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Coupled Characterization of Stratigraphic and Geo-Properties Uncertainties and Evaluation of the Influences on Geotechnical Performance

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Abstract: The primary purpose of site characterization is to characterize the subsurface stratigraphic configuration and the associated geo-properties. Due to the inherent spatial variability of geological bodies and limited site investigation efforts, uncertainty is often unavoidable in the derived subsurface stratigraphic configuration and the associated geo-properties. In previous studies, substantial research has been undertaken to address geo-properties uncertainty and evaluate its effects on the performance and design of geotechnical systems. In contrast, characterization of the stratigraphic uncertainty and analyses of its impact on the predicted performance of geotechnical systems have been relatively limited. Further, the literature has hardly reported the studies on a coupled characterization of stratigraphic and geo-properties uncertainties. The authors have recently developed a conditional random field approach for the coupled characterization of stratigraphic and geo-properties uncertainties to fill this technology gap. In this coupled approach, the spatial correlation of the stratum existence between different subsurface elements and that of geo-properties are captured by two independent autocorrelation functions; and, the coupling between the stratigraphic configuration and the spatial distribution of geo-properties is realized by updating the autocorrelation matrix and statistics of geo-properties in non-borehole elements based on the sampled stratigraphic configuration. This paper reviews the key components of the coupled characterization approach, including methodology and implementation procedure. Then, the effectiveness of this characterization approach is demonstrated through a series of examples, and the superiorities over the existing characterization approaches are discussed. Finally, case studies are undertaken to examine the influences of the stratigraphic and geo-properties uncertainties on the geotechnical performance evaluation.

Keywords: Site Characterization; Stratigraphic Uncertainty; Geo-properties Uncertainty; Random Field; Geotechnical Performance.

1 Introduction

Complete knowledge of the formation of a geological body is challenging to acquire; in a specific project, only limited site investigation efforts can be afforded. Thus, the subsurface stratigraphic information and geo-material properties (referred to as geo-properties) are known only at sparse locations. In contrast, such information at other positions cannot be known and must be interpreted from that collected through site investigations. The incomplete knowledge of the formation of the geological body, along with insufficient site investigation efforts, leads to uncertainty in the interpreted geological model (i.e., subsurface stratigraphic configuration and spatial distribution of geo-properties), which would lead to uncertainty in the predicted performance of the geotechnical system. As an outcome, the design of the geotechnical system could be compromised. Thus, the characterization of the uncertainty of the interpreted geological model and the evaluation of its effect on the predicted performance of the geotechnical system are essential tasks facing the geologist and the engineer (Juang et al., 2019).

The uncertainty of a built geological model at a site might be divided into stratigraphic uncertainty and geo-properties uncertainty (Wu and Wong, 1981; Li et al., 2016; Gong et al., 2021). The stratigraphic uncertainty mainly refers to the uncertainty of boundaries between different strata, and the geo-properties uncertainty refers to the inherent spatial variation of geo-properties within each stratum. It is noted that substantial research has been conducted to address the geo-properties uncertainty based on geostatistics, random variables, and random field theories (Vanmarcke, 1980; Fenton, 1999; Cao and Wang, 2013; Hicks et al., 2014; Gong et al., 2018), and the effects of the geo-properties uncertainty on the design and performance of geotechnical systems have been extensively investigated (Griffiths and Fenton, 2004; Baecher and Christian, 2005; Juang et al., 2013; Gong et al., 2014&2016). On the contrary, the characterization of the stratigraphic uncertainty and analyses of its effect on the predicted performance of geotechnical systems have been relatively limited (Li et al., 2016; Wang et al., 2016; Crisp et al., 2019; Shi and Wang, 2021a; Gong et al. 2020), even though the characterization of the stratigraphic configuration is also a significant issue in various geoscience disciplines.

In conventional geotechnical practices, the stratigraphic configuration at a site is usually obtained through spatial interpolations of borehole stratigraphies, coupled with local experience (e.g., sedimentology, surface geology, geologic/geomorphic setting, and structural geology). Various interpolation or geostatistical methods are available for constructing the subsurface stratigraphic configuration (Schloeder et al., 2001; Chilès et al., 2004). However, these approaches often estimate the most-probable stratigraphic configuration while the associated uncertainty is ignored. The early attempt in the characterization of stratigraphic uncertainty was reported by Evans (1982), Tang et al. (1989), and Halim (1991). However, broad applications of these methods are hindered by the challenge of determining the spatial correlation structures of the geological heterogeneity. To overcome this obstacle, the probabilistic modeling approaches, based upon the coupled Markov chain (CMC) (Carle, 2000; Hu and Huang, 2007; Li et al., 2016), stochastic Markov random field (MRF) (Norberg et al., 2002; Wang et al., 2016), random field theory (Gong et al., 2020&2021; Zhao et al., 2021), and iterative convolution eXtreme Gradient Boosting model (IC-XGBoost) (Shi and Wang, 2021a), have recently been proposed.

It is worth noting that a geotechnical system's performance is affected by both stratigraphic uncertainty and geo-properties uncertainty, and there is an apparent coupling between the stratigraphic configuration and the spatial distribution of geo-properties. However, studies that treat both stratigraphic uncertainty and geo-properties uncertainty in a single geological model are relatively limited (Deng et al., 2017; Liu et al., 2020; Gong et al., 2021; Shi and Wang, 2021b). Indeed, most of the past work adopted an un-coupled modeling strategy (i.e., simulating the stratigraphic configuration first and then simulating the spatial variability of geo-properties in each stratum). For ease of a coupled characterization of the stratigraphic and geo-properties uncertainties, the authors have recently proposed a conditional random field approach (Gong et al., 2021). This paper reviews the coupled characterization approach, analyzes its effectiveness, and evaluates the effects of the stratigraphic and geo-properties uncertainties on the geotechnical performance. The article is organized as follows. First, the methodology of the coupled characterization approach is introduced. Second, the effectiveness of the coupled characterization approach is illustrated. Third, the effects of the stratigraphic and geo-properties uncertainties on geotechnical performances are discussed. Finally, the conclusions are provided.

2 Methodology for the Coupled Characterization of Stratigraphic and Geo-properties Uncertainties

In this section, the random field theory for characterizing the spatial variation of geo-properties is first reviewed; then, the random field approach proposed by the authors (Gong et al., 2020; Zhao et al. 2021) for characterizing the stratigraphic uncertainty is reviewed, finally, the approach proposed by the authors (Gong et al., 2021) for a coupled characterization of stratigraphic and geo-properties uncertainties is presented.

2.1 Random field theory for characterizing the spatial variation of geo-properties

In the context of the random field theory, a geo-property at various locations at a site is correlated to some extent in both horizontal and vertical directions, such a spatial correlation often decreases with the distance (Vanmarcke, 1980; Fenton, 1999). The spatial correlation of the geo-properties can be captured by an autocorrelation function, and various autocorrelation functions (e.g., exponential, binary noise, and second-order Markov) are available in the literature, among which the squared exponential autocorrelation function, formulated below, is very popular.

$$\rho_1(i,j) = \exp\left(-\frac{\pi d_{h(ij)}^2}{{r_{h1}}^2} - \frac{\pi d_{v(ij)}^2}{{r_{v1}}^2} \right) \tag{1}$$

where $\rho_1(i,j)$ is the autocorrelation between the geo-property in element i (i = 1, 2, ⋯, n_e) and that in element j (j = 1, 2, ⋯, n_e) in a two-dimensional (2-D) problem, in which n_e is the number of subsurface elements discretized at the concerned site; $d_{h(ij)}$ and $d_{v(ij)}$ are the horizontal and vertical distances of these two elements, respectively; and, r_{h1} and r_{v1} are the scales of fluctuation in the horizontal and vertical directions, respectively.

For ease of applying the random field theory in characterizing the spatial variability of geo-properties, the geometry domain of concern is first discretized into a set of smaller sub-domains (or elements), thus permitting an assignment of different geo-properties to different elements. Then, the spatial correlations of geo-properties among the discretized elements are estimated based upon the autocorrelation function and statistical information adopted, from which an autocorrelation matrix can be formulated. It should be informed that the property within a specified element is usually captured by a fixed value, and thus, the spatial averaging of the spatial correlations should be considered in the formulation of the autocorrelation matrix and the statistics of geo-properties within each element (Xiao et al., 2016). Based on the obtained autocorrelation matrix and statistics of the geo-properties, the geo-properties in the discretized elements are readily sampled with a variety of sampling methods such as the covariance matrix decomposition method, local average subdivision method, turning-band method, and fast Fourier transformation method (Fenton, 1994).

2.2 Random field approach proposed for characterizing the stratigraphic uncertainty

It is noted that the spatial distribution of the strata and that of the geo-properties within a stratum at a site should share similar features, as both are products of common deposit histories and tectonic activities. Inspired by the concept of the random field theory, the spatial correlation of the stratum existence between two elements could also be characterized by an autocorrelation function, such as the squared exponential autocorrelation in a 2-D site characterization problem (see Fig. 1a).

$$\rho_2(i,j) = \exp\left(-\frac{\pi d^2_{p(ij)}}{{r_{p2}}^2} - \frac{\pi d^2_{v(ij)}}{{r_{v2}}^2}\right) \tag{2}$$

where $\rho_2(i,j)$ is the spatial correlation between the existence of a stratum in element i (i = 1, 2, …, n_e) and that in element j (j = 1, 2, …, n_e); r_{p2} and r_{v2} represent the scales of fluctuation of the stratum existence that are parallel and vertical to the stratigraphic dip, respectively; and, $d_{p(ij)}$ and $d_{v(ij)}$ are the distances between these two elements that are parallel and vertical to the stratigraphic dip, respectively (see Fig. 1b). The calculation of the terms $d_{p(ij)}$ and $d_{v(ij)}$ is similar to that commonly adopted in hydrology engineering (Zhao et al., 2021).

