Proceedings of the 33rd European Safety and Reliability Conference (ESREL 2023)
Edited by Mário P. Brito, Terje Aven, Piero Baraldi, Marko Čepin and Enrico Zio
 Published by Research Publishing, Singapore.
doi: 10.3850/978-981-18-8071-1_P401-cd

Topography-based Fuzzy Assessment of Burning Area in Wildfire Spread Simulation

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Wildfire propagation simulation is a key asset for first responders in their operational management. Geographical features, such as the terrain, and meteorological features, such as wind, play a key role in the evolution of a wildfire. However, some aspects of the propagation are fuzzy by nature, e.g. velocity regarding weather conditions and terrains. In this paper, we propose a topography-based fuzzy approach to wildfire simulation. Without loss of generality, we illustrate this assessment with some examples from an area of interest in Southern France.

Keywords: Topographic fuzzy modeling, uncertain fire propagation, spatial reasoning, crisis management.

1. Introduction

Wildfires are becoming more destructive and less predictable, especially due to human activities and climate change that interact with fire dynamics, *e.g.* the vegetation distribution (Garbolino, Sanseverino-Godfrin, and Hinojos-Mendoza 2016). Some softwares assist fire fighters in wildfire prevention (Kalabokidis et al. 2016). However, fire propagation simulation is a key asset to help first responders prioritising the tasks during crisis management. We can distinguish two kinds of contributions that predict fire propagation.

On the one hand, the first approaches rely on physics modeling. Most of the modern models are based on Rothermel's fire spread model (Rothermel 1972). These reliable equations describe the chemical and physical processes of fire, considering the type of fuels. To decrease the computational cost, newer models use a grid to represent the terrain (Adou et al. 2010). Indeed, recent contributions suggest having a less precise simulation that can be provided faster (Grasso and Innocente 2020).

On the other hand, recent methods use Artificial Intelligence (AI). A majority of contributions assess the risk of fire (Sakr et al. 2010), mainly due to the availability of corresponding data or knowledge. Nevertheless, AI has also been applied to other related tasks (Jain et al. 2020) like prevention and crisis management. In recent years, different data driven approaches have been developed to predict the spread of wildfires (Subramanian and Crowley 2018; Radke, Hessler, and Ellsworth 2019).

Even if these methods differ on the approach, they nevertheless agree on the difficulty of the problem due to a strong uncertainty (Thompson and Calkin 2011), and the importance of the slope and the wind velocity and direction.

In this paper, we use a knowledge-based approach to predict the spread of wildfires. To be as general as possible, we rely only on data that can be accessed seamlessly, *i.e.* weather forecast and Digital Terrain Model (DTM). Indeed, in our experience, it is very difficult to get historic data, except in certain locations. Our method does not rely neither on the fuels: we do no consider the vegetation and the soles. We use fuzzy logic (Zadeh 1965) to capture the uncertainty of our predictions on a spatial context (Carniel and Schneider 2021), in particular DTMs (Iphar, Boudet, and Poli 2021).

We designed a method so that we can distinguish three states of the areas: unburnt, burning and burnt. Since our goal is to use the fire propagation estimation as a tool to estimate risks, we rather need a tool that can be used in different locations than a precise model. This explains we do not need any specific knowledge (*e.g.* fuels).

The next section gives an overview and the intuition behind our contribution.

2. General overview

We designed a new approach to estimate the fire propagation regarding the main parameters that are always available: the weather forecast, in particular the wind velocity and direction, and the terrain as a DTM to provide with the slopes of the area of interest.

The process of fire evolution can be decomposed in three different states that evolve with time: firstly unburnt, then currently burning and finally burnt. Our approach consists in representing these three different states with 2D fuzzy sets (Zadeh 1965) on the area of interest, named respectively U (as Unburnt), F (as Fire, for currently burning) and B (as Burnt). It allows considering the uncertainty of the spread of fire.

