

## Effect of Spatial Variability in Asphalt Layer on Critical Pavement Response

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**Abstract:** Spatial variability of the soil or structural materials has been proven to induce harmful and non-negligible effects on the structural response in geotechnical engineering. Whereas only limited studies have incorporated spatial variability into the probabilistic analysis of pavement engineering as opposed to many other geotechnical areas. This study aims to assess the impact of spatial variability of Young's modulus of the asphalt layer on the critical pavement strain by using the random finite difference method (RFDM). The scale of fluctuation (SOF), as a key influencer, and the role of weak subgrade in the statistical effect caused by asphalt spatial variability is especially focused on by conducting a parametric study. Several conclusions are drawn. (a) The spatial variability in the asphalt layer can have adverse effects on the pavement response by increasing the mean value and inducing considerable variability (in terms of coefficient of variation). (b) A value of SOF is found to have the most unfavourable statistical effect on the critical strain. (c) The presence of a weak subgrade can aggravate the adverse effect of spatially variability.

Keywords: pavement; spatial variability; Random finite difference method; scale of fluctuation; Young's Modulus.

### 1. Introduction

Spatial variability is one of the main sources of material uncertainties (Phoon & Kulhawy, 1999). In the past years, many studies have been undertaken to better understand the impact of the spatial variability on the structural response, for instance in the case of foundations (Fenton & Griffiths, 2005), tunnels (Huang et al., 2017), and slopes (Gong, et al., 2020). In these studies, the probabilistic finite element method (PFEM) or the probabilistic finite difference method (PFDM) have been used to characterize the spatially variable materials and explore their effect on the structural response. However, their application to pavement structure has been comparatively limited. Lua & Sues (1996) presented the first RFEM application to study the impacts of spatial variability on pavement life. Ali et al. (2013) and Vaillancourt et al. (2014) investigated the effect of spatially varied modulus in subgrade on the pavement responses. Titi et al. (2014) investigated the spatial variability of the base layer from FWD (falling weight deflectometer) tests and assessed their impact on pavement performance from existing pavement data. These studies have demonstrated that the structural response of pavement could be adversely affected by the spatial variability of the base and subgrade layers. However, few studies have assessed the effect of the spatial variability in the asphalt layer on pavement performance. The SOF is the distance beyond which the properties at two points can be assumed to be independent. Its effects on pavement responses remain unclear.

The goal of this study is to investigate the effect of spatial variability in the modulus of the asphalt layer on the pavement response by using the random finite difference method (RFDA). The horizontal tensile strain at the bottom of the asphalt layer (denoted by  $\varepsilon_h^{ac}$ ) is selected as the critical pavement response variable. The locations of this critical strain are assumed to occur along the vertical axis directly below the tire area. To avoid the sophisticated coupling effect of different sources of uncertainties, only the modulus of the asphalt layer is treated as a spatially varied property. All other parameters are set as deterministic. The SOF, as a key influencer, and the role of weak subgrade in the statistical effect caused by asphalt spatial variability are especially focused on by conducting a parametric study.