It is expected that if two elements exhibit a high spatial correlation of the stratum existence, the chance that these two elements share the same stratum could be high. As such, the probability of the presence of a particular stratum in an element could be assessed based on the estimated spatial correlation of the stratum existence. For example, the probability of the presence of a stratum k in element i, denoted as $P_{2k}(i)$, could be approximated as follows (Gong et al., 2020; Zhao et al., 2021).

$$P_{2k}(i) = \frac{\rho_{2k}(i)}{\rho_2(i)} = \frac{\sum_{l=1}^{l=n_B}\left[\rho_{2k}(i,l)\cdot Index(l,k)\right]}{\sum_{h=1}^{h=m}\left\{\sum_{l=1}^{l=n_B}\left[\rho_{2h}(i,l)\cdot Index(l,h)\right]\right\}} \tag{3}$$

where $\rho_{2k}(i)$ is the accumulated spatial correlations of the stratum existence between element i (i = 1, 2, …, n_e) and all borehole elements that yield stratum k (k = 1, 2,…, m), in which m is the number of strata revealed at this site; $\rho_2(i)$ is the accumulated $\rho_{2k}(i)$ within all the strata; n_B is the number of borehole elements; and, $Index(l, k)$ is an indicator function of borehole element l (l = 1, 2, …, n_B), the value of which is equal to 1.0 when the stratum observed in borehole element l yields stratum k, otherwise $Index(l, k)$ = 0.

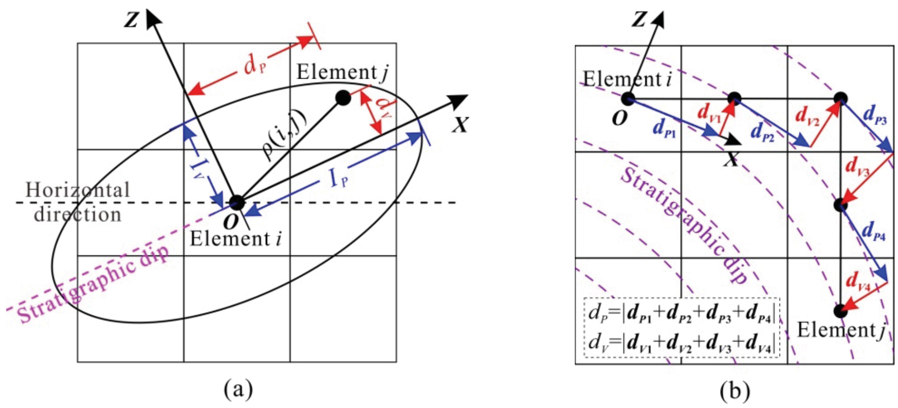


Figure 1. Illustration of the spatial correlation of stratum existence and that of geo-properties (modified from Zhao et al., 2021): (a) Squared exponential autocorrelation function-based spatial correlation; (b) Distances calculation between two elements in the random field approach for characterizing the stratigraphic uncertainty.

According to the calculated probability of the existence of each stratum in every element, the stratigraphic configurations are readily sampled with the modified random field approach (Zhao et al., 2021). In this modified random field approach, the initial stratigraphic configurations are first sampled with the conditional random field theory, based upon the calculated probability of the stratum existence. Then, the maximum-a-posteriori (MAP) estimates of these initial stratigraphic configurations are derived using the Markov Chain Monte Carlo (MCMC)-based updating algorithm (Wang et al., 2016) and which are taken as the final stratigraphic realizations.

2.3 Approach for coupled characterization of stratigraphic and geo-properties uncertainties

To explicitly consider the influence of the stratigraphic configuration on the spatial distribution of geo-properties, the spatial correlations of the geo-properties must be updated based on the sampled realizations of the subsurface stratigraphic configuration. For simplicity, the criterion established below can be adopted for updating the spatial correlation of geo-properties (Gong et al., 2021).

$$\rho_1'(i,j) = \begin{cases} \rho_1(i,j) & \text{if } s(i) = s(j) \\ 0 & \text{if } s(i) \neq s(j) \end{cases} \tag{4}$$

where $\rho_1'(i, j)$ is the updated spatial correlation of geo-properties between element i ($i = 1, 2, \ldots, n_e$) and element j ($j = 1, 2, \ldots, n_e$); and, $s(i)$ and $s(j)$ are the stratum sampled in element i and that in element j, respectively. Based on the updated spatial correlations of geo-properties, the geo-properties at the site could readily be sampled with the conditional random field theory (Liu et al., 2017; Gong et al., 2018).

Note that a prerequisite for the coupled characterization of stratigraphic and geo-properties uncertainties is an accurate characterization of the spatial correlation of the stratum existence and that of the geo-properties. The maximum likelihood principle (Fenton, 1999; Gong et al., 2014) is used to calibrate the spatial correlation of the stratum existence and that of the geo-properties, based on the stratigraphies and the geo-properties collected at borehole positions, respectively. For simplicity, the cross-correlation between the stratum existence and the geo-properties is ignored, and calibrations of the spatial correlation of the stratum existence and that of the geo-properties are conducted separately. The readers could be referred to Zhao et al. (2021) for more details on the characterization of the spatial correlation between stratum existence and that of the geo-properties. Further, the information entropy of the sampled stratigraphic configurations and the standard deviation of the sampled geo-properties, formulated below, are computed to characterize the stratigraphic and geo-properties uncertainties.

$$E_E(i) = -\sum_{k=1}^{k=m}\left[p_{2k}(i)\log p_{2k}(i)\right] \tag{5}$$

$$\sigma_g(i) = \sqrt{\sum_{j=1}^{j=n_R}\left\{g^j(i) - \sum_{j=1}^{j=n_R}\left[g^j(i)\right]\Big/n_R\right\}^2 \Big/ (n_R - 1)} \tag{6}$$

where $E_E(i)$ is the information entropy in non-borehole element i ($i = 1, 2, \ldots, n_e - n_B$); $p_{2k}(i)$ is the probability of stratum k ($k = 1, 2, \ldots, m$) assigned to non-borehole element i, which could be estimated based on the sampled realizations of the stratigraphic configuration; $\sigma_g(i)$ is the standard deviation of the sampled geo-properties in non-borehole element i ($i = 1, 2, \ldots, n_e - n_B$); and, $g^j(i)$ is the geo-property sampled in non-borehole element i in the jth ($j = 1, 2, \ldots, n_R$) realization of the geo-properties, in which n_R is the number of sampled realizations of the geo-properties (or stratigraphic configurations). According to the convergence of the information entropy of the stratigraphic configurations and that of the standard deviation of the geo-properties, the number of the samples adopted in the characterization of the stratigraphic and geo-properties uncertainties can be determined.

From there, the coupled characterization of stratigraphic and geo-properties uncertainties at a site could be implemented with the following steps: 1) estimate the spatial correlation structure of the stratum existence and that of the geo-properties with the maximum likelihood principle (Gong et al., 2021); 2) sample the stratigraphic configurations using the modified random field approach (Zhao et al., 2021); 3) update the spatial correlations of the geo-properties, based on the stratigraphic configurations sampled; 4) sample the spatial distributions of the geo-properties with the conditional random field theory, based on the updated spatial correlation of geo-properties.

3 Effectiveness of the Approach for Characterizing Stratigraphic and Geo-properties Uncertainties

To demonstrate the effectiveness of the reviewed approach, four examples are studied in this section. In the first example, a fold stratigraphic structure is analyzed to demonstrate the effectiveness of the modified random field approach in modeling the stratigraphic uncertainty. In the second and third examples, comparative analyses are conducted to depict the advantages of the modified random field approach over the coupled Markov chain (CMC) and stochastic Markov random field (MRF)-based approaches, respectively. In the fourth example, a case study of a seabed site for an offshore wind farm in central-western Taiwan is undertaken to illustrate the effectiveness of the reviewed approach for a coupled characterization of the stratigraphic and geo-properties uncertainties.

3.1 Example 1: Application to a complex fold stratigraphic structure

This example is concerned with a fold stratigraphic structure, as shown in Fig. 2(a). The dimension of this site is 100 m by 30 m, and five equally spaced boreholes, each with a length of 28 m, are conducted at this site. The leftmost borehole is located at the left boundary while the rightmost borehole is located at the right boundary. The results from this artificial site investigation depict the existence of five strata, and the thickness of the thinnest stratum layer is 5.0 m. The stratigraphic dip in this example is not fixed but varies with the coordinate.

For ease of applying the random field approach modified in Zhao et al. (2021), the subsurface domain is discretized into 12,000 square elements, each with a size of 0.5 m by 0.5 m. Based on the maximum likelihood analysis results, the single exponential autocorrelation function is selected in this example for characterizing the spatial correlation structure of the strata, and the horizontal and vertical scales of fluctuation are 18.1 m and 5.0 m, respectively. The number of iterations in the MCMC updating process is set at 200 to derive the MAP estimates of initial stratigraphic configurations, and the number of final stratigraphic configurations is set at 1,000 to meet the convergence criterion of COV[E_A] < 0.2% (see Fig. 2b), where COV[E_A] is the coefficient of variation (COV) of the average information entropy of the sampled stratigraphic configurations in non-borehole elements. Illustrated in Fig. 2(c) is the stratigraphic uncertainty (indicated by the information entropy E_E) characterized at this site, Fig. 2(d) illustrates the probability of the existence of a stratum (i.e., Formation 3). The plots in Figs. 2(c-d) demonstrate the validity of the modified random field approach in simulating this fold stratigraphic structure, and the results are consistent with the intuition and knowledge of the fold stratigraphic structure. Compared to the deterministic stratigraphic configuration obtained with traditional linear interpolation, the uncertainty in the obtained stratigraphic configuration can be explicitly characterized using the modified random field approach.