The 2D fuzzy sets are defined over a grid that covers the area of interest and whose size depends on the spatial resolution and the size of the area of interest. For the sake of simplicity, let us consider in this paper that the number of lines and the number of columns of the grid are equal, and takes the value $n \in \mathbb{N}^{+*}$. Let us note c a cell of this grid in the set of all the cells $\mathcal{C}$.

As the propagation is a temporal process that depends on the previous state, we use an iterative process (Figure 1). At each time step, the contributions of the slope and of the wind are first assessed. Then, those two individual contributions are combined to compute the fire area of the following time step. The iteration is then ended with an update of the membership score of each cell of the three fuzzy sets U, F and B, shown on the left-hand side of Figure 1. Once updated, those fuzzy sets can be used as parameters of the next iteration, and so on.

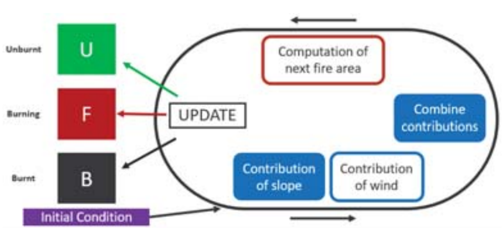

Fig. 1. Overview of the iterative approach.

As shown in Eq. (1), the 2D fuzzy sets form a strong fuzzy partition on the spatial space that is to say that, for all cell $c \in \mathcal{C}$, $U(c)$, $F(c)$ and $B(c)$ are fuzzy values that sum to one:

$$U(c) + F(c) + B(c) = 1. \qquad (1)$$

We detail the approach in the next sections.

3. Iterative process description

As stated before, in our approach, one iteration corresponds to the simulation of the evolution of the fire across one unit of time. This unit of time can be decided by the user according to the terrain, to wind conditions or with the size of the global computation, as more frequent iterations may lengthen the total computational time, particularly when the timeframe of interest is large.

Input: $F^0, B^0, U^0, W, \chi, T, m$
Output: F, B, U
$F \leftarrow F^0$ {Initial burning area}
$B \leftarrow B^0$ {Initial burnt area}
$U \leftarrow U^0$ {Initial unburnt area}
$k \leftarrow 1$ {Counter initialised}
$\tilde{\chi} = \tilde{f}(\chi)$ {Fuzzification of slope}
$E_s = f_1(\tilde{\chi}, T)$ {Slope contribution, *cf.* Eq. (5), invarient}
while $k < m$ **do**
  $\tilde{W} = \tilde{f}(W)$ {Fuzzification of wind}
  $E_w = f_2(\tilde{W}, T)$ {Wind contribution, *cf.* Eq. (6)}
  $S^k = f_3(F^{k-1}, E_w, E_s)$ {Fire Spread, current iteration}
  $B^k \leftarrow \{B^{k-1}, F^{k-1}\}$ {New burnt area, *cf.* Eq. (12)}
  $F^k \leftarrow \{F^{k-1}, S^k\}$ {Next burning area, *cf.* Eq. (13)}
  $U^k \leftarrow \{B^k, F^k\}$ {Still unburnt area, *cf.* Eq. (14)}
  $k \leftarrow k + 1$ {Counter incremented}
end while
$F \leftarrow F^{k-1}$ {Final burning area}
$B \leftarrow B^{k-1}$ {Final burnt area}
$U \leftarrow U^{k-1}$ {Final unburnt area}

Fig. 2. Algorithm for the computation of the wildfire spread area

Figure 2 shows the pseudo-code of our iterative process for the fire spread prediction. It takes as inputs an initial state, under the form of the three fuzzy sets named U^0, F^0 and B^0 (their nature will be further discussed in Section 5.4). On top of the initial state, the input parameters are W, the local state of the wind (direction and velocity) at the time of the simulation, χ, the local Digital Terrain Model, T, the time step between two iterations and m, the desired number of iterations. $\tilde{f}$, f_1, f_2

and f_3 are functions, which will be presented in upcoming sections.

The procedure outputs three fuzzy sets in their final form, denoted F, B and U.

In the procedure, a loop allows time to increment for a fixed number of iterations. Once this iteration limit has been reached, the fuzzy sets in their last updated form constitute the output.