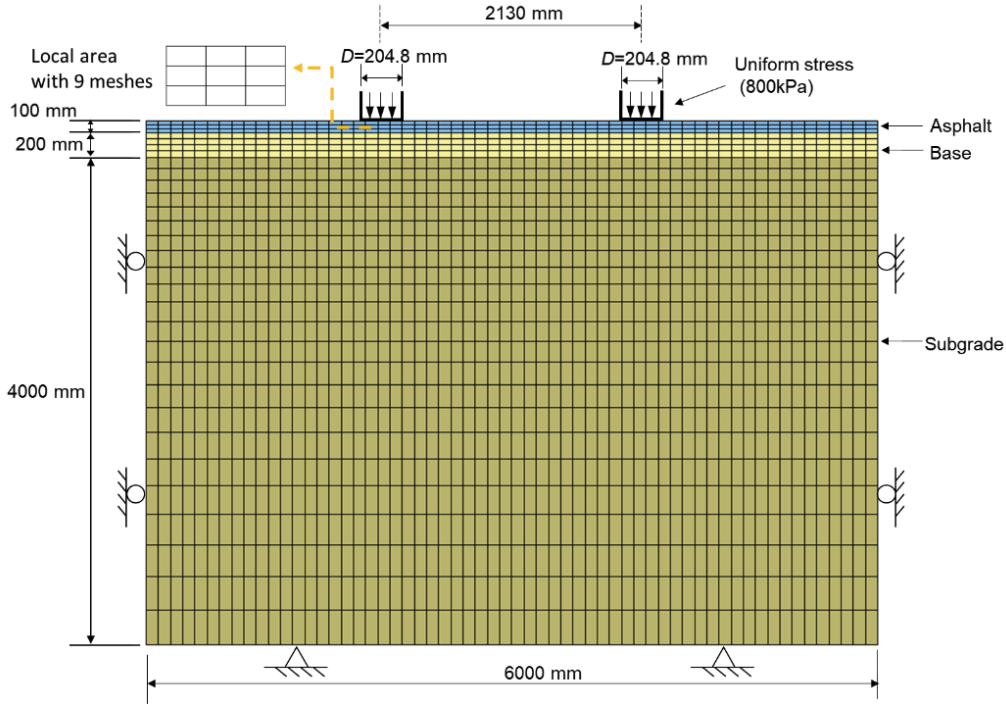
### 2. Random finite difference Method

#### 2.1 Numerical modelling

A typical three-layered pavement structure reported in Austroads (2017) guide is selected as the subject of this study. Figure 1 shows the three-layered pavement structure modelled with FLAC2D version 5.0 software. The asphalt layer material was set as linear elastic materials, and both the base and subgrade layers were set as the Mohr-Coulomb constitutive model. Table 1 lists the material properties.

**Table 1.** Material properties of the pavement model

| Material           | Asphalt layer        | Base layer           | Subgrade layer         |
|--------------------|----------------------|----------------------|------------------------|
| Constitutive model | Linear elastic       | Mohr-Coulomb         | Mohr-Coulomb           |
| Thickness          | 100 mm               | 200 mm               | 4000 mm                |
| Young's modulus    | 2000 MPa             | 200 MPa              | 30 MPa                 |
| Poisson's ratio    | 0.44                 | 0.35                 | 0.28                   |
| Cohesion           | -                    | 5kPa                 | 30 kPa                 |
| Friction angle     | -                    | 40°                  | 25°                    |
| Unit weight        | 24 kN/m <sup>3</sup> | 20 kN/m <sup>3</sup> | 18.2 kN/m <sup>3</sup> |

**Figure 1.** Finite difference model of the three-layered pavement

## 2.2 Modelling and discretization of random fields

As mentioned above, Young's modulus of the asphalt layer ( $E$ ) is set as spatially variable. The spatial variability is represented by a continuous and stationary random field as a vector of random variables  $E(\mathbf{X}_i)$  via the random field discretization process. The lognormal distribution is adopted to characterize the variability of  $E$  with its mean  $\mu_E = 2000$  MPa, and coefficient of variation  $COV_E = 30\%$ . As  $E$  is lognormally distributed,  $\ln(E)$  follows a normal distribution with its mean  $\zeta_{\ln E}$  and standard deviation  $\lambda_{\ln E}$  given by:

$$\zeta_{\ln E} = \sqrt{\ln(1 + COV_E^2)} \quad (1)$$

$$\lambda_{\ln E} = \ln \mu_E - \frac{1}{2} \zeta_{\ln E}^2 \quad (2)$$

The exponential autocorrelation function,  $\rho(x, y)$ , adopted by Huang et al. (2013) is used:

$$\rho(x, y) = \exp\left(\frac{-2|x|}{\delta_x}\right) \exp\left(\frac{-2|y|}{\delta_y}\right) \quad (3)$$

where  $\delta_x$  and  $\delta_y$  denote the horizontal and vertical scale of fluctuation (SOF). In this study, the asphalt modulus is set as isotropic random field. So  $\delta_x$  and  $\delta_y$  are equal (denoted by  $\delta$  or SOF hereinafter).