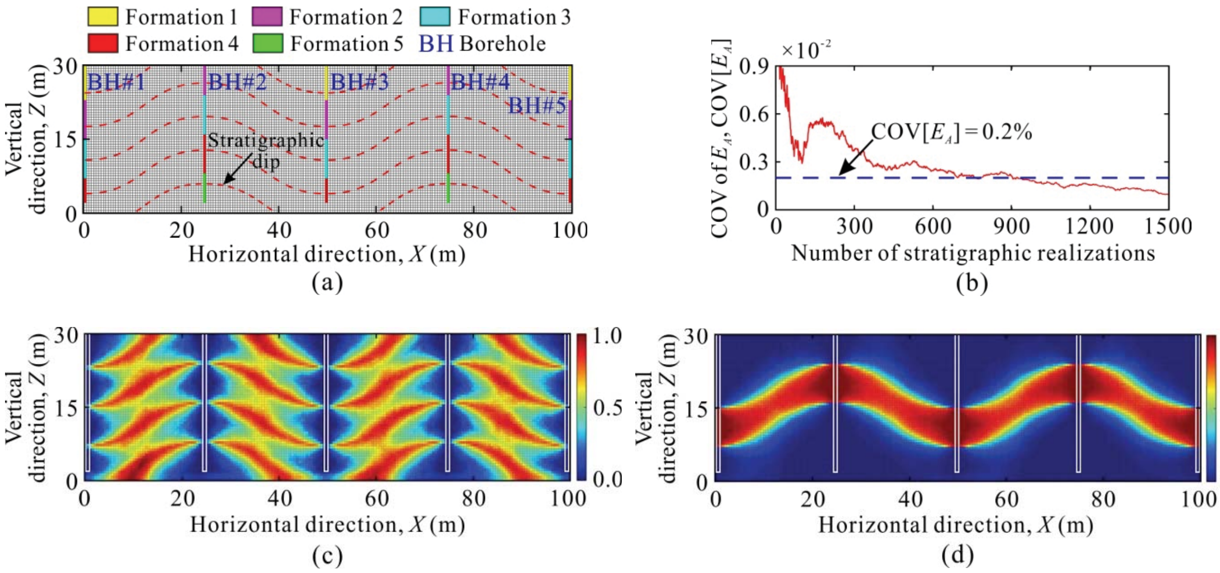


Figure 2. The geological setting, borehole exploration program, and the stratigraphic uncertainty characterization results in Example 1: (a) Geological setting and borehole exploration program; (b) Relationship between the COV of average information entropy and the number of stratigraphic realizations; (c) Spatial distribution of the information entropy; (d) Spatial distribution of the probability of the existence of Formation 3.

3.2 Example 2: Comparison with the coupled Markov chain (CMC)-based approach

The second example is concerned with a comparison with the CMC-based approach, the comparison between the reviewed approach and the CMC-based approach is undertaken by analyzing the case reported by Xiao et al. (2017). As shown in Fig. 3(a), this site is in a weathered ground, the dimension of which is 40 m by 55 m. This site is flat and no geological fault is reported. The lengths of five boreholes conducted at this site range from 44 m to 50 m, and five strata are revealed at this site, including clay alluvium, another alluvium, completely to highly decomposed granite (DG-C/H), highly to moderately decomposed granite (DG-H/M), and moderately to slightly decomposed granite (DG-M/S). The thinnest stratum layer thickness is 0.5 m while that of the thickest stratum layer is 25.5 m. It is noted that several ultra-thin layers (i.e., with a thickness of about 0.5 m) are detected through the boreholes. The distribution of these ultra-thin layers is not continuous; thus, these layers are taken as geological lenses in the stratigraphic configuration. The stratigraphic dips, as shown in Fig. 3(a), are predefined based on the borehole stratigraphies revealed and local experience.

The site is discretized into 8,800 rectangular elements, each with a size of 0.5 m by 0.5 m. According to the maximum likelihood analysis results, the single exponential autocorrelation function is selected in this example for characterizing the spatial correlation structure of the strata, the horizontal and vertical scales of fluctuation are 6.0 m and 2.0 m, respectively. The number of iterations in the MCMC updating process is set at 200 to derive the MAP estimates of initial stratigraphic configurations, and the number of final stratigraphic configurations is set at 1,400 to meet the convergence criterion of COV[E_A] < 0.2%. The comparison between the reviewed approach and the CMC-based approach focuses mainly on the spatial distribution of the strata in the sampled stratigraphic realizations, the probability of the existence of stratum DG-C/H, and the computational efficiency. Fig. 3(b) depicts one possible subsurface stratigraphic realization obtained with the reviewed approach, while Fig. 3(c) shows the result obtained with the CMC-based approach. As can be seen, the strata boundaries obtained with the reviewed approach are more consistent with the predefined stratigraphic dips, the strata boundaries are smoother. On the other hand, the strata boundaries obtained with the existing CMC-based approach tend to be linear in either horizontal or vertical direction; that is to say, abrupt changes in the spatial variation of strata are detected. Fig. 3(d) depicts the probability of the existence of stratum DG-C/H obtained with the reviewed approach, while Fig. 3(e) shows the result obtained with the CMC-based approach. Again, the probability of existence obtained with the reviewed approach is more consistent with the predefined stratigraphic dips and the intuition. The variation of the probability of existence obtained with the reviewed approach is smoother around the strata boundaries. Another observation from Figs. 3(b-e) is that the reviewed approach could characterize the strata in the subsurface elements beyond the rightmost borehole, while the CMC-based approach cannot. Finally, fewer realizations of the possible stratigraphic configuration are required by the reviewed approach for achieving the sampling convergence, in comparison to the CMC-based approach. For example, 1,400 realizations of the stratigraphic configuration are sufficient to reach the sampling convergence, while it could take the CMC-based approach 10,000 realizations to get the converged outcome (Xiao et al., 2017).

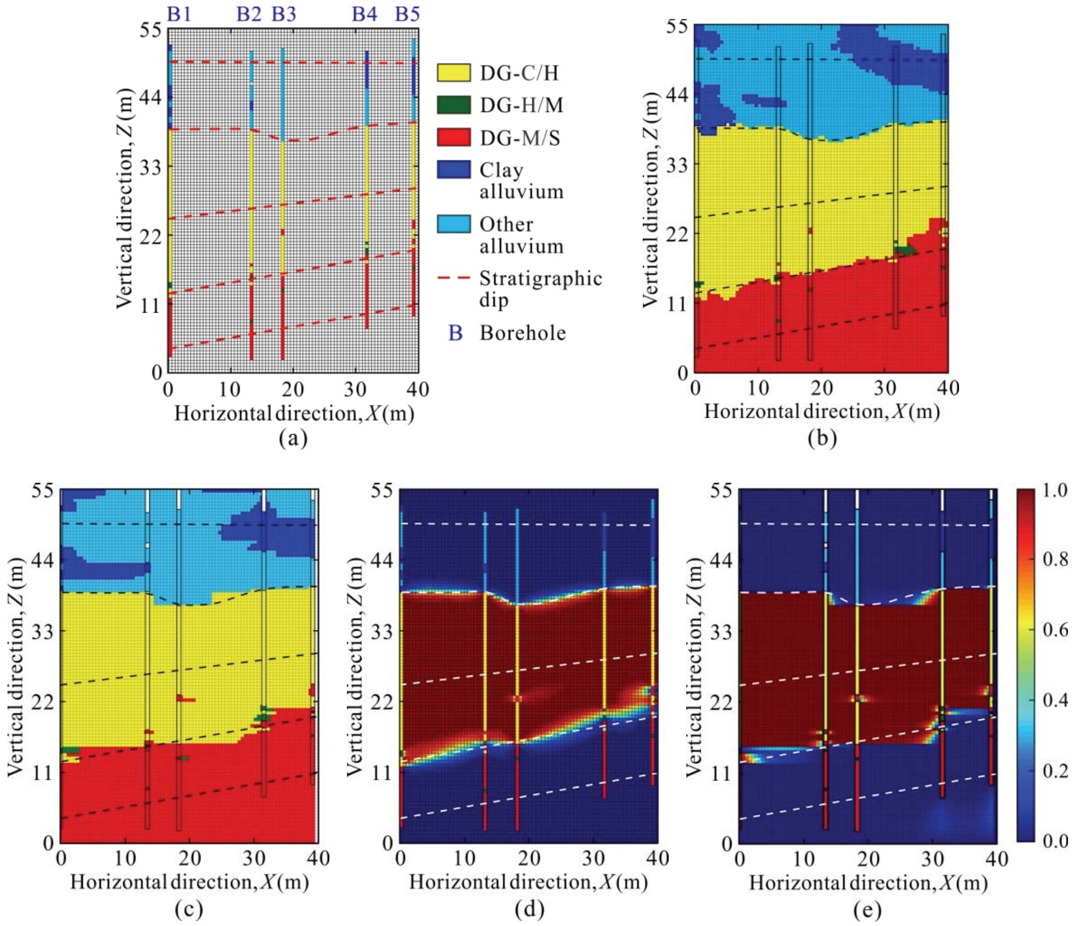


Figure 3. The geological setting, borehole exploration program, and the stratigraphic uncertainty characterization results in Example 2: (a) Geological setting and borehole exploration program; (b) One possible stratigraphic realization generated with the reviewed approach; (c) One possible stratigraphic realization generated with the CMC-based approach (from Xiao et al., 2017); (d) Probability of the existence of stratum DG-C/H generated with the reviewed approach; (e) Probability of the existence of stratum DG-C/H generated with the CMC-based approach (from Xiao et al., 2017).

