Proc. of the European Safety and Reliability Conference (ESREL 2026)
Edited by José C. Matos, Paulo B. Lourenço, Michael Beer, Daniel Oliveira,
Edoardo Patelli, Hélder S. Sousa, Rui A. Silva, Monica Santamaria and Chiara Turco

ISBN: 978-981-94-6718-1 | doi: 10.3850/ESREL2026061419_esrel26-p26335-cd

Online Model Discovery under Parameter Uncertainty for Risk and Reliability-Based Decision Support

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This paper presents an online identification framework for uncovering the governing equations of dynamical systems operating under degradation to support risk and reliability-informed decision making. To enable accurate discovery from large basis function libraries that capture the complexity of process systems, the wide-array of nonlinear dynamic approximation (WyNDA) method is employed and enhanced through the integration of recursive sparse regression (RSR) and recursive sparse Bayesian learning (RSBL). The proposed approach is validated through numerical simulations of a subsea system under varying degrees of sparsity. In addition, parameter drifts is introduced to emulate degradation effects and their estimates are utilized as evidence for decision supports. Specifically, remaining useful life (RUL) prediction based on a Wiener degradation process and the use of a Bayesian belief network (BBN) are demonstrated. The results demonstrate the potential of the proposed approach as a real-time solution for decision support in systems with limited data availability, while maintaining high accuracy.

Keywords: System identification, decision support, WyNDA, subsea seawater injection system, RUL, and BBN.

1. Introduction

Dynamic modeling is an integral part of a process system throughout its entire lifecycle. It is not only used to obtain an optimal system structure and control design configuration, but also plays a key role in demonstrating the safety and reliability of the configured system (Zhou et al., 2020). Process systems, such as subsea systems, are subject to continuous changes that result in evolving dynamics caused by aging, operational loads, fouling, and harsh environmental conditions (Dan and Liu, 2024). If these evolving dynamics are not properly addressed, uncertainties will impact the system behavior due to deviations from the nominal designs. Therefore, maintaining an accurate and up-to-date dynamic model is essential for safety and reliability assurance, as well as for supporting decision making throughout its lifecycle.

Recent technological advancements have enabled access to a wide range of computational and data-driven techniques for obtaining dynamic models of systems, showing strong potential as system identification tools. However, many of the developed approaches face challenges in capturing evolving dynamics. The most significant challenges are the high computational demand and the requirement for high quality training data (Pereira and Thomas, 2020). These limitations often restrict their application to offline analysis, which is insufficient for safety critical systems that require real time adaptation. In addition, these methods often provide limited interpretability, raising concerns about the credibility of the identified dynamic models when used for decision making.

This work addresses these challenges by developing a system identification methodology based on a wide-array of nonlinear dynamic approxi-

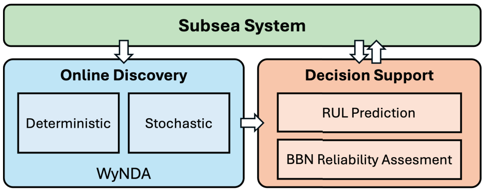

Fig. 1. WyNDA utilization in decision support.

mation (WyNDA) approach (Hasan, 2024). The use of a dynamic system estimation (DSE) algorithm as the main component of WyNDA enables online identification, while the use of a basis function library allows the governing equations of the dynamical system to be represented. The overall framework of the proposed methodology is depicted in Figure 1. To address the complexity of process systems such as subsea systems, the proposed method is further improved by incorporating recursive sparse estimation in both deterministic and stochastic settings. The improved WyNDA framework acts as a decision support tool for online model discovery of subsea systems, providing evidence in the form of mathematical representations as well as degradation estimates based on the parameter uncertainties. These outputs are subsequently utilized within a probabilistic reliability framework, such as remaining useful lifetime (RUL) prediction (Chang et al., 2021) and Bayesian belief network (BBN) (Cai et al., 2019). This approach specifically focuses on assessing the probability of a failure mode, meaning the probability of the loss of required system functionality.

The remainder of this paper is organized as follows: Section 2 presents the online model discovery formulation based on the WyNDA methodology. Section 3 describes in detail the sparse estimation approach developed for identifying complex systems. The validity of the proposed method is demonstrated through numerical simulations in Section 4. Finally, the paper is concluded in Section 5.

