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Uncertainty quantification in portfolio optimization

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The skepticism of investment practitioners towards the Mean-variance model (Markowitz, 1952) is that the optimal portfolios it generates are often vulnerable to changes in the problem's parameters, such as expected returns and covariance matrix. To make decisions despite errors in estimation, researchers have turned to robust optimization. How the uncertainty sets are structured plays a crucial role in formulating and solving robust portfolio selection problems. In general, uncertainty sets can be classified as "separable" and "joint," with separable uncertainty being more common. For instance, Tütüncü and Koenig (2004) proposed box uncertainty sets for expected return $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Goldfarb and Iyengar (2003) studied a factor model for the vector of random asset returns to propose uncertainty sets, and Schöttle and Werner Schöttle and Werner (2009) considered the use of the Wishart distribution to construct uncertainty sets for $\boldsymbol{\Sigma}$. Lu (2011) proposed a "joint" elliptic uncertainty set for the model parameters ($\boldsymbol{\mu}$, V) based on Goldfarb and Iyengar. Although the methods described above are diverse, many of them are complex, involving parametric, statistical tools, etc., and are computationally challenging in practice. Moreover, there are even fewer ways to create an uncertainty set for the covariance matrix alone. We therefore wish to propose a method to create uncertainty sets for covariance matrices in a simple and fast way.

In this paper, we introduce a novel method for uncertainty quantification in portfolio optimization. Instead of following the commonly used bootstrap method that constructs an uncertainty set directly for parameters of the portfolio optimisation problem, we propose constructing the uncertainty set for the covariance matrix via building the uncertainty around the dual multipliers of the constrained portfolio optimisation problem described in Jagannathan and Ma (2003). In this formulation, we use a box uncertainty set around the multipliers, ensuring that the covariance matrices correspond to each feasible point in this uncertainty set positive semidefinite. We call the worse-case robust optimal portfolio calculated from our formulation to be the *λ-Robust Portfolio*.

Since the constrained portfolio optimisation formulation is equivalent to an unconstrained problem with a shrinkage matrix, we believe that λ-Robust Portfolios will have lower short interest and higher out-of-sample diversification ratios in most cases, and therefore lead to better performance than existing models. Also, this new method should help investors avoid excessive losses in the face of contingencies when compared to portfolio strategies that rely on non-robust optimization techniques. To validate the performance of the model, we compare it with six other common investment strategies such as mean-variance model (Markowitz, 1952). For this purpose, we select six data-sets from public websites which include popular investment assets. Finally we calculate out-of-sample Sharpe ratios, variances, value-at-risk and cumulative returns in our numerical experiment. Our numerical experiments show that λ-Robust Portfolios often result in lower short sales, higher diversification ratios, higher cumulative returns, higher cumulative return and lower Value-at-Risk(VaR) in out-sample datasets compared to classical methods. At the same time, our new approach has a higher Sharpe ratio and a lower VaR in most of the data sets compared to other common investment approaches. In addition to the above contributions, we also understand that a fundamentally disruptive trend in the industry 4.0 is the innovative asset management services offered by robo-advisors (Tao et al., 2020). Inevitably, the Markowitz mean-variance optimization problem is involved in smart investment advice. However, as Bourgeron et al. (2018) observes constraints are inherent to mean-variance portfolio optimization. In practice, Quants must devote considerable time to adding and testing constraints, which makes it challenging to directly translate the original mean-variance model into an intelligent investment strategy. Therefore, in future research, we will also investigate whether the new robust portfolio model we propose can provide a new viable model for smart investing, as the model can change the constraints in disguise by controlling the multiplier vector instead.

Keywords: Portfolio selection, Global minimum variance portfolio, Robust optimization, Shrinkage method, Risk management, Quantitative finance

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