3.3 Example 3: Comparison with the stochastic Markov random field (MRF)-based approach

The third example is concerned with a comparison with the MRF-based approach, the comparison between the reviewed approach and the MRF-based approach is undertaken by characterizing the site reported by Wang et al. (2016), as shown in Fig. 4(a). The dimension of this site is 150 m by 50 m, and four equally spaced boreholes are conducted at this site, each with a length of 50 m. The borehole results reveal six strata, including miscellaneous fill, alluvial deposit, clay, decomposed granitic soil, weathered granitic rock, and hard granitic rock.

The site is discretized into 30,000 square elements, each with a size of 0.5 m by 0.5 m. According to the maximum likelihood analysis results, the single exponential autocorrelation function is selected in this example for characterizing the spatial correlation structure of the strata, the horizontal and vertical scales of fluctuation are 23.5 m and 5.0 m, respectively. The number of iterations in the MCMC updating process is set at 200 to derive the MAP estimates of initial stratigraphic configurations, and the number of final stratigraphic configurations is set at 800 to meet the criterion of COV[E_A] < 0.2%. The comparison between the reviewed approach and the MRF-based approach is mainly made through the information entropy E_E and the probability of the existence of stratum decomposed granitic soil characterized at this site. The map of information entropy E_E derived with the reviewed approach and that derived with the MRF-based approach are plotted in Figs. 4(b-c), respectively. The two approaches are comparable; however, a closer look at Figs. 4(b-c) suggests that the stratigraphic uncertainty characterized by the reviewed approach could be more consistent with the intuition. For example, the spatial distributions of the uncertain areas derived with the reviewed approach are consistent with the stratigraphic setting; whereas, the uncertain areas obtained with the MRF-based approach tend to be distributed horizontally in the elements near boreholes, which is not consistent with the stratigraphic setting. Furthermore, the number of elements with large information entropy can be smaller when the reviewed approach is employed, indicating less uncertainty in the stratigraphic characterization. Finally, in the work of Wang et al. (2016), in which the MRF-based approach is used, 1,000 samples of the stratigraphic configuration are required for achieving the convergence; whereas, with the reviewed approach, 800 samples are sufficient to reach the convergence.

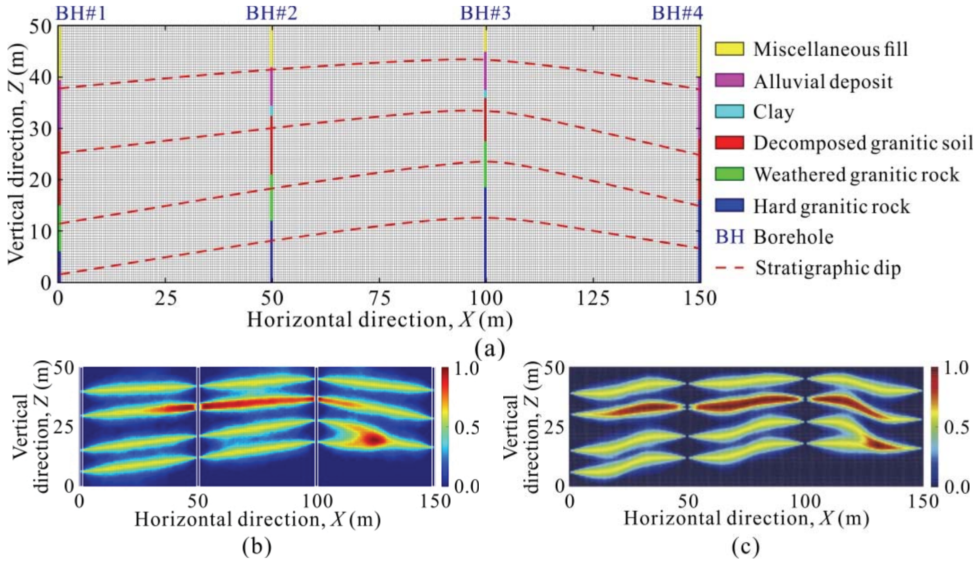


Figure 4. The geological setting, borehole exploration program, and the stratigraphic uncertainty characterization results in Example 3: (a) Geological setting and borehole exploration program; (b) Information entropy obtained with the reviewed approach; (c) Information entropy obtained with the MRF-based approach (from Wang et al., 2016).

3.4 Example 4: Coupled characterization of the stratigraphic and geo-properties uncertainties

The fourth example is concerned with a case study of a seabed site in central-western Taiwan (Kuo et al., 2020), which is conducted to illustrate the effectiveness of the proposed approach for a coupled characterization of the stratigraphic and geo-properties uncertainties. The site studied is in an inclined seabed with water depths ranging from 13 m to 22 m. The seabed deposit at this site is mainly the Quaternary alluvium. The surface soil is loose due to the effects of waves, currents, and river alluviation. In reference to Fig. 5(a), the cross-section of interest I-I′ has a size of 40,000 m by 40 m. Seven boreholes (BH#1 to BH#7) are undertaken along the cross-section I-I′. The depths of these boreholes range from 29 m to 36 m and these boreholes are spaced at 5 km to 7 km. In this site investigation, both stratigraphies and standard penetration test (SPT) blow counts (N values) are collected at borehole locations. As can be seen from Fig. 5(a), five strata, including silty sand (SM), poorly graded sand (SP), low plasticity clay (CL), low plasticity silt (ML), and silty poorly graded sand (SP-SM), are revealed at this site. The thickness of the thinnest stratum layer is 1.5 m while the thickness of the thickest stratum layer is 31.5 m. As the depositional environment of this site is relatively stable, an inclined stratigraphic structure as shown in Fig. 5(a), which is mainly determined based on visual inspections of the elevations of silty sand (SM) and low plasticity clay (CL) revealed at borehole locations, is taken in this example. Plotted in Fig. 5(b) are the SPT N values obtained at borehole locations, ranging from 2 to 41. As an example, in this coupled characterization of the stratigraphic and geo-properties uncertainties, the subsurface domain is discretized into 31,853 rectangular elements, with an element size of 200 m × 0.2 m.

Based on the data (strata and N values) collected at borehole locations, the spatial correlation of the stratum existence and that of the N values can readily be estimated with the maximum likelihood method. The maximum likelihood analysis indicates that the spatial correlation of the stratum existence could best be captured by the single exponential autocorrelation function, the corresponding scales of fluctuation are 4,116.7 m and 4.5 m in the directions parallel and perpendicular to the stratigraphic dip; whereas, the spatial correlation of the N values is best captured by the second-order Markov autocorrelation function, the corresponding scales of fluctuation are 3,711.0 m and 3.4 m in the directions parallel and perpendicular to the stratigraphic dip, respectively. Note that the spatial correlation of the stratum existence and that of the N values are comparable, which is consistent with the intuition and the state of knowledge on site characterization. The large scales of fluctuation in the direction parallel to the stratigraphic dip might be attributed to 1) a stable depositional environment that yields dominance in the direction parallel to the stratigraphic dip and 2) a much greater center-to-center spacing of boreholes at this site than those encountered in typical geotechnical practices. As can be seen from Fig. 5(c), both COV[E_A] and COV[σ_A] decrease with the number of the sampled geological models, where COV[σ_A] represents the COV of the average standard deviation of the sampled geo-properties in non-borehole elements; and, the criterion of COV[E_A] < 0.2% and COV[σ_A] < 0.2% is achieved when the number of the sampled geological models is greater than 1,400. Thus, the number of simulations (i.e., geological models) is taken as 1,400 in this example.

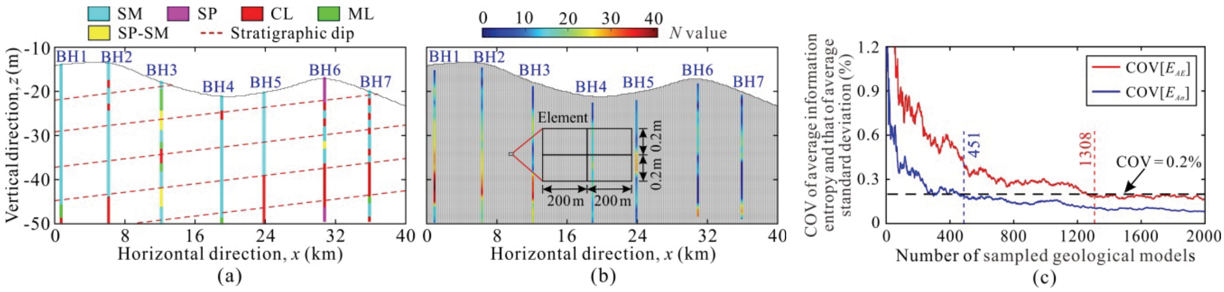
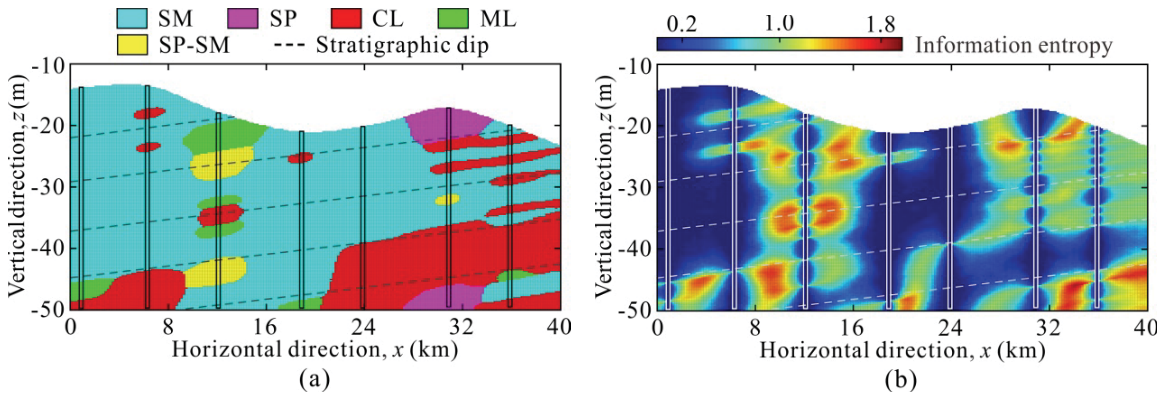


Figure 5. The geological setting and borehole exploration program in Example 4 and determination of the number of sampled geological models: (a) Strata revealed at borehole locations; (b) N values obtained at borehole locations and subsurface domain discretization; (c) Determination of the number of sampled geological models.