2. Online Discovery

The main contribution of the WyNDA methodology is the formulation of system identification in a way that enables the (online) recursive discovery of the governing equations of a dynamical system. The identification is achieved through the use of a DSE algorithm, which allows the identification process to be performed online while remaining interpretable. A nonlinear dynamical system can generally be described by a first order ordinary differential equation (ODE) as follows:

$$\dot{\boldsymbol{x}}_t = \boldsymbol{f}(\boldsymbol{x}_t, \boldsymbol{u}_t). \tag{1}$$

Here, $\boldsymbol{x}_t \in \mathbb{R}^n$ represents the system state at time t, which is driven by the control input $\boldsymbol{u}_t \in \mathbb{R}^m$ through the nonlinear function $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$. The system state is observed through sensor measurements in discrete time. In the current development, a full state measurement is assumed, such that:

$$\boldsymbol{y}_k = \boldsymbol{x}_k + \boldsymbol{v}_k \tag{2}$$

where $\boldsymbol{y}_k \in \mathbb{R}^n$ denotes the measured output at time step $k = 0, \ldots, N$ and $\boldsymbol{v}_k \in \mathbb{R}^n$ represents white Gaussian measurement noise with covariance $\boldsymbol{R} \in \mathbb{R}^{n \times n}$.

The system identification problem for online model discovery aims to reconstruct the nonlinear dynamics in Eq. (1) based on the measurements obtained in Eq. (2). This is achieved by expressing the system in the following discrete time approximation form:

$$\boldsymbol{x}_{k+1} = \boldsymbol{\Psi}(\boldsymbol{y}_k, \boldsymbol{u}_k)\boldsymbol{\theta}_k + \boldsymbol{w}_k. \tag{3}$$

Here, $\boldsymbol{w}_k \in \mathbb{R}^n$ represents white Gaussian process noise with covariance $\boldsymbol{Q} \in \mathbb{R}^{n \times n}$. The matrix $\boldsymbol{\Psi} \in \mathbb{R}^{n \times r}$ denotes the array of basis library functions, where each basis function is associated with a parameter in the vector $\boldsymbol{\theta}_k \in \mathbb{R}^r$ These parameters indicate the contribution of each basis function to the approximated dynamic model. The approximation array is constructed in a diagonal structure as:

$$\boldsymbol{\Psi}(*) = \mathrm{diag}\left(\left[\boldsymbol{\Phi}_1(*) \cdots \boldsymbol{\Phi}_n(*)\right]\right) \tag{4}$$

where $\boldsymbol{\Phi}_i(*) = \boldsymbol{\Phi}_i(\boldsymbol{y}_k, \boldsymbol{u}_k) \in \mathbb{R}^{r_i}$ is a vector of scalar basis functions selected to best represent the dynamics of the corresponding state $i = 1, \ldots, n$.

The complexity of the identified dynamical system directly affects the challenges associated with online model discovery. Achieving a higher probability of identifying the correct system model generally requires considering a large number of basis library functions in Eq. (4). This, in turn, increases the number of parameters that must be estimated. Conventional DSE algorithm used in WyNDA approach for this purpose typically relies on adaptive Kalman filter (AKF) methods (Hasan, 2024). However, in practice, the underlying mathematical model of such dynamical systems is often governed by only a limited number of dominant functions. This motivates the use of ℓ_1 regularization to enforce sparsity in the parameter estimation, allowing a compact and interpretable model to be identified while preserving the capability for online discovery.

3. Sparse Identification

This section explains the design of sparsity regularization within the estimation algorithm to obtain a accurate model identification. Sparse regularization requires the solution of an optimization problem defined by the following loss function:

$$\min_{\boldsymbol{\theta}_k} \sum_{l=0}^{k} \|\boldsymbol{y}_l - \boldsymbol{\Psi}(*)\boldsymbol{\theta}_l\|_2^2 + \rho\|\boldsymbol{\theta}_k\|_1 \tag{5}$$

where the first term represents the quadratic reconstruction error of the system state, and the second term represents the ℓ_1-norm penalty used to promote sparsity in the parameter vector. The parameter ρ controls the level of sparsity enforced in the estimation. Based on this formulation, we develop recursive sparse estimation methodologies for both deterministic and stochastic settings, which are incorporated into the WyNDA framework and described in the following sections.

