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Confidence intervals for RUL: a new approach based on time transformation and reliability theory

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Abstract

This work describes a new analytical approach to derive confidence intervals on Remaining Useful Life (RUL) estimators. The new method applies a time transformation to make the Mean Residual Life (MRL) a linearly decreasing function of the transformed time. Then, explicit confidence bounds for the RUL are derived in the linear MRL case and mapped back to the physical space with the inverse time transformation. A reliability assessment problem of Light-emitting diodes (LEDs) that have undergone accelerated degradation tests, and for which confidence bound and RUL must be provided, demonstrates the new approach. LEDs fail when the luminous flux depreciation exceeds a maximum threshold and the time to failure corresponds to the first hitting time. In this case, Weibull distributed first hitting time is a realistic assumption, which allows the above time transformation method to be carried out explicitly. It is then shown that, if an alternative model (a Gamma distribution, with an appropriate shape factor) had been adopted instead, the results would be quite close to those obtained initially. The key parameter is the slope of the MRL in the transformed time; that parameter can be explicitly related to the shape factor of the Weibull or Gamma distribution. Comprised between 0 and 1, it is used to build the confidence interval; the larger its value (the steeper the slope, i.e., the faster the degradation), the lower the variance of the RUL is, and the narrower the confidence interval. Similar results can be obtained with a Wiener or a Gamma process, which suggests a distributional robustness of this approach. We believe this time transformation approach to be advantageous when the computational efficiency and robustness of the RUL estimation are of primary importance. It also provides useful insights into the dynamics of RUL, which are linked to the ageing characteristics.

Keywords: RUL, Time Transformation, Confidence Interval, Mean Residual Life, LED, Degradation.

1. Background

Remaining useful life (RUL) estimation is a critical need for predictive maintenance and system health management. RUL is inherently uncertain due to operational and environmental conditions and different ageing and wear-out speeds, Dersin (2023). Uncertainty quantification (UQ) is therefore essential for reliable RUL estimation. Despite the increasing interest in UQ for RUL, most existing approaches are based on simplifying assumptions and fail to account sufficiently for epistemic and aleatory uncertainties in RUL estimation. This work introduces a new framework for RUL estimation and UQ. By applying a time-warping function, we can linearize the average loss of RUL over time and formally analyse its natural variability and statistical uncertainty analytically. We demonstrate the efficiency of the proposed method on an RUL estimation problem for LEDs, Van Driel et al (2012). The proposed method enables quantification of uncertainty for the RUL by means of closed-form expressions for confidence bounds, functions of time. This can provide invaluable support for predictive maintenance, e.g., a risk reduction regarding system failures, and data collection decision-making.

2. The method

In Dersin (2023), a time transformation $\tau = g(t)$ is introduced to map the lifetime variable (t) to a warped lifetime (τ) so that the MRL linearly decreases in the transformed domain. Formally, the transformed MRL is defined as, $v(\tau) = m - k\tau = m - kg(t)$, where m is the mean time to failure and k is the angular coefficient determining the speed of ageing (loss of mean residual life) in the transformed space.

The transformation g(t) is given in terms of the reliability function $R(t)$ by,

$$g(t) = \frac{m}{k}[1 - R(t)^{\frac{k}{1-k}}] \quad (1)$$

and the parameter k is expressed in terms of the coefficient of variation σ/m by,

$$k = \frac{1-(\sigma/m)^2}{1+(\sigma/m)^2} \quad (2)$$

Eq. (2) shows that, the smaller σ/m, the steeper the slope k. In other words, a faster degradation corresponds to less uncertainty. Also, at the limit $t \to \infty$, the transformed time $\tau \to \frac{m}{k}$. The g(t) function is usually S-shaped and has an inflection point $t^\star$ where d^2g/dt^2 vanishes. The time derivative can be seen as a metric for the speed of ageing, and $t^\star$ marks the transition from fast to slow ageing. Before the inflection point, g(t) is convex (faster ageing) whilst after the inflection point it is concave (slower aging). The parameter k turns out to be an upper bound on the time derivative of the MRL after the inflection point:

