Variogram analysis for the γ_d data (Shell-2)

3.2 Uncertainty modelling of *c'* and *ϕ'*

The shear strength parameters *c'* and *ϕ'* can be determined by triaxial shear tests. In total, 8 consolidated-undrained triaxial shear tests with pore water pressure measurements are available for the considered dam (5 tests for the Shell zone and 3 for the Core zone). Such a low quantity of tests does not allow a meaningful statistical study for determining the *c'* and *ϕ'* distributions. A possible solution is to estimate the uncertainties of *c'* and *ϕ'* by further exploiting the Mohr circles derived from the test records (Guo et al., 2018). This consists of using the first two statistical moments of *In* and *tanα* (Y-intercept and slope of the regression line in Figure 4.a – an example for the Shell zone) which can be obtained by performing a regression analysis on the peak points of all the Mohr circles, to generate a large number of these two parameters and then compute the corresponding *c'* and *ϕ'*. It is noted that 3 or 4 values of confining pressures were considered in each triaxial test and 16 effective Mohr circles were obtained for the Shell zone. The generated artificial *c'*-*ϕ'* data are then fitted to the Beta distribution to determine the necessary parameters. Figure 4.b shows an example of these data and the fitted CDF for the Shell zone.

The above-mentioned solution cannot provide the location information for *c'* and *ϕ'*. Therefore, it is not possible to estimate the autocorrelation structure for these two soil properties with the 8 tests. However, one can use physical or empirical relations to obtain *c'* and *ϕ'* RFs by transforming the RFs of γ_d. The Caquot's relation (Mouyeaux et al,. 2018) is adopted here to obtain the *ϕ'* RFs since it links the friction angle with the soil void ratio *e* (thus indirectly with dry density). No effective transformation equations exist for estimating *c'* from γ_d as indicated in Li et al., (2014). For this reason, the *c'* variability is only represented by RVs.



(a) (b)

Figure 4. (a) Mohr circles of the triaxial tests (Shell); (b) Histogram and CDF of the generated *c'* data (Shell)

3.3 Summary of the parameters

Table 1 presents a summary of the distribution parameters and the autocorrelation distances. Due to the assumption of the statistical method, the generated *c'* - *ϕ'* values are uncorrelated or weakly correlated. Therefore, in this work, the correlation between *c'* - *ϕ'* is not considered. Moreover, this table indicates that a considerable homogeneity can be found in the Shell-1 zone, while the soils in the Shell-2 and Core are likely to have more remarkable spatial variations. This difference can be explained by the better selection of the material composing the upstream zone and the greater attention which has given to its construction.

Table 1. Summary of the parameters for soil variability modelling

Zones	Autocorrelation distance (m)			Beta distribution parameters				Statistics	
	Horizontal	Vertical		A*	B*	Min+	Max+	Mean	CoV# (%)
Shell-1	78.1 m	7.8 m	γ_d (g/cm^3)	15.7	18.0	1.63	2.40	1.99	3.21
			c' (kPa)	1.48	2.78	0	30	10.55	57.63
			ϕ' (°)	28.71	29.61	25	45	34.85	3.72
Core	13.0 m	1.5 m	γ_d (g/cm^3)	22.4	27.5	1.44	2.32	1.83	3.33
			c' (kPa)	4.07	5.22	0	30	13.23	34.21
			ϕ' (°)	231.16	192.28	15	50	34.11	2.48
Shell-2	4.9 m	1.9 m	γ_d (g/cm^3)	26.7	22.2	1.63	2.40	2.05	2.65
			c' (kPa)	1.48	2.78	0	30	10.55	57.63
			ϕ' (°)	28.71	29.61	25	45	34.85	3.72

Notes of Table 1: *shape parameters; +extreme values; #coefficient of variation

4 Reliability analysis using Monte Carlo Simulation

This section aims to estimate the dam *Pf* considering the uncertainties quantified above by using the MCS.