3.1. *Sparse Regression*

In the context of enforcing sparsity for system identification, sparse regression has become a popular approach for balancing model complexity and accuracy, based on the optimization problem in Eq. (5). Several efforts have been made to extend sparse regression methods into a recursive form suitable for online implementation. In this work, we adapt a sparse regularized recursive least squares (RLS) based estimation methodology by eliminating the spatial correlation penalty and restructuring the formulation, originally proposed in (Baldi and Liu, 2024), to serve as an efficient approximation of sparse regression within the WyNDA framework and to improve its overall performance.

First, the sparsity formulation is reconfigured to better approximate an ℓ_0-norm penalty to enhance its enforcement. This is achieved by introducing a reweighting strategy for the penalty term in Eq. (5), using a weight vector $\boldsymbol{\lambda}_k \in \mathbb{R}^r$. Since both the ℓ_0-norm and the ℓ_1-norm are non differentiable, a smoothing procedure is required for each weight. This results in the following weighting expression:

$$\lambda_{j,k}^{(i)} = \frac{\left(\|\theta_{j,k}^{(i)}\|_1 + \varepsilon\right)^{-1}}{\sqrt{\left(\theta_{j,k}^{(i)}\right)^2 + \varepsilon_d}} \tag{6}$$

where (i) denotes the subset of elements in the parameter vector associated with the basis function vector $\boldsymbol{\Phi}_i(*)$ and $j \in \mathcal{I}_i$ denotes the index of an element within that subset. The constants $\varepsilon, \varepsilon_d > 0$ are small positive values introduced to avoid numerical singularities.

The recursive sparse regression (RSR) approach avoids solving a batch optimization problem at each measurement update by performing the optimization sequentially, using a RLS-based approximation of the gradient of the loss function (Baldi and Liu, 2024). To compute the sparse regularized estimation gains, the following update equations are used:

$$\boldsymbol{U}_{k+1} = (\rho\boldsymbol{\Lambda}_k)^{-1} \tag{7}$$

$$\boldsymbol{G}_{k+1} = \boldsymbol{G}_k - \frac{\boldsymbol{G}_k\boldsymbol{\Phi}_i^\intercal(*)\boldsymbol{\Phi}_i(*)\boldsymbol{G}_k}{1 + \boldsymbol{\Phi}_i(*)\boldsymbol{G}_k\boldsymbol{\Phi}_i^\intercal(*)} \tag{8}$$

$$\boldsymbol{V}_{k+1} = (\boldsymbol{U}_{k+1} + \boldsymbol{G}_{k+1})^{-1} \tag{9}$$

$$\boldsymbol{P}_{k+1} = (\boldsymbol{I}_{r_i} - \boldsymbol{U}_{k+1}\boldsymbol{V}_{k+1})\boldsymbol{U}_{k+1} \tag{10}$$

where $\boldsymbol{\Lambda}_k = \mathrm{diag}(\boldsymbol{\lambda}_k^{(i)})$ represents the reweighting matrix, $\boldsymbol{U}_k, \boldsymbol{G}_k, \boldsymbol{V}_k \in \mathbb{R}^{r_i \times r_i}$ are auxiliary matrices, while $\boldsymbol{P}_k \in \mathbb{R}^{r_i \times r_i}$ is the estimated parameters covariance matrix. The resulting sparse parameter estimates are updated based on the available measurements as follows:

$$\begin{aligned}\bar{\boldsymbol{\theta}}_{k+1}^{(i)} &= \bar{\boldsymbol{\theta}}_{k}^{(i)} + \boldsymbol{P}_{k+1}\boldsymbol{\Phi}_i^{\intercal}(*)\bar{e}_k \\ &\quad - \boldsymbol{P}_{k+1}\left(\boldsymbol{U}_{k+1}^{-1} - \boldsymbol{U}_{k}^{-1}\right)\bar{\boldsymbol{\theta}}_{k}^{(i)}\end{aligned} \tag{11}$$

$$\bar{x}_{i,k+1} = \boldsymbol{\Phi}_i\bar{\boldsymbol{\theta}}_{k+1}^{(i)} \tag{12}$$

where $\bar{e}_k = (y_{i,k} - \bar{x}_{i,k})$. These gain calculations and parameter updates are performed recursively for all n system states at each time step.