Since the local DTM does not change over time, the contribution of the slope is computed once before the loop begins. However, since the wind could change between iterations, the contribution of wind is computed again at each iteration. Both fuzzified contributions are then combined for the generation of the fire spread area for the current iteration.

Then, the updated forms of the fuzzy sets are computed, in this order: first the burnt area, from the former burnt and the former burning areas ; second the burning area, from the previous burning area and the fire spread area ; and last the unburnt area, from the newly computed burnt and burning areas.

4. Wind and topography fuzzy contributions

In this section, the two contributions of the wind and of the slope to the propagation of fire are described, along with their combination and their fuzzification.

4.1. *Topography contribution*

In this section, we denote as $\overrightarrow{E_s}$ the contribution of the slope of the terrain to the evolution of the wildfire. Fire tends to spread uphill, therefore the direction of the line of greatest dip must be determined in order to compute $\overrightarrow{E_s}$. The steeper the slope, the fastest the fire spreads, therefore the norm of $\overrightarrow{E_s}$ will also evolve with the value of the slope.

Let us denote $\overrightarrow{S}$, a unit vector co-linear to the upslope direction and s the value of the slope, in percent. $\overrightarrow{S}$ and s have been retrieved from formulas that we previously stated in (Iphar, Boudet, and Poli 2021), while the evolution of fire speed is ruled by a doubling of its speed every 10 degrees in slope (Barjak and Hearne 2002; Sharples, Gill, and Dold 2010):

In addition to the effect of the slope *stricto sensu*, the fire also propagates when there is no (or little) slope, at a speed that is denoted S_f, in m/s, and that is retrieved from the literature (*cf.* section 4.4):

$$\overrightarrow{E_s} = T \cdot S_f \cdot 2^{\frac{s}{10}} \cdot \overrightarrow{S}. \quad (2)$$

In Eq. (2), T represents the number of units of time that corresponds to an iteration, in seconds, and the resulting vector $\overrightarrow{E_s}$ corresponds to the contribution of the slope for each iteration. Since the terrain is irregular, points will display various values for both the uphill direction and the steepness of the slope. Therefore, the resulting field of local $\overrightarrow{E_s}$ vectors will vary spatially. However, since the terrain does not evolve over time, this field remains still across various iterations. If the local terrain is flat, the fire will then spread in the direction of the wind. Figure 3 shows an example of a field of slope contributions.

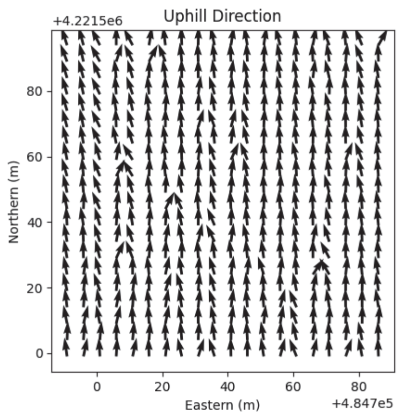

Fig. 3. A field of uphill vectors, in this case the terrain has a positive slope towards North

4.2. *Wind contribution*

In this section, we denote as $\overrightarrow{E_w}$ the contribution of the wind to the evolution of the wildfire. Fire tends to spread in the direction of the wind, therefore this direction must be known in order to compute $\overrightarrow{E_w}$. Let us denote $\overrightarrow{W}$ the direction of the wind, with $W = \|\overrightarrow{W}\|$ the speed of the wind, in km/h.

Thus, $\overrightarrow{E_w}$ is colinear to $\overrightarrow{W}$, in the same direction, and its norm is determined as a fraction η of wind speed as shown in the following formula :

$$\overrightarrow{E_w} = T \cdot \eta \cdot \overrightarrow{W}/3.6. \quad (3)$$

In Eq. (3), T represents the number of units of time that corresponds to an iteration, in seconds, and the resulting vector $\overrightarrow{E_w}$ corresponds to the contribution of the wind for this iteration. The value of η is to be determined by expert knowledge, in the frame of the research project we are involved in (discussed in section 4.4).