4.1 Validation of the LEM model

The LEM model is preferred to the SRM model due to its high computational efficiency. Firstly, a comparison study is carried out in order to validate the performance of the LEM model for the analyses with soil spatial variations. It involves generating a number of RFs for ϕ' and γ_d, and RVs for c', and then estimating the dam FoS for each set by using both models. In total, 150 random input sets are considered in the comparison study. It is noted that the RFs in this work are generated by using the Karhunen-Loeve Expansions (Sudret and Der Kiureghian, 2000). Figure 5.a presents a direct comparison of the FoS values computed by the two models for the 150 sets. Additionally, a comparison concerning the failure surface location determined by the two models is conducted, and the results are presented in Figure 5.b for two cases as an illustration. In summary, the performance of the LEM model for slope stability analyses involving spatially varying soils is validated.

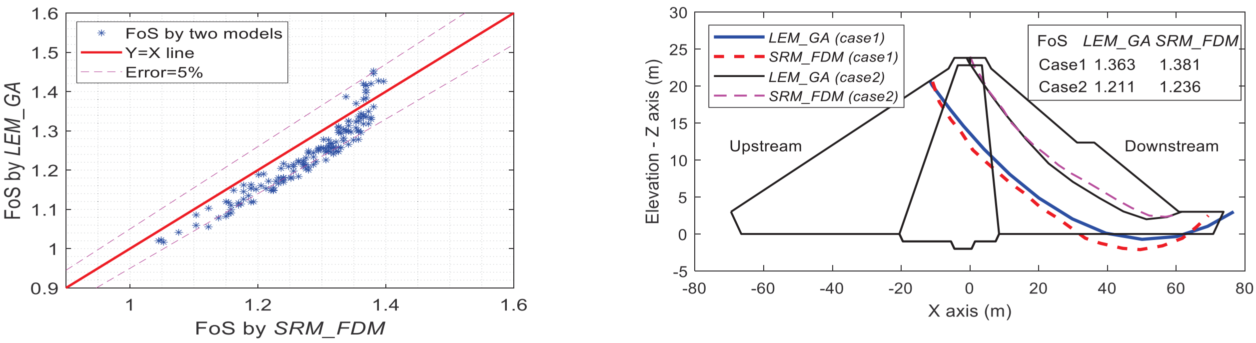


Figure 5. (a) FoS comparison of the two models for the 150 sets; (b) Comparison of failure surfaces for two random cases

4.2 Reliability analysis results

The LEM model is used in this section to estimate the dam Pf within the MCS framework. The number of deterministic calculations in the MCS is 20000, which can lead to an accurate enough estimation for the Pf.

Table 2 shows the Pf and the statistical moments of the FoS obtained by MCS. The direct failure probability Pf_{con} of the dam under a pseudo-static acceleration of 2.4 m.s^{-2} is estimated to be equal to 0.022. This value is then multiplied by the occurrence probability of the earthquake (simply assumed to be 1/5000 which is related to the return period of the considered acceleration (Loudière et al., 2014)) and becomes equal to 4.4×10^{-6}. As for the statistical moments of the FoS values, the mean is about 1.23 with a standard deviation of 0.11.

Table 2. Reliability analysis results of the studied dam using the MCS

Failure probability			FoS statistics	
Pf_{con}	$Pfsce$	CoV_{Pf}	Mean	Standard deviation
0.022	4.4×10^{-6}	4.7%	1.23	0.11

5 Evaluation of different reliability methods

This section aims to compare different reliability methods using the current dam problem given that its dimension (i.e., number of input variables) is very high due to the RF discretization and very low autocorrelation distances in the Shell-2 zone.