3.2. *Sparse Bayesian Learning*

As a second dynamic system estimation approach, we develop a recursive sparse Bayesian learning (RSBL) method as a stochastic alternative for handling measurement uncertainties during the identification process. The proposed method is based on a hierarchical Bayesian inference framework, which involves the selection of appropriate prior distributions and the recursive estimation of posterior distributions (Tipping, 2001). In addition, hyperparameter optimization is performed to act as a regularization mechanism, promoting sparsity in the identified model while accounting for uncertainty.

A Gaussian prior is assumed for the parameter estimation, given by:

$$p(\boldsymbol{\theta}|\boldsymbol{\gamma}) = \mathcal{N}(\boldsymbol{\theta}|0, \boldsymbol{\gamma}) \tag{13}$$

where $\boldsymbol{\gamma} \in \mathbb{R}^r$ is a vector containing the variance associated with each corresponding parameter and $\boldsymbol{\Gamma} = \mathrm{diag}(\boldsymbol{\gamma})$. The posterior distribution, along with its mean and covariance, is given by (Tipping, 2001):

$$p(\boldsymbol{\theta}|\boldsymbol{Y}, \boldsymbol{\gamma}, \boldsymbol{R}) = \mathcal{N}(\boldsymbol{\mu_\theta}, \boldsymbol{\Sigma}) \tag{14}$$

$$\boldsymbol{\Sigma} = \left(\boldsymbol{\Psi}^{\intercal}\boldsymbol{R}\boldsymbol{\Psi} + \boldsymbol{\Gamma}^{-1}\right)^{-1} \tag{15}$$

$$\boldsymbol{\mu_\theta} = \boldsymbol{\Sigma}\boldsymbol{\Psi}^{\intercal}\boldsymbol{R}\boldsymbol{Y}. \tag{16}$$

Here, $\boldsymbol{\mu_\theta} \in \mathbb{R}^r$ denotes the posterior mean of the parameter vector, with covariance $\boldsymbol{\Sigma} \in \mathbb{R}^{r\times r}$. The batch of measurement data is denoted by $\boldsymbol{Y} = \left[\boldsymbol{y}_0^{\intercal} \cdots \boldsymbol{y}_N^{\intercal}\right]^{\intercal}$ and the corresponding library of basis functions is given by $\boldsymbol{\Psi} = \boldsymbol{\Psi}(\boldsymbol{Y}, \boldsymbol{U})$.

As noted earlier, the posterior distribution is obtained using a batch of data, which is not suitable for online model discovery where measurements arrive sequentially. For computational efficiency, the estimation process must therefore be performed recursively, without requiring access to historical data. A common approach is to propagate the parameter estimates and their covariances using a Kalman filter framework. The recursive computation of the Kalman gain and parameter covariance is given by:

$$\boldsymbol{S}_{k+1} = \left(\boldsymbol{\Psi}(*)\boldsymbol{\Sigma}_k\boldsymbol{\Psi}^{\intercal}(*) + \boldsymbol{R}\right)^{-1} \tag{17}$$

$$\boldsymbol{K}_{k+1} = \boldsymbol{\Sigma}_k\boldsymbol{\Psi}^{\intercal}(*)\boldsymbol{S}_{k+1}^{-1} \tag{18}$$

$$\boldsymbol{\Sigma}_{k+1} = \left(\boldsymbol{I}_r - \boldsymbol{K}_{k+1}\boldsymbol{\Psi}^{\intercal}(*)\right)\boldsymbol{\Sigma}_k. \tag{19}$$

Now, as the Kalman gain is available for the measurement update, the next step for sparsity enforcement is to compute the optimal hyperparameters.