Since weather conditions can evolve over time and over space, an accurate field of wind contribution can be built with meteorological data, the higher the spatial and temporal granularity and update rate, the more precise the resulting field. In this paper, for the sake of simplicity, we consider a homogeneous wind field over both space and time. Thus, $\overrightarrow{E_w}$ can be computed once before the "while" loop. Figure 4 shows two examples of wind contribution fields.

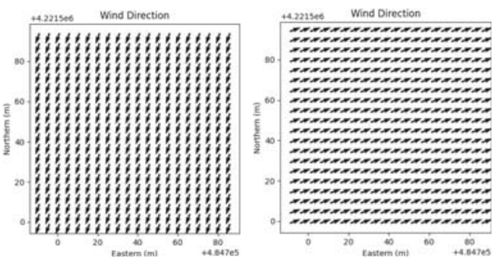

Fig. 4. Two examples of wind contribution fields. Only the direction is shown, not the norm. Left 4(a) direction is 200 degrees, right 4(b) direction is 70 degrees

4.3. *Combination of both contributions*

At each point of interest, we want to consider both the contribution of the wind and the contribution of the slope of the terrain.

The purpose is to determine the direction that the fire will take, when departing from the point of interest, which point will be reached by the end of the iteration and by extension the area that will be impacted by the evolution of the fire, during the duration of the iteration, from the specific point of interest.

The evolution of the fire, denoted $\overrightarrow{E}$, is computed as the vectorial sum of $\overrightarrow{E_s}$ (Section 4.1) and $\overrightarrow{E_w}$ (Section 4.2), following the model shown in (Viegas 2004) and used in (Boboulos and Purvis 2009). A visual interpretation of this vectorial sum Eq. (4) is shown in Figure 5:

$$\overrightarrow{E} = \overrightarrow{E_w} + \overrightarrow{E_s} \quad (4)$$

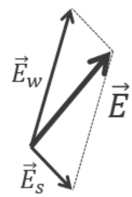

Fig. 5. The combination of wind and slope contributions, in the crisp case, in the computation of the evolution of a wildfire from (Viegas 2004)

Figure 6 shows the application of this vectorial sum. Only the direction of the vectors are shown in the picture, not their norm. It shows both resulting vector fields of the sum of the slope contribution of Figure 3 and both wind contributions of Figure 4, with wind speed of 40 km/h in both cases.

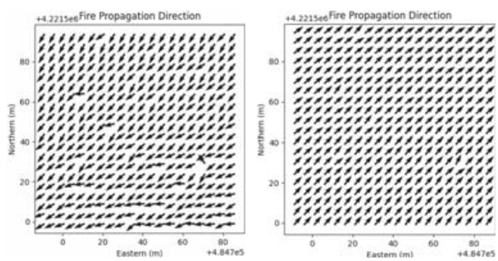

Fig. 6. Vector fields coming as a the vectorial sum of both wind and slope combinations. Only direction is shown, not the norm of vectors. Left: combination of the upslope field of Figure 3 and the wind field shown 4(a). Right : combination of the upslope field of Figure 3 and the wind field shown 4(b).

4.4. *Fuzzification*

When assessing the evolution of a fire, some parameters have to be considered during the computation. Since their value are not well known and can vary depending on different factors such as the type of fuel (the local vegetation type), it is useful to consider them as fuzzy variables to cover the various possibilities that can be encountered. These fuzzy variables are defined by two bounds: a lower bound for which the fire will spread surely with that value, and an upper bound for which the fire may spread with that value in one unit of time. In this article, we focus on two variables: the speed of fire on flat land and the evolution of fire with wind.

The speed of fire on flat land is a variable we can find in Eq. (2), denoted S_f. We take lower and upper bounds of 3 and 5 m/s for the evolution of the fire on a flat terrain, following the values found in (McAlpine 1988) and (Adou et al. 2010).

The evolution of fire with wind is a variable we can find in Eq. (3), denoted η. We take lower and upper bounds of 5% and 15% for η, representing the contribution of the wind in percentage of local wind speed, following the values found in (Beer 1991).