5.1 The considered reliability methods for the comparison

For a reliability analysis, the MCS is always considered as a standard reference to test other methods. However, it suffers from a very low computational efficiency. Based on the MCS, one advanced sampling method, Subset Simulation (SS) (Au and Beck, 2001), is proposed to reduce the variance of the MCS estimator with a limited number of deterministic model calls. It is applicable for the problems with spatially varying soils as the method is independent of the input dimension. Another reliability method is the Moment Method (MM) which uses approximation by Taylor series to estimate the first moments of a system response and then to compute the reliability index with the estimated moments. Alternatively, the first moments can be determined by performing an MCS until convergence is reached. The MM, based on the MCS, is independent to the input dimension, so it is also applicable for the present problem. Furthermore, the First/Second Order Reliability Method are also commonly used for reliability analyses. Unfortunately, these methods are not able to handle too many RVs.

During the last decades, meta-modelling techniques have received much attention in reliability analysis and they have been widely applied to geotechnical engineering (Zhou et al., 2020; Guo et al., 2020;). In the context of high dimensional stochastic problems, some dimension reduction techniques were introduced and combined with the meta-modelling technique to improve its performance, such as the SPCE combined with the GSA (termed as SPCE/GSA) (Al-Bittar and Soubra, 2014) and the SPCE combined with the Sliced Inverse Regression (termed as SPCE/SIR) (Pan and Dias, 2017).

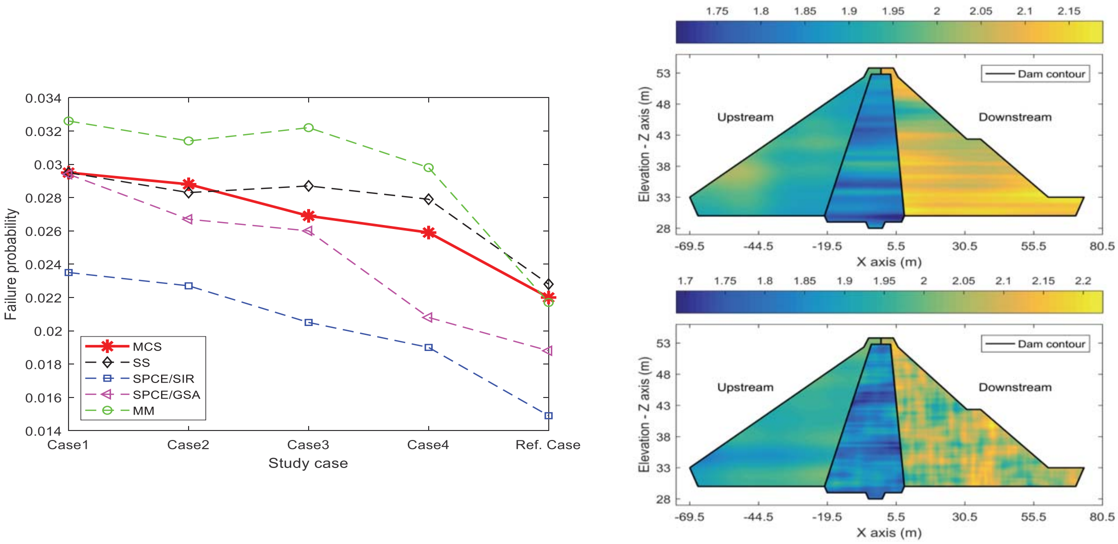
In summary, the selected reliability methods for the comparative study are thus: a reference method (MCS), a variance reduced MCS (SS), an MCS-based moment method (MM), and two meta-modelling approaches (the SPCE/GSA and the SPCE/SIR). Please refer to Guo et al., (2019) for the parameter settings of these methods.

5.2 Results of the comparative study

Five cases are considered for the comparison with a reference case using the parameters from Table 1 and four complementary cases using higher autocorrelation distances. The obtained results (Pf_{con}) from the five reliability methods for each case are plotted in Figure 6.a, and some randomly generated RFs of Case 1 and the Reference Case are presented in Figure 6.b for illustration. The numbers of calls to the deterministic model (N_{call}) for each case are summarized in Table 3. In this table, the number (N_{RV}) of the required RVs for representing the three soil properties by means of RFs or RVs is given as well.