A normally distributed random field  $G(\mathbf{X}_i)$  is first generated using the Karhunen-Loève expansion with zero mean, unit variance, an autocorrelation function  $\rho(x, y)$  and Monte-Carlo runs of 100. Then the lognormally distributed random field  $E(\mathbf{X}_i)$  is obtained by:

$$E(\mathbf{X}_i) = \exp[\lambda_{\ln E} + \zeta_{\ln E} G(\mathbf{X}_i)] \quad (4)$$

## 2.3 Parametric study

The RFD is developed by combining the finite difference method with the random field based on Monte-Carlo simulations. A parametric study was conducted by adopting different SOFs and subgrade moduli ( $E_{sub}$ ). Six

cases with different SOFs shown in Table 2 are used to explore the effect of SOF on  $\epsilon_h^{ac}$ . To minimize the influence of the mesh size and aspect ratio, the SOF in most cases (except for Case-1) is no smaller than the largest dimension of the meshes (0.1m). A length ratio  $R_L = \delta/D$  is defined to measure the relative length of SOF over the diameter of the loading area ( $D$ ). To determine the role of weak subgrade in the statistical effect caused by asphalt spatial variability, each case in Table 2 is conducted with 3 different  $E_{sub}$ : 30MPa, 10MPa, and 5MPa. Figure 2 shows 6 realizations of random fields with different SOFs. It is obvious that as SOF increases, the modulus of the asphalt in the vicinity of the loading area begins to vary slowly and tends to be homogeneous at  $D$  scale.

**Table 2. Scale of fluctuations of 6 Cases**

| Case   | $\delta$ (m) | $R_L = \delta/D$ |
|--------|--------------|------------------|
| Case-1 | 0.05         | 0.24             |
| Case-2 | 0.1          | 0.49             |
| Case-3 | 0.2          | 0.98             |
| Case-4 | 0.3          | 1.46             |
| Case-5 | 0.4          | 1.95             |
| Case-6 | 0.5          | 2.44             |

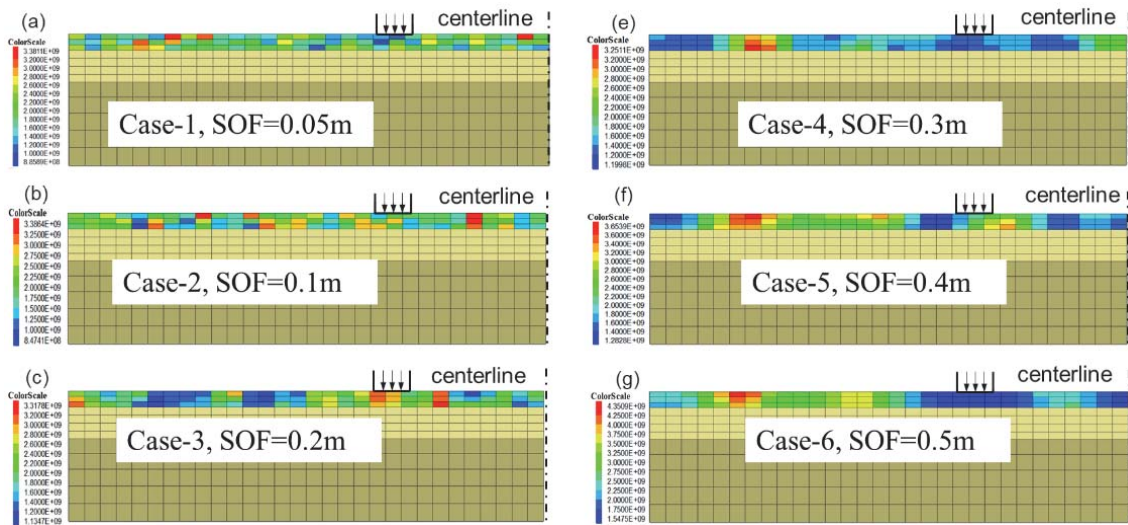


Figure 2. Random fields of Young's modulus in the asphalt layer for 6 cases.

### 3. Influence of the scale of fluctuation and the subgrade modulus

#### 3.1 Effect of SOFs and weak subgrades on the mean value of $\epsilon_h^{ac}$

Figure 3 (a) shows the results of the parametric evaluation illustrating the effect of SOF on the mean value of  $\epsilon_h^{ac}$ , for 3 kinds of subgrade moduli. The non-dotted lines denote the deterministic (DT) strains. As shown in Figure 3 (a), all the mean values in the spatially variable (SV) cases are higher than the corresponding deterministic strains. This indicates that the assumption of asphalt heterogeneity can underestimate the critical strains.