The characterized stratigraphic configuration and its uncertainty are depicted in Figs. 6(a-b). Based on the sampled 1,400 possible stratigraphic realizations, the most probable (MP) stratigraphic configuration is obtained, as shown in Fig. 6(a). The overall trend of the spatial distribution of the strata in this MP configuration agrees well with those obtained with the conventional linear interpolation. This outcome validates the effectiveness of the reviewed approach. Certainly, producing stratigraphic configurations comparable to those obtained with the conventional linear interpolation is not the objective of the reviewed approach. The latter has several advantages over the traditional interpolation method, such as the ability to characterize the uncertainty in the stratigraphic configuration obtained. Fig. 6(b) illustrates the spatial distribution of the characterized stratigraphic uncertainty at this site, represented by the map of the information entropy. As can be seen, the areas with large stratigraphic uncertainty, bracketed by the elements with large information entropy, are mainly distributed along the potential boundaries between adjacent strata (see Fig. 6b). Further, the widths of uncertain areas tend to increase with the distance from boreholes, indicating that the constraint of boreholes on the stratum assignment for non-borehole elements decreases with the distance away from boreholes. Based on the sampled 1,400 realizations of the N values, the mean and standard deviation of N values in each subsurface element can be estimated. The results are shown in Figs. 6(c-d). In Fig. 6(c), the spatial distribution of the means of the N values is in good agreement with those observed at borehole locations. Meanwhile, it agrees with the boundaries between adjacent strata identified in the MP stratigraphic configuration. In Fig. 6(d), the standard deviations of the sampled N values increase with the distance away from boreholes, indicating that the constraint from boreholes on the characterization of geo-properties in non-borehole elements decreases with the distance away from the boreholes.



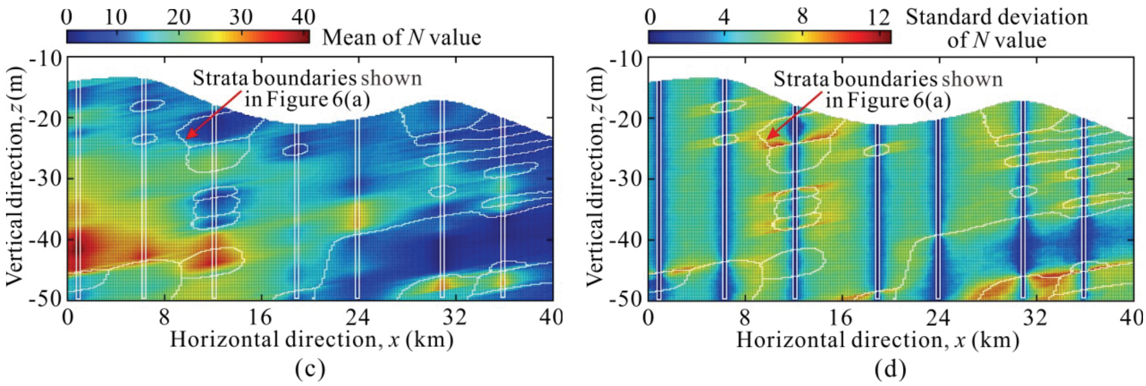


Figure 6. Results of the coupled characterization of the stratigraphic and geo-properties uncertainties in Example 4: (a) Most probable (MP) stratigraphic configuration; (b) Information entropy of the sampled stratigraphic configurations; (c) Mean of the sampled N values; (d) Standard deviation of the sampled N value.

4 Influences of Stratigraphic and Geo-properties Uncertainties on Geotechnical Performance

In this section, two case studies are presented to investigate the influences of the stratigraphic and geo-properties uncertainties on the geotechnical performance. In the first case study, the probabilistic stability analysis of a slope is conducted, based on the coupled characterization of the stratigraphic and geo-properties uncertainties. In the second case study, a shallow foundation problem is analyzed to investigate the effects of different sources of uncertainty on the predicated differential settlement between two footings.

4.1 Case study 1: Probabilistic stability analysis of a slope

The first case study is concerned with the slope problem reported in Deng et al. (2017), in which the influence of the stratigraphic and geo-properties uncertainties on the stability of the slope is addressed. In reference to Fig. 7, the dimension of the site is 70 m by 28 m. For illustration purposes, only the spatial variability of the stratum existence and that of the cohesion are considered in this case study; whereas, other geo-properties are assumed as fixed values. For ease of comparison, an artificial geological model consisting of the stratigraphic configuration shown in Fig. 7(a) and the cohesion shown in Fig. 7(b) is taken as the benchmark geological model. It is noted that this benchmark model is a realization of the geological model sampled by the approach reviewed above. As shown in Fig. 7(a), six boreholes are undertaken at this site with borehole lengths ranging from 12.5 m to 27.5 m, and these boreholes are spaced at 10 m to 20 m. Three strata are revealed at this site, including clay, sand, and silt. Fig. 7(b) shows the cohesion obtained at borehole locations, ranging from 0.6 kPa to 25.3 kPa. For ease of applying the reviewed approach, this site is discretized into 10,723 elements, each with a size of 0.5 m by 0.3 m.

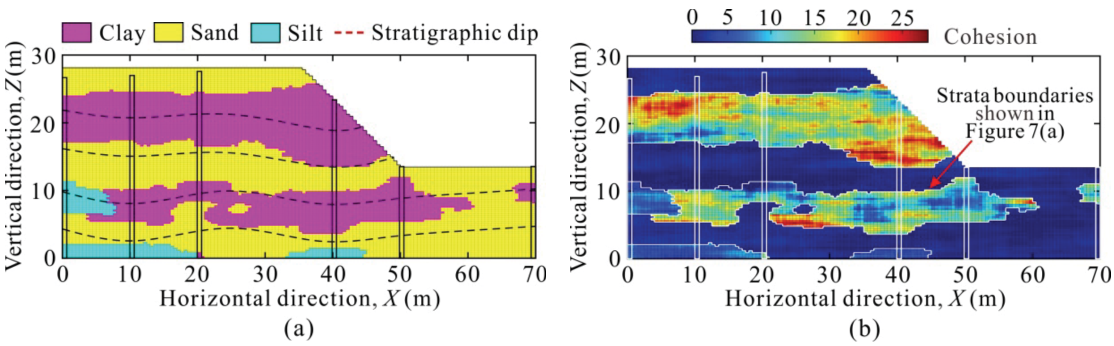


Figure 7. The benchmark geological model and borehole exploration program in Case study 1: (a) Strata revealed at borehole locations; (b) Cohesion obtained at borehole locations.

Based on the data collected at borehole locations, the spatial correlation of the stratum existence and that of cohesion could readily be characterized with the maximum likelihood method. The maximum likelihood analysis indicates that the spatial correlation of the stratum existence could best be captured by the single exponential autocorrelation function, the corresponding scales of fluctuation are 23.0 m and 2.7 m in the directions parallel and perpendicular to the stratigraphic dip, respectively; whereas, the spatial correlation of the cohesion can best be captured by the single exponential autocorrelation function, the corresponding scales of fluctuation are 27.2 m and 2.1 m in the directions parallel and perpendicular to the stratigraphic dip, respectively. With the aid of the coupled characterization approach reviewed above, multiple realizations of the stratigraphic configuration and the associated cohesion can be systematically sampled, and the number of the sampled geological models is set up as 900 in this case according to the convergence criterion mentioned above. Through comparing with the benchmark geological model (see Fig. 7), the accuracy of the stratigraphic configuration characterization and the error of the geo-property characterization in each element could be evaluated. For ease of comparison, the accuracy of the stratigraphic configuration characterization, denoted as S_A, is defined as follows.

$$S_A(i)=\frac{\sum_{j=1}^{j=n_R} IndexStra(i)^j}{n_R}\times 100\% \tag{7}$$

where $S_A(i)$ represents the stratigraphic configuration characterization accuracy in element i (i = 1, 2, …, n_e); $IndexStra(i)^j$ is an indicator function of the stratum assignment in element i in the jth (j = 1, 2, …, n_R) realization of the stratigraphic configuration, the value of which is equal to 1.0 when the stratum sampled in element i in the jth sample of the stratigraphic configuration yields the true stratum observed in element i, otherwise $IndexStra(i)^j$ = 0; and, n_R is the number of sampled realizations of the stratigraphic configuration.