Bounds for both variables S_f and η are parameters, thus their values are taken for the sake of the example and can be replaced with any value an expert might think better suits.

Since both $\overrightarrow{E_w}$ and $\overrightarrow{E_s}$ are fuzzy, $\overrightarrow{E}$ becomes fuzzy and its bound values are defined by $\overrightarrow{E}_{\min} = \overrightarrow{E_w}_{\min} + \overrightarrow{E_s}_{\min}$ and $\overrightarrow{E}_{\max} = \overrightarrow{E_w}_{\max} + \overrightarrow{E_s}_{\max}$, as shown in Figure 7.

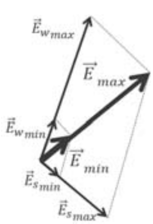

Fig. 7. Combination of wind and slope fuzzy contributions, adaptation of Figure 5 with fuzzy values for both S_f and η

Let us consider the cell c in all the cells $\mathcal{C}$. Once a couple of vectors $(\overrightarrow{E}^c_{\min}, \overrightarrow{E}^c_{\max})$ has been computed for each cell c, the propagation step can take place. We associate values $d^c_{\min} = \|\overrightarrow{E}^c_{\min}\|$, $d^c_{\max} = \|\overrightarrow{E}^c_{\max}\|$ and g_c, corresponding to the circular mean of the directions of $\overrightarrow{E}^c_{\min}$ and $\overrightarrow{E}^c_{\max}$.

5. Spread of burning area

In this section, the iterative spread of the fire, given the local wind and terrain conditions, is shown, leading to the update of the three fuzzy sets that define the situation in terms of burning and burnt areas at a given time.

5.1. *Propagation from a single point*

We want to allocate to every single cell p in $\mathcal{C}$ a score, noted $S^k_c(p)$, corresponding to the evolution of the fire from the cell of interest c at iteration k. So for each cell p, we first have to assess whether or not is it in the direction g_c from the cell c. Then, the score $S^k_c(p)$ is allocated with respect to the distance between p and c: if the distance is lower than $d^c_{\min}$, then the score is maximal, if the distance is greater than $d^c_{\max}$, then the score is minimal and if the distance is in between $d^c_{\min}$ and $d^c_{\max}$, the score evolves linearly from maximal to minimal.

The score for each cell p, as computed in Eqs. (6) and (7) is then multiplied by the membership value of the cell c to the class *burning*, corresponding to the value of $F^{k-1}(c)$, such that $\forall p \in \mathcal{C}$:

$$S^k_c(p) = F^{k-1}(c) \cdot \xi(c,p) \cdot \mu(c,p) \tag{5}$$

where

$$\xi(c,p) = \begin{cases} 1 \text{ if } p \text{ is in direction } g_c \text{ from } c \\ 0 \text{ otherwise} \end{cases} \tag{6}$$

which means that $\exists \Xi \in p$ such as $\overrightarrow{c\Xi}$ and an unit vector that originated in c and has a direction g_c are colinear and in the same direction, and where

$$\mu(c,p) = \begin{cases} 1 & \text{if } d(c,p) < d^c_{\min} \\ 0 & \text{if } d(c,p) > d^c_{\max} \\ \frac{d(c,p)-d^c_{\max}}{d^c_{\min}-d^c_{\max}} & \text{else} \end{cases} \tag{7}$$

where $d(c,p)$ is the Cartesian distance between c and p. The values taken by μ, as computed in Eq. (7), are shown in Fig. 8.

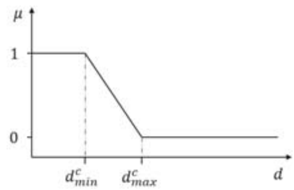

Fig. 8. Membership function for the spatial predicate "*is in the range of fire evolution*"

The result of the propagation from a cell c is shown in Figure 9.