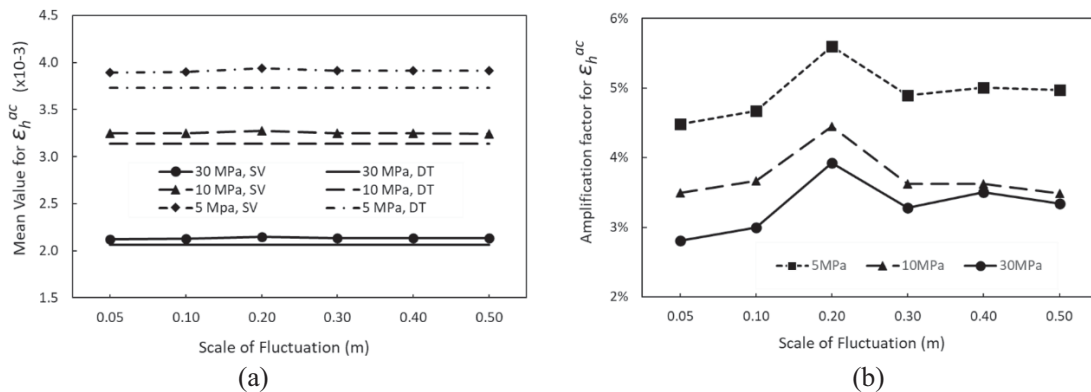


Figure 3. Effect of scales of fluctuation on: (a) the mean  $\epsilon_h^{ac}$ ; and (b) the amplification factor for different subgrades.

Figure 3 (a) also shows that the variation of SOF seems to have marginal effects on the mean  $\varepsilon_h^{ac}$ . To further explore this, an amplification factor  $R_{mean}$  is defined to quantify the relative increase of the mean  $\varepsilon_h^{ac}$  in spatial variable cases as opposed to the deterministic strains:

$$R_{mean} = \left( \frac{\bar{\varepsilon}}{\varepsilon_{det}} - 1 \right) \times 100\% \quad (5)$$

where  $\bar{\varepsilon}$  is the mean  $\varepsilon_h^{ac}$  in a spatial variable case, and  $\varepsilon_{det}$  is the corresponding deterministic strain.

Figure 3 (b) shows the variation of  $R_{mean}$  with SOF under 3 kinds of subgrade conditions. The most unfavourable amplification factor for  $\varepsilon_h^{ac}$  appears when SOF=0.20m (i.e., when SOF=  $D$ ) under all subgrade conditions. It also indicates that the weaker the subgrade, the greater the amplification factors for  $\varepsilon_h^{ac}$ . This implies that a weak subgrade can aggravate the adverse effect of asphalt spatial variability on  $\varepsilon_h^{ac}$  by increasing the amplification effect.

### 3.2 Effect of SOFs and weak subgrades on extreme values of critical strain

All the spatially variable cases are simulated with 100 realizations based on Monte-Carlos simulations. For a specific case, although the mean  $\varepsilon_h^{ac}$  is higher than the deterministic strain, the critical strain of one certain realization is either higher or lower than the corresponding deterministic strain. Especially, the maximum and minimum strain value in an ensemble of realizations reflects the range of the strain distribution and to some extent, the scatter of the strain resulting from the spatial variability. To measure the relative deviation of the extreme strains (i.e. maximum and minimum strain) from the deterministic strains, two deviation ratios,  $R_{max}$  and  $R_{min}$  are defined as follows:

$$R_{max} = \frac{\varepsilon_{max}}{\varepsilon_{det}} - 1 \quad (6)$$

$$R_{min} = 1 - \frac{\varepsilon_{min}}{\varepsilon_{det}} \quad (7)$$

where  $\varepsilon_{max}$  and  $\varepsilon_{min}$  are the maximum and minimum  $\varepsilon_h^{ac}$  for a specific case.