The error of the geo-property characterization (i.e., cohesion in this case), denoted as P_E, is defined below:

$$P_E(i)=\frac{\sum_{j=1}^{j=n_R} \left|g(i)^j-g(i)^T\right|/g(i)^T}{n_R}\times 100\% \tag{8}$$

where $P_E(i)$ represents the characterization error of the geo-property in element i (i = 1, 2, …, n_e); $g(i)^j$ is the geo-property sampled in element i in the jth sample; and, $g(i)^T$ is the true geo-property observed in element i.

Based on the sampled realizations of the stratigraphic configuration and those of the cohesion, the average accuracy of the stratigraphic configuration characterization and the average error of the cohesion characterization of all subsurface elements at this site can be calculated, respectively. The results show that the average accuracy of the stratigraphic configuration characterization is 86.7%, which is larger than that obtained with the traditional deterministic approach (i.e., 83.7%). In the traditional deterministic approach, the stratigraphic configuration is constructed based on linear interpolations of the borehole stratigraphies while cohesion in each stratum is taken as a fixed value (i.e., the mean of cohesion derived at borehole locations). And, the average error of the cohesion characterization obtained with the reviewed approach is 69.0%, which is much smaller than that obtained with the traditional deterministic approach (i.e., 90.7%). As such, the coupled characterization approach could yield higher accuracy in characterizing the geological model at this benchmark site.

With the sampled geological models as inputs, the stability of this slope could be analyzed probabilistically. Here, the 2-D explicit finite difference program FLAC version 7.0 (Itasca, 2011) is taken as the solution model for evaluating the stability of the slope. Within FLAC version 7.0, the strength reduction method is adopted for estimating the slope factor of safety (FS); and, the plane-strain condition is adopted for the stability analysis. The geometry domain of this slope is discretized into 7,840 elements with unequal sizes, and the size of the smallest element is 0.1 m by 0.1 m. The bottom boundary is set at the vertical coordinate of Z = 0 m, the left boundary is set at the horizontal coordinate of X = 0 m, and the right boundary is set at the horizontal coordinate of X = 70 m. The bottom boundary is restrained vertically, and the left and right boundaries are restrained horizontally. The geo-materials are simulated with the Mohr-Coulomb model, and the geo-properties in the FLAC elements are determined according to the input realization of the geological model. The geo-properties of the three strata shown in Fig. 7(a) are listed in Table 1.

Table 1. Parameters involved in the slope stability evaluation in Case study 1.

Stratum	Statistic	Cohesion (kPa)	Friction angle (°)	Unit weight (kN/m^3)	Young's modulus (MPa)	Poisson's ratio
Clay	Mean	16	25	20	30	0.3
	COV	0.3	-	-	-	-
	Distribution	Normal	-	-	-	-
Sand	Mean	2	31	20	50	0.3
	COV	0.3	-	-	-	-
	Distribution	Normal	-	-	-	-
Silt	Mean	6	25	20	30	0.3
	COV	0.3	-	-	-	-
	Distribution	Normal	-	-	-	-

Note: the symbol ''-'' denotes the associated parameter is taken as a fixed value.

The probabilistic analysis results of the slope stability are shown in Fig. 8. Fig. 8(a) depicts the distribution of the evaluated FS of the slope, which varies from 0.85 to 1.24. The mean and the standard deviation of the FS, denoted as μ_{FS} and σ_{FS}, respectively, are then computed from these FS values: μ_{FS} = 1.057 and σ_{FS} = 0.093. Note that the FS estimated with the benchmark geological model (see Fig. 7) as inputs is taken as the true FS of this slope, which is also provided in Fig. 8(a). As can be seen, the true FS is in the range of the FSs obtained with the probabilistic analysis. Also shown in Fig. 8(a) is the single FS obtained with the deterministic geological model.

In comparison to the single FS value obtained from the deterministic geological model, the distribution of the FS shown in Fig. 8(a) allows the engineers to acquire the FS value with more confidence. Apart from the FS values, the failure region in the slope can also be identified from the FLAC analysis results. For example, the potential failure region can be indicated by the map of the shear strain increment in the FLAC elements. Fig. 8(b) shows the mean of the shear strain increment calculated in the FLAC elements within this slope domain.

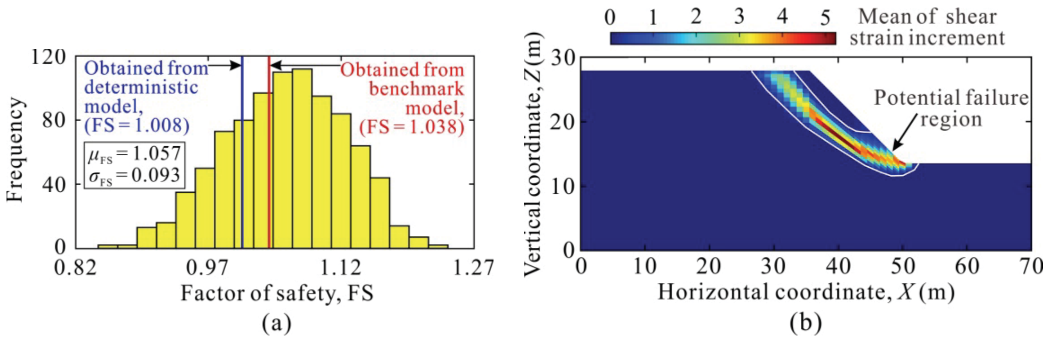


Figure 8. Probabilistic analysis results of the slope stability in Case study 1: (a) Histogram of the calculated FS; (b) Mean of the shear strain increment within the slope region.

4.2 Case study 2: Probabilistic analysis of differential settlement between two shallow foundations

The second case study is concerned with the shallow foundation problem reported in Wu et al. (2020), in which the effects of different sources of uncertainty on the differential settlement between two footings are compared. As shown in Fig. 9, the dimension of the concerned site is 30 m by 20 m and two footings are constructed. The width of each footing is 2.8 m, and the center-to-center distance between these two footings is 8.4 m. The load (F) applied on the top of the footings is set to be 300 kN. As the settlement of a shallow foundation mainly relies upon Young's modulus, only the spatial variability of Young's modulus (and that of the stratum existence) are considered in this case study; whereas, other geo-properties are assumed as fixed values. Similarly, an artificial geological model consisting of the stratigraphic configuration shown in Fig. 9(a) and Young's modulus shown in Fig. 9(b) is taken as the benchmark geological model. In this case study, three boreholes are undertaken, the depths of these boreholes are 20 m, and these boreholes are spaced at 12 m to 18 m, as shown in Fig. 9(a). Three strata are revealed at this site, including clay, sand, and silt. Illustrated in Fig. 9(b) is Young's modulus obtained at borehole locations, ranging from 1.0 MPa to 53.0 MPa. For ease of applying the reviewed approach, this site is discretized into 10,000 square elements, each with a size of 0.3 m by 0.2 m.

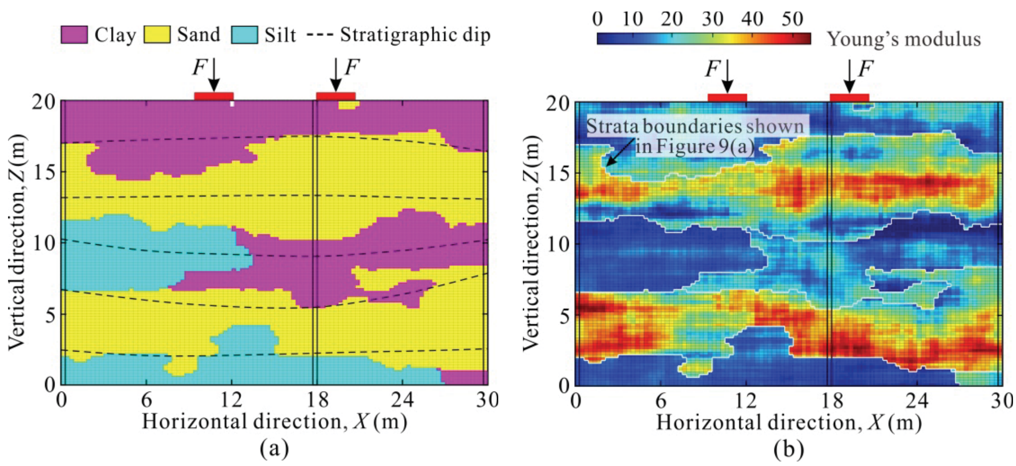


Figure 9. The benchmark geological model and borehole exploration program in Case study 2: (a) Strata revealed at borehole locations; (b) Young's modulus obtained at borehole locations.

The maximum likelihood analysis indicates that the spatial correlation of the stratum existence can best be captured by the single exponential autocorrelation function, the corresponding scales of fluctuation are 37.1 m and 2.9 m in the directions parallel and perpendicular to the stratigraphic dip, respectively; and, the spatial correlation of Young's modulus could also be captured by the single exponential autocorrelation function, the corresponding scales of fluctuation are 50.5 m and 2.2 m in the directions parallel and perpendicular to the stratigraphic dip, respectively. Based on the estimated spatial correlation of the stratum existence and that of Young's modulus, realizations of the stratigraphic configuration and the associated Young's modulus can be systematically sampled with the approach reviewed above, and the number of the sampled geological models is taken as 900 in this case study according to the convergence criterion mentioned above. The characterization results are depicted in Fig. 10, and the plots in Fig. 10 confirm the effectiveness of the reviewed approach for the coupled characterization of the stratigraphic and geo-properties uncertainties.