The optimal values of the hyperparameters associated with the parameter variances can, in principle, be obtained by maximizing the marginal likelihood, which maximizes the probability of the model given the available evidence, as derived in (Qiao and Wang, 2022). However, this optimization is typically performed in a batch setting rather than sequentially. To enable online implementation, we propose a suboptimal yet recursive update strategy as follows:

$$\gamma_{j,k}^{(i)} = \left(\Phi_{i,j}^2(*) + \epsilon\right)^{-1}\left(y_{i,k} - R_{ii}^2\right) \tag{20}$$

where $\epsilon > 0$ is a small positive constant and R_{ii} denotes the i-th diagonal element of the measurement noise covariance matrix. To preserve historical information and smooth the hyperparameter updates, an exponential moving average (EMA) is applied:

$$\boldsymbol{\Gamma}_{k+1} = \alpha\boldsymbol{\Gamma}_k + (1-\alpha)\,\mathrm{diag}(\boldsymbol{\gamma}_k) \tag{21}$$

where $\alpha \in [0, 1]$ is a learning rate. Based on the updated hyperparameters, the RSBL parameter and state estimates are updated as:

$$\begin{aligned}\hat{\boldsymbol{\theta}}_{k+1} &= \hat{\boldsymbol{\theta}}_k + \boldsymbol{K}_{k+1}\hat{\boldsymbol{e}}_k \\ &\quad - \boldsymbol{K}_{k+1}\boldsymbol{\Psi}(*)\boldsymbol{\Sigma}_{k+1}\Delta\boldsymbol{\Gamma}_k\hat{\boldsymbol{\theta}}_k\end{aligned} \tag{22}$$

$$\hat{\boldsymbol{x}}_{k+1} = \boldsymbol{\Psi}(*)\hat{\boldsymbol{\theta}}_{k+1}. \tag{23}$$

Here, $\hat{\boldsymbol{e}}_k = \boldsymbol{y}_k - \hat{\boldsymbol{x}}_k$ denotes the measurement residual and $\Delta\boldsymbol{\Gamma}_k = \boldsymbol{\Gamma}_{k+1} - \boldsymbol{\Gamma}_k$ represents the change in the regularized covariance.

4. Numerical Simulation

This section is presented to validate the proposed approaches in terms of its system identification capability by integrating RSR and RSBL into WyNDA, with particular focus on online governing equation discovery and its role in risk and reliability based decision support. The numerical simulations are conducted, in the absence of experimental data, using a modelled subsea system, specifically a submersible cascade pump system, for which a simplified schematic is shown in Figure 2. This system is planned to be deployed as part of a subsea seawater injection system and consists of a feed pump that compensates for head losses across the nanomembrane module, as well as an injection pump that provides the required injectivity index for the reservoir. Downstream of the nanomembrane module, a reject flow branch is regulated by a control valve to discharge impurities back to the sea.

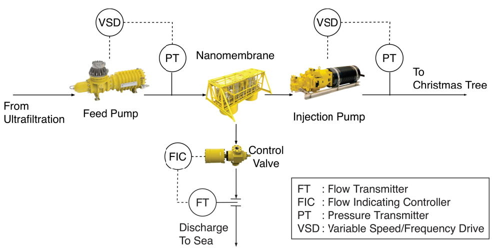

Fig. 2. Subsea cascade pump system flow diagram.

4.1. *Online Estimation Capability*

The first objective is to demonstrate the online discovery of the governing equations for the submersible cascade pump system, in order to evaluate the system identification capability of the proposed approach. The dynamic model discovery is based on a quasi steady state model of the system presented in (Hilmi and Lundteigen, 2026), which describes the feed pressure p_f, injection pressure p_i, and reject flow rate q_r. Based on this model, a total of 11 parameters must be identified. To challenge the sparsity mechanism, the basis function library is augmented with K artificial functions for each true function. In this case, $K = 1$ results in 11 additional fake functions, where these additional functions have little to no correlation with the actual system dynamics. Further details regarding the system model and the basis function library are provided in Appendix A.

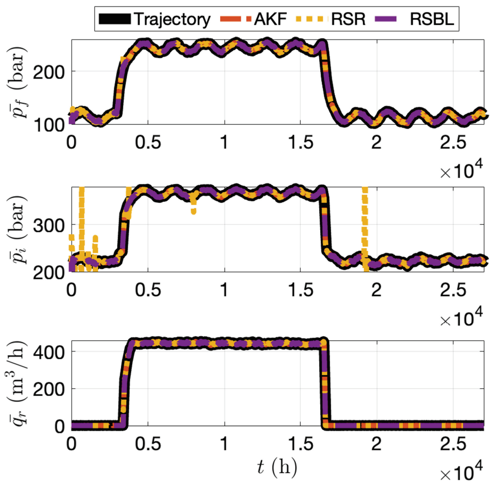

Fig. 3. State trajectory estimation by WyNDA.

