

A multivariate Poisson model with flexible dependence structure

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Multivariate distributions are indispensable tools for modeling complex data structures with multiple dependent variables. Despite extensive research on discrete multivariate distributions, the multivariate Poisson distribution remains inadequately defined. However, multivariate Poisson counts are not rare and have gained considerable attention in scientific fields such as reliability engineering. Accurately specifying the dependence structure presents a significant challenge in analyzing such data. Although several methods have been proposed in the literature to address this issue, they have limitations in satisfying all feasible correlations. Currently, there is an outstanding question regarding the development of a multivariate Poisson model that is easily interpretable and effectively handles dependent Poisson counts.

In this study, we present a novel multivariate Poisson model that leverages multivariate reduction techniques (MRT) to enable greater flexibility in the dependence structure, particularly for negative correlations, than classical constructions. Our proposed model generalizes existing MRT-based methods by having the same parameters when some of our model's parameters are preset. We demonstrate the feasible regions of correlations and show that our model overcomes the limitations of previous methods, making it ideal for analyzing multivariate Poisson counts. Furthermore, we establish several probabilistic properties, including the probability mass function, the probability-generating function, and the Pearson correlation coefficient. We also provide a detailed discussion of maximum likelihood estimation and an algorithm for generating multivariate Poisson random variables. Our model's superiority is demonstrated through simulations and a real-world example.

Keywords: Multivariate Poisson model, dependence structure, Pearson correlation coefficient, multivariate reduction, maximum likelihood estimation, random variable generation.