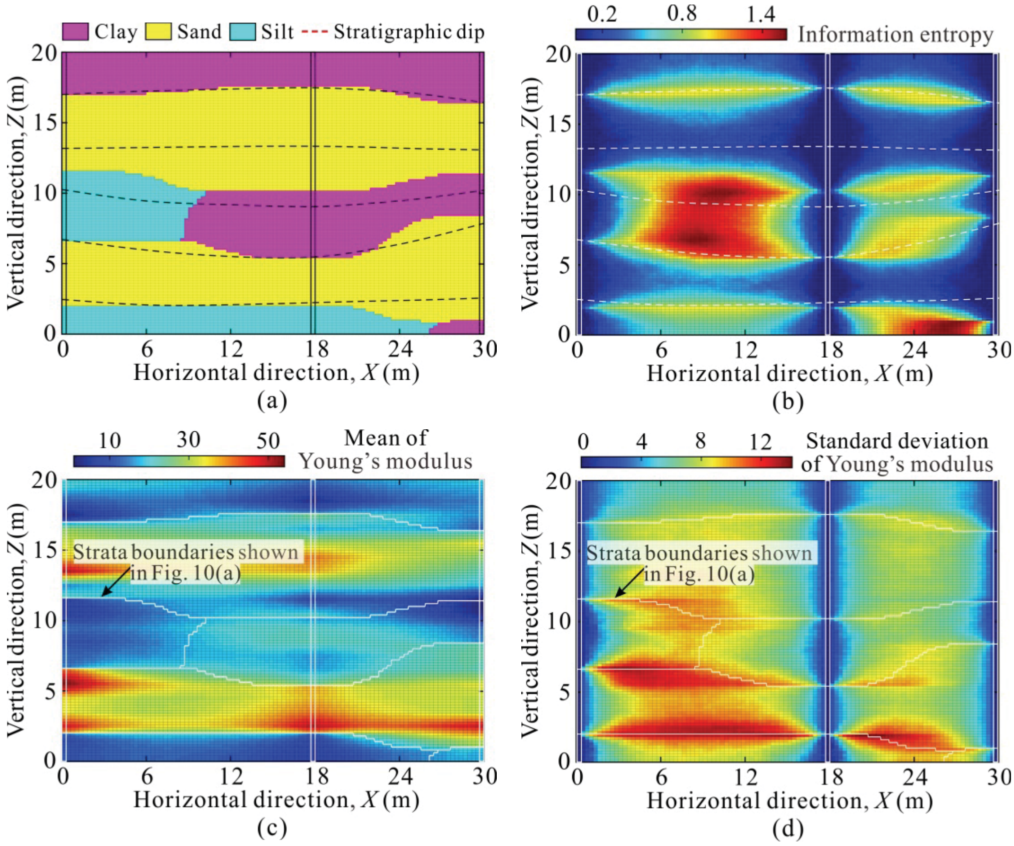


Figure 10. Results of the coupled characterization of the stratigraphic and geo-properties uncertainties in Case study 2: (a) MP stratigraphic configuration; (b) Information entropy of the sampled stratigraphic configurations; (c) Mean of the sampled Young's modulus; (d) Standard deviation of the sampled Young's modulus

To analyze the influences of different sources of uncertainty on the site characterization accuracy and those on the concerned geotechnical performance, four geological models are constructed in this case study: 1) Model 1, the stratigraphic configuration is constructed based on linear interpolations of the borehole stratigraphies while Young's modulus in each stratum is taken as a fixed value (i.e., mean of Young's modulus derived at borehole locations); 2) Model 2, the stratigraphic configuration is sampled with the modified random field approach while Young's modulus in each stratum is taken as the deterministic value; 3) Model 3, the stratigraphic configuration is obtained with the linear interpolation method while Young's modulus in each stratum is sampled with the conditional random field theory; 4) Model 4, the stratigraphic configuration and the associated Young's modulus are characterized with the approach reviewed above. Based on the sampled realizations of the stratigraphic configuration and those of Young's modulus, the accuracy of the stratigraphic configuration characterization and the error of the geo-property characterization could be evaluated, respectively; and, the evaluation results are illustrated in Fig. 11(a). The plots in Fig. 11(a) indicate that Model 4 could yield the highest accuracy of the stratigraphic configuration characterization and the lowest error of Young's modulus characterization at this site; whereas, Model 1 and Model 3 can yield the lowest accuracy of the stratigraphic configuration characterization, Model 1 and Model 2 yield largest error of Young's modulus characterization. From there, the effectiveness and superiority of the coupled characterization approach reviewed above can be demonstrated.

With the sampled geological models as inputs, the differential settlement between these two footings, denoted as Δ, at this site is readily studied probabilistically. Here, the 2-D explicit finite difference program FLAC version 7.0 (Itasca, 2011) is taken as the solution model for predicting the settlements of the footings, in which the plane-strain condition is assumed for the foundation settlement analysis. In the built numerical model, the geometry domain is discretized into 10,000 elements with unequal sizes, and the size of the smallest element is 0.1 m by 0.1 m. The bottom boundary is set at the vertical coordinate of $Z = 0$ m, the left boundary is set at the horizontal coordinate of $X = 0$ m, and the right boundary is set at the horizontal coordinate of $X = 30$ m. The bottom boundary is restrained vertically, while both left and right boundaries are restrained horizontally. The geo-materials are all modeled with the Mohr-Coulomb model, and the geo-properties in the FLAC elements are determined according to the input realization of the geological model. The geo-property information of the three strata involved in this case study is listed in Table 2.

Table 2. Parameters involved in the foundation settlement analysis in Case study 2.

Stratum	Statistics	Young's modulus (MPa)[a]	Poisson's ratio[b]	Unit weight (kN/m^3)[b]	Cohesion (kPa)[b]	Friction angle (°)[b]
Clay	Mean	20	0.3	20	16	25
	COV	0.3	-	-	-	-
	Distribution	Normal	-	-	-	-
Sand	Mean	30	0.3	20	2	31
	COV	0.3	-	-	-	-
	Distribution	Normal	-	-	-	-
Silt	Mean	10	0.3	20	6	25
	COV	0.3	-	-	-	-
	Distribution	Normal	-	-	-	-

Note: [a] data from Deng et al. (2017);[b] data from engineering experience; and, the symbol "-" denotes the associated parameter is taken as a fixed value.

The obtained differential settlements, with Models 1 to 4 as inputs, are illustrated in Fig. 11(b). Note that the true differential settlement, with the benchmark geological model shown in Fig. 9 as inputs, is also given in Fig. 11(b). As can be seen, the discrepancy between the mean of the differential settlement obtained with Model 4 and the true differential settlement is the smallest, which is consistent with the highest accuracy of Model 4 in site characterization. Meanwhile, the differential settlement obtained with Model 4 yields the largest variation. In other words, the geological model that considers only stratigraphic uncertainty or geo-properties uncertainty may underestimate the variation of the concerned geotechnical performance. Another observation in Fig 11(b) is that the true differential settlement can be bracketed by the 68% confidence interval (i.e., mean ± standard deviation) of the differential settlement obtained with Model 3 (considers only geo-properties uncertainty) and Model 4 (consider both stratigraphic uncertainty or geo-properties uncertainty), whereas, the true differential settlement could not be bracketed by the 68% confidence interval of the differential settlement obtained with Model 2 (considers only stratigraphic uncertainty). Further, Model 3 yields a larger variation in the differential settlement than Model 2. Therefore, in this shallow foundation problem, the variation of the predicted settlement tends to be dominated by the geo-properties uncertainty, compared to the stratigraphic uncertainty. From there, the coupled characterization approach could yield a more comprehensive characterization of the geological uncertainty and thusly a more comprehensive evaluation of the geotechnical performance.

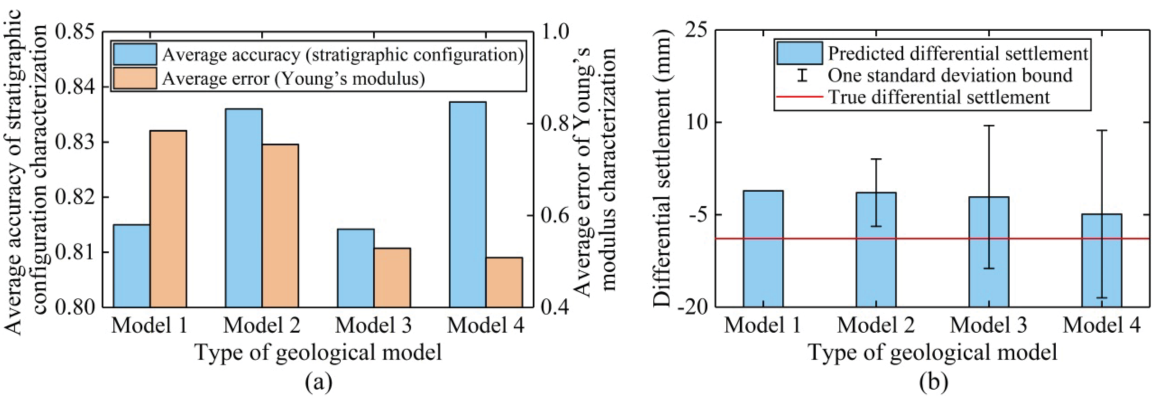


Figure 11. Influences of different sources of uncertainty on site characterization and those on the geotechnical performance evaluation: (a) Average characterization accuracy of subsurface stratigraphic configuration and average characterization error of Young's modulus; (b) Differential settlement between two footings.