Figure 4 presents the deviation ratios of the extreme  $\varepsilon_h^{ac}$  and illustrates the effect of SOF on the two deviation ratios under different subgrade conditions. Both  $R_{max}$  and  $R_{min}$  changes with the variation of SOFs but not in strictly monotone trends. The comparison of the deviation ratios for different subgrades reveals that the weaker the subgrade, the further the extreme  $\varepsilon_h^{ac}$  deviates from the corresponding deterministic strain. In other words, a weaker subgrade tends to induce a wider distribution (and probably greater variability) of  $\varepsilon_h^{ac}$ . In addition, for a specific SOF,  $R_{max}$  is more sensitive to the decrease of subgrade modulus than  $R_{min}$  does. Therefore, from a statistical point of view, a weaker subgrade not only increases the mean value but also the standard deviation of  $\varepsilon_h^{ac}$  by significantly increasing large strains.

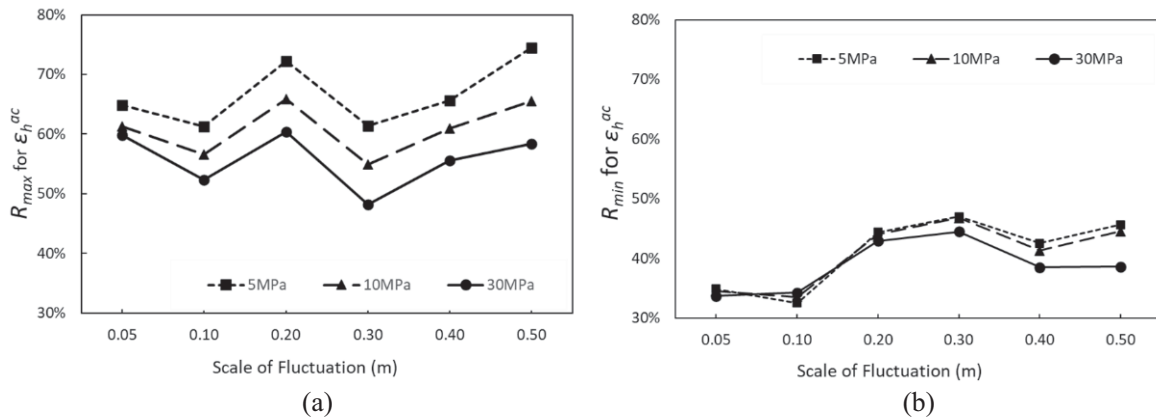


Figure 4. Effect of SOFs on: (a)  $R_{max}$ , and (b)  $R_{min}$  of  $\varepsilon_h^{ac}$  for different subgrades.

Figure 4 (a) also shows that the largest  $R_{max}$  generally occurs at SOF=0.2m. Figure 4 (b) indicates that  $R_{min}$  is generally larger when SOF ranges from 0.2m to 0.3m. So, comparatively large  $R_{max}$  and  $R_{min}$  both occur at the SOF of 0.2m. This implies that the SOF of 0.2m can result in the widest distribution of  $\varepsilon_h^{ac}$ . Figure 4 also reveal that for every case, the  $R_{max}$  is higher than the  $R_{min}$ . This implies that  $\varepsilon_h^{ac}$  have a skewed rather than a symmetrical distribution. Another important implication is that the effective modulus is low-modulus dominated and thus the softer asphalt zones have a greater influence on the critical strain than the stiffer asphalt zones do. This low-modulus dominating effect provides a reasonable explanation for why the mean values in the spatial variable cases are higher than the corresponding deterministic strain.

**3.3 Effect of SOFs and weak subgrades on the COV of critical strain**

Figure 5 shows the influence of SOF on the COV of  $\epsilon_h^{ac}$  for different subgrades. For every subgrade, the maximum COV of  $\epsilon_h^{ac}$  is observed at SOF=D (i.e., 0.2m). For a specific SOF, the weaker the subgrade, the larger the COV of  $\epsilon_h^{ac}$ . This confirms the finding from Figure4 that the presence of a weak subgrade can aggravate the magnitude of variability in the  $\epsilon_h^{ac}$ . It is also worth noting that the COVs of  $\epsilon_h^{ac}$  in all cases (ranging from 18% to 25%) are lower than the COV of asphalt modulus (30%). So in a probabilistic analysis considering spatial variability, the COV of the material properties can not be directly adopted as the COV of the critical structural response due to the interaction between the pavement layers.