$$\left|\frac{dv}{dt}\right| \leq k \quad \text{for } t > t^\star \quad (3)$$

For 2-parameter Weibull (W) and Gamma (G) distributed lifetimes the value for k can be explicitly derived as $k = \frac{2\Gamma^2(1+\beta^{-1})}{\Gamma(1+2\beta^{-1})} - 1$, and $k = \frac{\beta-1}{\beta+1}$, where β is the shape parameter for the W and G, respectively. Also note that, for Weibull distributed lifetimes, the inflection point $t^\star$ can be formally derived as follows:

$$t^\star = \eta\left(\frac{\beta-1}{\beta}\frac{1-k}{k}\right)^{\frac{1}{\beta}} \quad (4)$$

3. Application and results

The method presented in Section 2 is applied to a LED durability assessment problem. The run-to-failure data, flux degradation trajectories $\phi(t)$, of 100 LEDs have been gathered with 4 combinations of accelerated current and temperature. For each combination, 25 LEDs are available, and $\phi(t)$ collected till 9072 hours with a monthly sampling frequency. A LED failure occurs when the luminous flux depreciation exceeds a 2% threshold, i.e., $\phi(t) < 0.98\phi(0)$. This failure threshold, relatively harsh compared to the most common industry best practice of 50 to 90 %, was selected to reduce the high number of right-censored failures. Table 1 presents the results of the lifetime and RUL analysis, and an example for the time transformation $g(t)$ and its derivative dg/dt are plotted in Fig. 1 for the stress level I =0.7 A and T = 85 °C. Note that for the lower stress levels (I=0.35 A), $t^\star$ =4455 h, which indicates that $|dv/dt| \leq 0.749$ for t > 4455 h. On the other hand, for the same temperature but higher current 0.7 A, see Fig. 1, $k = 0.84$ and change from fast to slow ageing occurs at $t^\star$ =2940 h, which indicates the flux degradation stabilizes earlier: $|dv/dt| \leq 0.84$ for t> 2940h

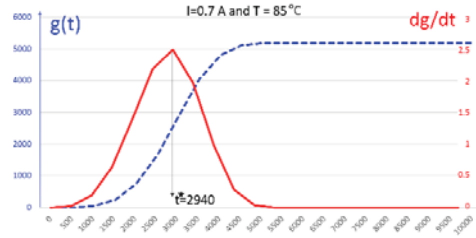

Fig. 1. The warping function $g(t)$, dashed line, and $\frac{dg}{dt}$, solid line, for $(T, I) = (85\,°C, 0.7\,A)$.

The method enables the derivation of confidence intervals for the RUL (see Table 2 in the Weibull case for the same stress combinations), which can support risk-based maintenance optimization. Note that a Gamma model (for the same value of k) would lead to very close results as the warping function would be very close.

Table 1 Result of the lifetime analysis and resulting best-fitting distributions according to the AIC.

T [°C]	85		105	85	105
I [A]	0.35		0.35	0.7	0.7
Best-fit	G	W	G	W	W
η	6.89	7581	7.04	4934	3347
β	6.98	2.86	3.88	3.85	3.58
m [kh]	6.8	6.7	4.4	4.4	3.0
k	0.749	0.75	0.59	0.84	0.82
$t^\star$ [kh]	-	4.4	-	2.9	1.9

Table 2: RUL bounds for 80% confidence level (h)

t	3000	4000	5000
RUL+	3227	2342	1757
RUL-	451	213	125

Also, under continuous monitoring conditions, the k parameter could be updated with time, leading to narrower confidence intervals as increased degradation would result in higher values of k over time. The degradation (loss of LED luminous flux) could also be modelled by a Gamma or a Wiener process.

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