A quick review of Figure 6.a reveals that the four methods can all give relatively accurate Pf_{con} estimates compared to the MCS results. The values vary within a similar order of magnitude for the different methods. For example, the Pf_{con} is between 0.015 and 0.024 for the Reference case according to the five methods.

Concerning the efficiency comparison presented in Table 3, it is found that all the approximated methods need fewer calls of the deterministic model than the direct MCS. Besides, the N_{RV} value is increased from Case 1 to Case 4, and the N_{RV} of the Reference Case is the biggest value, due to the decreased autocorrelation distances from Case 1 to the Reference Case. It is observed that the N_{call} of the MCS, SS and MM methods is almost constant irrespective of N_{RV}. In other words, the efficiency of these three methods is not related to the number of input RVs, but in fact depends on the value of the target failure probability. However, the N_{call} of the two meta-modelling methods (SPCE/GSA and SPCE/SIR) increases rapidly with increasing N_{RV}. This indicates that the efficiency of the meta-modelling methods strongly depends on the number of input RVs. Indeed, more input variables implies that more information is needed. Thus, a higher N_{call} will be required for constructing a meta-model which will be used to replace the original mechanical model.



(a) (b)

Figure 6. (a) Pf estimates for the five cases; (b) Illustrative γ_d RFs for Case 1 (upper) and Reference Case (lower)

Table 3. Approximated number of calls (N_{call}) to the deterministic model for the five reliability methods

	Case 1	Case 2	Case 3	Case 4	Reference case
N_{RV}	225	370	647	1207	2110
MCS	20000	20000	20000	20000	20000
SS	3000	3000	3000	3000	3000
SPCE/SIR	1000	5000	8000	10000	15000
SPCE/GSA	3000	5000	8000	13000	18000
MM	1900	1800	2100	2000	2000

5.3 Discussions on the comparative results

It seems that the MM and SS outperform the two surrogate-based methods as similar Pf_{con} results are obtained but the two sampling techniques need significantly less calls to the deterministic model. However, both the MM and SS cannot provide a monotone decreasing trend estimation for Pf_{con} from Case 1 to the Reference case, which is an expected result due to the decrease of autocorrelation distance. This feature is well captured by the two metamodeling methods. The fluctuation of the estimated Pf_{con} in the SS is due to the difficulty of effectively generating conditional samples in the context of many variables, while a similar observation in the MM arises from the fact that the tail area of a distribution (failure probability) is highly sensitive to the statistical moments used for the distribution approximation. Additionally, it is observed that the two metamodeling methods always underestimate the Pf_{con} values. This is not surprising as they reduced the input dimension leading to the loss of some input uncertainties. The underestimation is slight as presented in Figure 6.a with the SPCE/GSA results being the most accurate set among the four tested methods.

6 Conclusions

In this article, a probabilistic stability analysis of an existing earth dam is presented. The uncertainties in three soil properties are considered in the analysis and quantified by exploiting the project-specific data. The MCS is adopted for performing the reliability analysis. By benefiting from the results of the deterministic simulations collected in the performed MCS, a comparative study is carried out. It aims at evaluating the performance of different reliability methods for very high dimensional stochastic problems. Both the accuracy and efficiency are considered in the comparison. The results show that the most accurate method is the SPCE/GSA and the most efficient method is the SS. The efficiency of the SS and MM methods are independent to the number of input variables, while the necessary N_{call} of the SPCE/GSA and SPCE/SIR methods can be very important (close to the N_{call} of an MCS) when a large number of random variables are involved. For a first order estimate, the SS and MM methods are sufficient to give relatively accurate results. Nevertheless, it should be noted that these two methods were not sufficiently accurate for the parametric study of autocorrelation distance since the obtained values of Pf fluctuate.

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