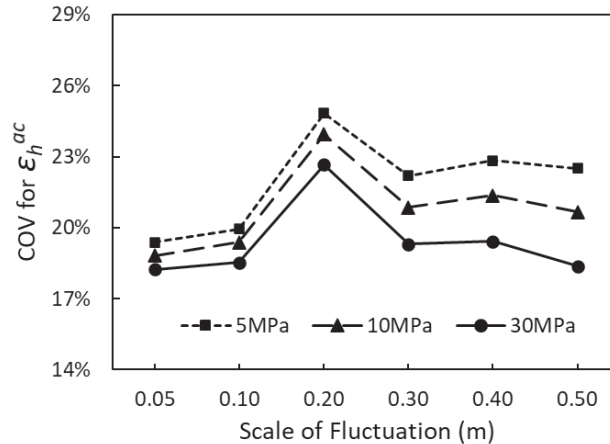


Figure 5. Effect of SOFs on the COVs of  $\epsilon_h^{ac}$  for different subgrades.

**3.4 Statistical explanation for the worst SOF of  $\epsilon_h^{ac}$**

As seen in Figure 3, Figure 4 and Figure 5, there exists a SOF which leads to the most unfavourable statistical effect on  $\epsilon_h^{ac}$ . The maximum amplification factors of mean value, the maximum deviation ratios of extreme values, and the maximum COVs always occur at SOF=D (D is the dimension of the tyre loading areas). To explore the reasons, a local modulus  $E_{local}$  and two kinds of COVs about  $E_{local}$  are introduced.

As illustrated in Figure 1, the asphalt layer under the tyre loading footprint is simulated with 9 meshes in the finite difference model. The average modulus of the loading area is defined as the local modulus  $E_{local}$ . This definition is based on the assumption that the material spatial variability at elements away from the tire load has little effect on key pavement responses in the asphalt layer (Lua & Sues,1996).

Figure 6 illustrates the fluctuation of  $E_{local}$  and  $\epsilon_h^{ac}$  among different realizations when SOF is 0.3m with  $E_{sub}=10\text{MPa}$  (It shows similar results when SOF=0.2m or other values). It shows that a large  $E_{local}$  always corresponds to a small  $\epsilon_h^{ac}$  (or vice versa). This not only indicates a negative correlation between the  $E_{local}$  and  $\epsilon_h^{ac}$ , but also confirms the validity of using the local area below the tyre pressure as the dominating area for the  $\epsilon_h^{ac}$ . Therefore, the effect of the spatial variability in the asphalt layer on the  $\epsilon_h^{ac}$  must be exerted by influencing the modulus distribution in the defined local area in Figure1. In other words, the magnitude and the degree of the variation of modulus in the local area dictate how much the  $\epsilon_h^{ac}$  is affected by the asphalt spatial variability.

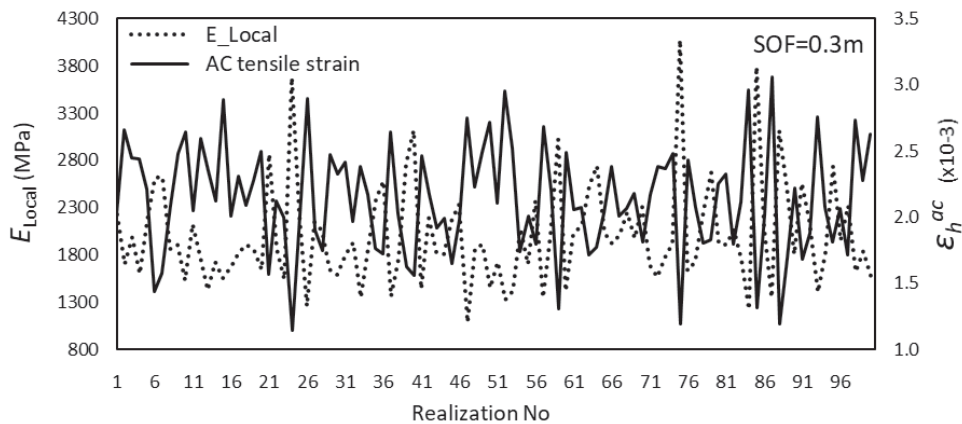


Figure 6. Fluctuation of  $E_{local}$  and  $\epsilon_h^{ac}$  among different realizations with SOF=0.3m and  $E_{sub}=10\text{MPa}$ .