The overall results of the online discovery for all sparsity levels $K = 0, 1, 2, 3$ are shown in Figure 3 for state estimation, while the corresponding parameter estimation results are summarized in Table 1. The two proposed sparse methodologies, RSR and RSBL, are compared against the state-of-the-art AKF approach used in the WyNDA framework in (Hasan, 2024). For state estimation, all methods demonstrate satisfactory performance. In particular, the norm root mean square error (RMSE) of the state estimation is 0.7211 for AKF and 0.7527 for the proposed sparse approaches. This indicates a slight performance advantage of the state-of-the-art method in terms of state estimation accuracy. In addition, distortion effects on the estimation signals can be observed in Figure 3 for both RSR and RSBL.

However, when considering parameter estimation performance, as given in Table 1, a different result emerges. As the degree of sparsity increases by introducing a larger number of artificial basis functions, both RSR and RSBL significantly out-

Table 1. Norm RMSE of parameter estimation by WyNDA under varying degree of parameter sparsity.

Method	$K=0$	$K=1$	$K=2$	$K=3$
AKF	7.7424	7.7455	10.5258	7.2188
RSR	2.0813	2.5285	2.4730	2.4166
RSBL	2.1453	2.3844	2.3843	2.3846

perform the AKF approach, with RSBL showing a slightly better overall performance. This discrepancy between state and parameter estimation performance can be explained by the underlying estimation algorithm. The state-of-the-art method performs joint estimation of states and parameters, which can be interpreted as separate RLS-like estimations for states in addition to only parameters, resulting in robust overall performance (Hilmi et al., 2025). In contrast, RSR and RSBL reconstruct the system states from the estimated parameters, as shown in Eq. (12) and (23). Since the primary objective of this work is accurate parameter discovery, parameter estimation performance is considered the most critical metric. Nevertheless, these results highlight the potential for further improvement of the WyNDA framework by extending RSR and RSBL to a joint state and parameter estimation formulation.

4.2. *Application to Decision Support*

The next step is to validate the capability of the proposed online discovery methodology for decision support applications. In this study, a parameter drift is simulated for the feed pump, which is tracked online using the WyNDA framework. This drift provides observable evidence through the estimated parameter evolution, which is then used to support risk and reliability related decision making. This capability is demonstrated through RUL prediction of the pump based on the identified degradation trends. In a more advanced case, a simple BBN is constructed, where operational measurements and WyNDA based estimates are used as multiple sources of evidence to evaluate the probability of pump failure.

The parameter drift introduced in the simulation, as shown in Fig. 3, is designed to exhibit a higher degradation rate under low operating conditions and a more moderate degradation rate under high operating conditions. This behavior reflects the increased load experienced by the feed pump during some regimes, while the overall degradation rate remains consistent with available industry standards (Ottermo et al., 2021).

For remaining useful life (RUL) prediction, the degradation variable d is defined based on pump efficiency loss, with evidence provided by the estimated parameter drift tracked by the WyNDA framework. The degradation process is modeled using a Wiener process:

$$d(t) = d_0 + \mu_d t + \sigma_d W(t) \tag{24}$$

where μ_d and σ_d denote the drift and diffusion parameters of the process, respectively, and are estimated using maximum likelihood estimation based on the evidence obtained from WyNDA, following (Hu et al., 2018). The parameter d_0 represents the initial degradation level and $W(t) \sim \mathcal{N}(0, \Delta t)$ denotes standard Brownian motion. The RUL prediction is shown in Fig. 4. The zoomed in view highlights the change in degradation rate caused by variying operating conditions. Based on this prediction, the estimated RUL is 42961 h.

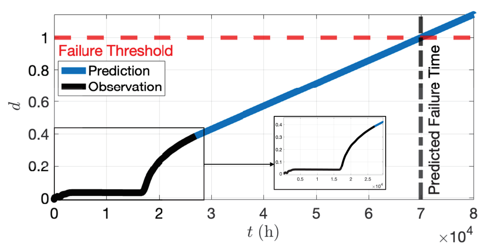

Fig. 4. Wiener-based RUL prediction by WyNDA.