5 Concluding Remarks

This paper reviewed a recently developed conditional random field-based approach for the coupled characterization of stratigraphic and geo-properties uncertainties. In this approach, the coupling between the inherent spatial variability of stratigraphic configuration and that of geo-properties within each stratum is explicitly considered. In the context of the coupled characterization approach, the spatial correlation of stratum existence between different subsurface elements and that of geo-properties are captured by two independent autocorrelation functions. Then the coupling between the stratigraphic configuration and the spatial distribution of geo-properties is realized by updating the autocorrelation matrix and statistics of geo-properties in non-borehole elements based on the sampled stratigraphic configurations. The other noticeable feature of the coupled characterization approach is the modified random field approach that involves three main steps. First, the probability of the existence of each stratum in every non-borehole element is derived based on the estimated spatial correlations of the stratum existence. Then, initial stratigraphic configurations are sampled with the conditional random field theory. Finally, the maximum-a-posteriori (MAP) estimates of these initial stratigraphic configurations are derived utilizing Markov Chain Monte Carlo (MCMC). The effectiveness and superiority of this approach in characterizing the stratigraphic uncertainty were first demonstrated through a series of illustrative examples. Then, the effectiveness of the coupled characterization approach was illustrated through a case study of a seabed site in central-western Taiwan. Finally, two case studies are presented to analyze the effects of the stratigraphic and geo-properties uncertainties on the geotechnical performance. The impact of different sources of uncertainty on the predicated geotechnical performance was further discussed. Although the coupling approach was effective, further research on probabilistic site characterizations with continuous subsurface and multi-source data is warranted. Finally, a more efficient computational algorithm is also needed to implement the coupled characterization approach in practical applications.

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References

Baecher, G.B., & Christian, J.T. (2005). Reliability and statistics in geotechnical engineering. *John Wiley & Sons*.

Cao, Z.J., & Wang, Y. (2013). Bayesian approach for probabilistic site characterization using cone penetration tests. *Journal of Geotechnical and Geoenvironmental Engineering, 139*(2), 267-276.

Carle, S.F. (2000). Use of a transition probability/Markov approach to improve geostatistical simulation of facies architecture. *In AAPG Hedberg Symposium., Applied Reservoir Characterization Using Geostatistics,* The Woodlands, Texas, 3-6.

Chilès, J.P., Aug, C., Guillen, A., & Lees, T. (2004). Modelling the geometry of geological units and its uncertainty in 3D from structural data: the potential-field method. *In Proceedings of International Symposium on Orebody Modelling and Strategic Mine Planning,* Perth, Australia, *22*, 24.

Crisp, M.P., Jaksa, M., Kuo, Y.L., Fenton, G.A., & Griffiths, D.V. (2019). A method for generating virtual soil profiles with complex, multi-layer stratigraphy. *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 13*(2), 154-163.

Deng, Z.P., Li, D.Q., Qi, X.H., Cao, Z.J., & Phoon, K.K. (2017). Reliability evaluation of slope considering geological uncertainty and inherent variability of soil parameters. *Computers and Geotechnics, 92*, 121-131.

Evans, S.G. (1982). Landslides and surficial deposits in urban areas of British Columbia: a review. *Canadian Geotechnical Journal, 19*(3), 269-288.

Fenton, G.A. (1994). Error evaluation of three random-field generators. *Journal of Engineering Mechanics, 120*(12), 2478-2497.

Fenton, G.A. (1999). Estimation for stochastic soil models. *Journal of Geotechnical and Geoenvironmental Engineering, 125*(6), 470-485.

Gong, W., Juang, C.H., Khoshnevisan, S., & Phoon, K.K. (2016). R-LRFD: Load and resistance factor design considering robustness. *Computers and Geotechnics, 74*, 74-87.

Gong, W., Juang, C.H., Martin II, J.R., Tang, H., Wang, Q., & Huang, H. (2018). Probabilistic analysis of tunnel longitudinal performance based upon conditional random field simulation of soil properties. *Tunnelling and Underground Space Technology, 73*, 1-14.

Gong, W., Luo, Z., Juang, C.H., Huang, H., Zhang, J., & Wang, L. (2014). Optimization of site exploration program for improved prediction of tunneling-induced ground settlement in clays. *Computers and Geotechnics, 56*, 69-79.

Gong, W., Zhao, C., Juang, C.H., Tang, H., Wang, H., & Hu, X. (2020). Stratigraphic uncertainty modelling with random field approach. *Computers and Geotechnics, 125*, 103681.

Gong, W., Zhao, C., Juang, C.H., Zhang, Y., Tang, H., & Lu, Y. (2021). Coupled characterization of stratigraphic and geo-properties uncertainties-A conditional random field approach. *Engineering Geology, 294*, 106348.

Griffiths, D.V., & Fenton, G.A. (2004). Probabilistic slope stability analysis by finite elements. *Journal of Geotechnical and Geoenvironmental Engineering, 130*(5), 507-518.

Halim, I.S. (1991). Reliability of Geotechnical Systems Considering Geological Anomaly (Ph.D. thesis). *The University of Illinois at Urbana-Champaign,* Urbana, Illinois, USA.

Hicks, M.A., Nuttall, J.D., & Chen, J. (2014). Influence of heterogeneity on 3D slope reliability and failure consequence. *Computers and Geotechnics, 61*(3), 198-208.

Hu, Q.F., & Huang, H.W. (2007). Risk analysis of soil transition in tunnel works. *In Proceedings of the 33rd ITA-AITES World Tunnel Congress - Underground Space the 4th Dimension of Metropolises,* 209-215.

Itasca Consulting Group, Inc. (2011). *Fast Lagrangian Analysis of Continua version 7.0*. Minneapolis, USA: Itasca Consulting Group, Inc.

Juang, C.H., Wang, L., Liu, Z., Ravichandran, N., Huang, H., & Zhang, J. (2013). Robust geotechnical design of drilled shafts in sand: New design perspective. *Journal of Geotechnical and Geoenvironmental Engineering, 139*(12), 2007-2019.

Juang, C.H., Zhang, J., Shen, M., & Hu, J. (2019). Probabilistic methods for a unified treatment of geotechnical and geological uncertainties in the geotechnical analysis. *Engineering Geology, 249*, 148-161.

Kuo, Y.S., Chong, K.J., Tseng, Y.H., Hsu, C.W., & Lin, C.S. (2020). Assessment on liquefaction potential of seabed soil in Chang-Bin Offshore wind farm considering parametric uncertainty of standard penetration tests. *Engineering Geology, 267*, 105497.

Li, D.Q., Qi, X.H., Cao, Z.J., Tang, X.S., Phoon, K.K., & Zhou, C.B. (2016). Evaluating slope stability uncertainty using coupled Markov chain. *Computers and Geotechnics, 73*, 72-82.

Liu, L.L., Cheng, Y.M., & Zhang, S.H. (2017). Conditional random field reliability analysis of a cohesion-frictional slope. *Computers and Geotechnics, 82*, 173-186.

Liu, L.L., Cheng, Y.M., Pan, Q.J., & Dias, D. (2020). Incorporating stratigraphic boundary uncertainty into reliability analysis of slopes in spatially variable soils using one-dimensional conditional Markov chain model. *Computers and Geotechnics, 118*, 103321.

Norberg, T., Rosén, L., Baran, A., & Baran, S. (2002). On modelling discrete geological structures as Markov random fields. *Mathematical Geology, 34*, 63-77.

Schloeder, C.A., Zimmerman, N.E., & Jacobs, M.J. (2001). Comparison of methods for interpolating soil properties using limited data. *Soil Science Society of America Journal, 65*(2), 470-479.

Shi, C., & Wang, Y. (2021a). Development of subsurface geological cross-section from limited site-specific boreholes and prior geological knowledge using iterative convolution XGBoost. *Journal of Geotechnical and Geoenvironmental Engineering, 147*(9), 04021082.

Shi, C., & Wang, Y. (2021b). Assessment of Reclamation-induced Consolidation Settlement Considering Stratigraphic Uncertainty and Spatial Variability of Soil Properties. *Canadian Geotechnical Journal*.

Tang, W.H., Sidi, I., & Gilbert, R.B. (1989). Average property in random two-state medium. *Journal of Engineering Mechanics, 115*(1), 131-144.

Vanmarcke, E.H. (1980). Probabilistic stability analysis of earth slopes. *Engineering Geology, 16*(1-2), 29-50.

Wang, X., Li, Z., Wang, H., Rong, Q., & Liang, R.Y. (2016). Probabilistic analysis of shield-driven tunnel in multiple strata considering stratigraphic uncertainty. *Structural Safety, 62*, 88-100.

Wu, T.H., & Wong, K. (1981). Probabilistic soil exploration: A case history. *Journal of the Geotechnical Engineering Division, ASCE, 107*, 1693-1712.

Wu, Y., Gao, Y., Zhang, L., & Yang, J. (2020). How distribution characteristics of a soil property affect probabilistic foundation settlement - from the aspect of the first four statistical moments. *Canadian Geotechnical Journal, 57*(4), 595-607.

Xiao, J., Gong, W., Martin II, J.R., Shen, M., & Luo, Z. (2016). Probabilistic seismic stability analysis of slope at a given site in a specified exposure time. *Engineering Geology, 212*, 53-62.

Xiao, T., Zhang, L.M., Li, X.Y., & Li, D.Q. (2017). Probabilistic stratification modeling in geotechnical site characterization. *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 3*(4), 04017019.

Zhao, C., Gong, W., Li, T., Juang, C.H., Tang, H., & Wang, H. (2021). Probabilistic characterization of subsurface stratigraphic configuration with modified random field approach. *Engineering Geology, 288*, 106138.