The coefficient of variation of the moduli of the nine meshes in the local area is defined as  $COV_{local}$ . Both  $E_{local}$  and  $COV_{local}$  vary with different realizations. The coefficient of variation of  $E_{local}$  of the 100 realizations

is defined as  $COV_1$ . The average  $COV_{local}$  of the 100 realizations is defined as  $COV_2$ . Both  $COV_1$  and  $COV_2$  measure the degree of the variation of modulus in the local area.

Figure 7 shows how  $COV_1$  and  $COV_2$  change with the variation of SOF:  $COV_1$  increases and  $COV_2$  decreases as SOF increases. Both  $COV_1$  and  $COV_2$  changes rapidly when SOF ranges from  $0.5D$  to  $1.5D$  and tend to intersect at  $SOF=D$ . As illustrated in Figure 7,  $COV_1$  and  $COV_2$  can not reach their own maximum at the same SOF because they show opposite trends versus SOF. But both  $COV_1$  and  $COV_2$  are above their average when SOF approximates  $D$ . This explains why  $D$  is the worst SOF that induces the most serious variation of modulus in the local area and thus leads to the most unfavourable statistical effect on  $\varepsilon_h^{ac}$ .

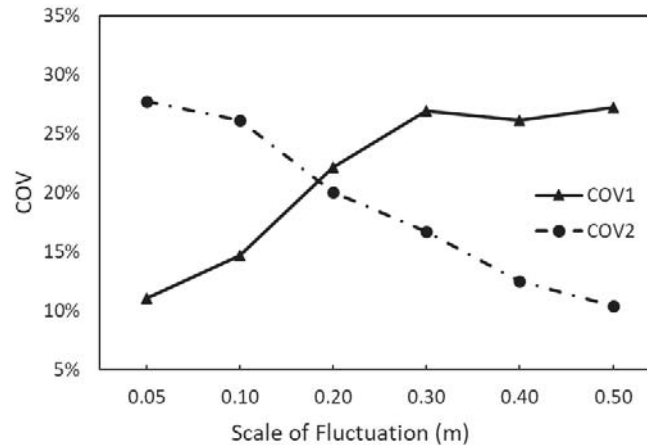


Figure 7. Effect of SOF on two kinds of COV of  $E_{local}$

It should be noted that when SOF is extremely small or extremely large, the asphalt modulus tends to be homogeneous at  $D$  scale. So the two extreme situations are similar to a random variable analysis. As shown in Figure 3(b), Figure 5 and Figure 7, when SOF is very small or large compared with the value of  $D$ , the effects of spatial variability in terms of the amplification factor  $R_{mean}$  and the COV of  $\varepsilon_h^{ac}$  tend to be stabilized and remain at this level. But when SOF is in the similar order of  $D$ , the variation of SOF brings obviously different magnitude of effects on the statistical results of  $\varepsilon_h^{ac}$ . So the probabilistic analysis based on the conventional random variable model is not adequate to fully explore the effect of spatial variability. As the conventional random variable model assumes material properties to be homogeneous and only considers the variations in the intensity of the properties

#### 4. Conclusions

This study investigates the impact of spatial variability of the modulus in the asphalt layer on the key pavement response in terms of  $\varepsilon_h^{ac}$  by using Random finite difference methods. The scale of fluctuation (SOF), as a key influencer, and the role of weak subgrade in the statistical effect caused by asphalt spatial variability are especially focused on by conducting a parametric study. Based on the results obtained in this research, it has been found that the spatial variability of Young's modulus in the asphalt layer can increase the critical strain in the asphalt layer. The most unfavourable statistical effect on the tensile strain in the asphalt layer occurs when the SOF equals the size of the loading area. The stiffness of the subgrade can greatly affect the propagation of uncertainties in the critical strain of the asphalt layer. Weaker subgrades tend to amplify the uncertainties. The preliminary findings may help with future studies on the performance of flexible pavement on spatially variable base course and subgrade layers.

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