To further demonstrate the capability of WyNDA, we examine the evolution of the RUL prediction over the observation period. Since its advantage lies in providing degradation information in real-time, which is particularly challenging to obtain. Using the estimated Wiener process parameters, the probability density function (PDF) can be expressed as (Hu et al., 2018):

$$f(t) = \frac{L_d}{\sqrt{2\pi\sigma_d^2 t^3}} e^{-\frac{(L_d - d_0 - \mu_d t)^2}{2\sigma_d^2 t}}. \tag{25}$$

Here, L_d denotes the failure threshold. When the degradation variable exceeds this threshold, the feed pump is considered to have failed. The evolution of the PDF over the observation period (i.e., the parameter tracking period) is shown in Fig. 5. The results illustrate how the mean-time-to-failure (MTTF) increases or decreases in response to favorable and unfavorable operating conditions of the feed pump. This behavior is captured as a direct consequence of WyNDA's online capability to continuously provide degradation evidence.

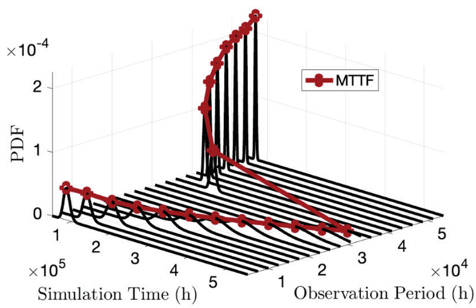

Fig. 5. PDF evolution of the feed pump degradation.

One limitation of the RUL prediction presented earlier is that the degradation model is solely time-based and relies on approximated model parameters. However, several other factors can significantly influence the reliability of the feed pump, most notably the operational load. As illustrated previously by the zoomed-in plots in Fig. 4 and Fig. 5, variations in operating conditions directly affect the degradation rate. When such effects are not explicitly accounted for, the resulting reliability prediction may become inaccurate.

To address this limitation, we present an additional decision-support application of WyNDA in a more advanced setting using a BBN. Unlike purely time-based degradation models, a BBN can incorporate multiple influencing factors and evidences to assess system reliability. In this study, a simple BBN is constructed to account for the operational load of the feed pump, including the flow rate Q and the motor speed ω, both of which directly affect pump degradation. In addition to the pump efficiency loss used in the RUL prediction, the BBN framework allows the inclusion of further evidences obtained via WyNDA, such as loss of head and the generated hydraulic power. These multiple sources of evidence enable a more comprehensive and context-aware assessment of pump reliability. A schematic representation of the proposed BBN is shown in Fig. 6.

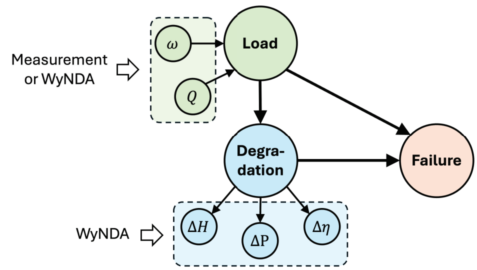

Fig. 6. BBN diagram for the feed pump failure model.

The BBN design shown in Fig. 6 gives the probability of failure (PoF) of the feed pump as:

$$P(F=1) = \sum_{i=1}^{M} P(F=1|D=d_i, L) \times P(D=d_i|\Delta H, \Delta P, \Delta\eta, L) \tag{26}$$

where $F \in \{0,1\}$ denotes the failure state, $D \in \{\text{low}, \text{medium}, \text{high}\}$ represents the degradation state, and $L \in \{\text{normal}, \text{abnormal}\}$ denotes the operating load condition. The conditional probability tables are constructed based on available system knowledge and engineering judgment.

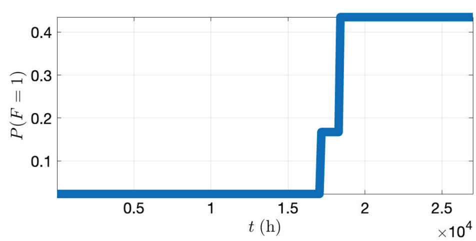

Fig. 7. PoF of the feed pump by BBN-WyNDA.

Since WyNDA provides degradation-related evidences in real time, the resulting PoF can be updated continuously as new measurements become available. The evolution of the PoF over the observation period is shown in Fig. 7. It can be observed that the PoF increases as the feed pump spends a significant amount of time operating under abnormal conditions, leading to the accumulation of degradation. In contrast to the initial phase, although the operating condition is abnormal, the PoF remains low due to the limited accumulated degradation. By the end of the observation period, the PoF reaches a value of 0.4349.

5. Conclusion

This paper proposes a sparse WyNDA for system identification, aimed at discovering the mathematical representation of dynamical systems while tracking parameter drift caused by degradation, thereby extending its role as a decision support tool. Numerical simulations based on a subsea cascade pump system are used to demonstrate the capabilities of the proposed approach. By incorporating the improved RSR and RSBL estimation algorithms into the WyNDA framework, the performance of online identification is significantly enhanced under varying degrees of sparsity. In addition, reliability-based decision support from the estimated parameter drift as evidence is provided through a Wiener-based RUL prediction and a BBN. These illustrate how online model discovery can be directly linked to risk and reliability assessment for safety critical systems.

Future work will focus on enhancing the proposed methods by introducing temporal penalties for more accurate parameter drift tracking in both RSR and RSBL, as well as developing joint state and parameter estimation schemes to improve robustness. Additionally, more comprehensive applications of WyNDA to subsea seawater injection systems, covering degradation management from prevention to mitigation.

Acknowledgement

This work was carried out as a part of SUBPRO-Zero, a Research Centre at the Norwegian University of Science and Technology (project code 100589 and CristinID 2720670). The authors gratefully acknowledge the project support from SUBPRO-Zero, which is financed by major industry partners and NTNU.

Appendix A. Additional Information

To support this paper, we provide the data and code utilized in the following GitHub repository:
`https://github.com/hilmi-ica/OD-SCPS.git`

References

Baldi, S. and D. Liu (2024). A recursive implementation of sparse regression. In *2024 IEEE 63rd Conference on Decision and Control (CDC)*, pp. 3551–3556.

Cai, B., X. Kong, Y. Liu, J. Lin, X. Yuan, H. Xu, and R. Ji (2019). Application of bayesian networks in reliability evaluation. *IEEE Transactions on Industrial Informatics 15*(4), 2146–2157.

Chang, M., X. Huang, F. P. Coolen, and T. Coolen-Maturi (2021). Reliability analysis for systems based on degradation rates and hard failure thresholds changing with degradation levels. *Reliability Engineering & System Safety 216*, 108007.

Dan, E. and Y. Liu (2024). Performance assessment of subsea safety systems subject to heterogeneous failure modes and repair delays. *Frontiers in Energy Research Volume 12 - 2024*.

Hasan, A. (2024). Discovering state-space representation of dynamical systems from noisy data. *IEEE Access 12*, 108744–108754.

Hilmi, M., A. Hasan, and M.-A. Lundteigen (2025). Self-tuning kalman filter for fault estimation of nonlinear systems. In *2025 IEEE 64th Conference on Decision and Control (CDC)*, pp. 502–508.

Hilmi, M. and M. A. Lundteigen (2026). Hierarchical resilient control framework on subsea seawater injection systems. In *23rd IFAC World Congress*. Manuscript under review.

Hu, Y., H. Li, P. Shi, Z. Chai, K. Wang, X. Xie, and Z. Chen (2018). A prediction method for the real-time rul of wind turbine bearings based on the wiener process. *Renewable Energy 127*, 452–460.

Ottermo, M. V., S. Hauge, and S. Håbrekke (2021). *Reliability Data for Safety Equipment: PDS Data Handbook*. SINTEF Digital.

Pereira, A. and C. Thomas (2020). Challenges of machine learning applied to safety-critical cyber-physical systems. *Machine Learning and Knowledge Extraction 2*(4), 579–602.

Qiao, X. and Y. Wang (2022). Recursive sparse bayesian learning. In *2022 China Automation Congress (CAC)*, pp. 2692–2697.

Tipping, M. E. (2001, September). Sparse bayesian learning and the relevance vector machine. *J. Mach. Learn. Res. 1*, 211–244.

Zhou, D., E. Pan, X. Zhang, and Y. Zhang (2020). Dynamic model-based saddle-point approximation for reliability and reliability-based sensitivity analysis. *Reliability Engineering & System Safety 201*, 106972.