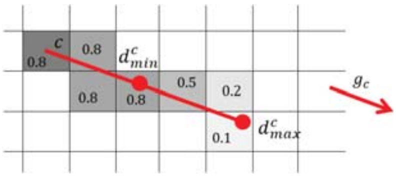

Fig. 9. Propagation of burning area during one iteration, in the direction g_c. Original burning cell is c with a membership score of 0.8. In light grey, all cells that will have a non-null burning score from the propagation from c are shown with their membership score.

5.2. *Burning area propagation*

The evolution of the whole burning area is determined by merging all areas impacted by the evolution of the fire from each and every burning point. Thus, we perform a fuzzy union on all the burning cells at the given iteration to update the *burning* fuzzy set. As shown in Eq. (8), we perform a fuzzy union between the scores from all cells. We use the approach of (Bloch and Maitre 1995) using t-norms and t-conorms as respectively fuzzy intersection and fuzzy union:

$$S^k(p) = \underset{c\in\mathcal{C}}{\perp} S^k_c(p) \tag{8}$$

where $\perp$ is a t-conorm. In this paper, for the sake of simplicity, and without loss of generality, we use the $\max$ function as t-conorm.

5.3. *Update of the fuzzy sets*

Once S^k has been computed for all cells, we can update the three fuzzy sets, namely the *burning*, *burnt* and *unburnt* fuzzy sets. As a reminder, we denote F^k, B^k and U^k those three fuzzy sets at iteration number k, and $F^k(c)$, $B^k(c)$ and $U^k(c)$ the fuzzy values taken in a cell c in $\mathcal{C}$. The computations of these fuzzy sets are shown in Eqs. (9), (10) and (11).

First, we update the values of the *burnt* fuzzy set: at iteration k, the burnt land corresponds to the addition of the burning land and the land already burnt both at the previous iteration. So, B^k is computed from B^{k-1} and F^{k-1} as, $\forall c \in \mathcal{C}$,

$$B^k(c) = B^{k-1}(c) + F^{k-1}(c). \tag{9}$$

According to Eq. (1), $B^k(c)$ is a fuzzy value.

Then, we update the values on the *burning* fuzzy set: at iteration k, Eq. (8) gives the burning land, unless the newly computed value creates a violation of Eq. (1), then only the complement to 1 of the local value of B (already computed by Eq. (9)) is taken:

$$F^k(c) = \begin{cases} S^k(c) & \text{if } S^k(c) + B^k(c) \leq 1 \\ 1 - B^k(c) & \text{otherwise.} \end{cases} \tag{10}$$

Last, we update the values on the *unburnt* fuzzy set by applying Eq. (1) to iteration k, so using the complement to 1 with values associated with both the *burning* and the *burnt* fuzzy set at the same iteration:

$$U^k(c) = 1 - B^k(c) - F^k(c). \tag{11}$$

5.4. *Initial condition*

At the beginning of the computation, a first state of the fuzzy sets U, F and B must be input, so that the first iteration can take place. However in most cases a fire is lit from a unique point of ignition, it is possible to consider any kind of initial condition, with a fire already underway of any shape, or with several points of ignition.

In this paper, we propose a default setting of an unique point of ignition located at the center of the area of interest (see Eq. (12) for U, Eq. (13) for F and Eq. (14) for B), which therefore are defined as, $\forall c \in \mathcal{C}$

$$U^0(c) = \begin{cases} 1 \text{ if } c = \left(\lfloor \frac{n}{2} \rfloor, \lfloor \frac{n}{2} \rfloor\right) \\ 0 \text{ else} \end{cases} \tag{12}$$

$$F^0(c) = \begin{cases} 0 \text{ if } c = \left(\lfloor \frac{n}{2} \rfloor, \lfloor \frac{n}{2} \rfloor\right) \\ 1 \text{ else} \end{cases} \tag{13}$$

$$B^0(c) = 0 \tag{14}$$

where n is the number of lines and columns of the grid in $\mathbf{N}^{+*}$.

The reader is invited to remember that any set of burnt, unburnt and burning terrain can be retrieved or handcrafted before being included in the computation, thus enabling to consider any scenario that may occur or the simulation of which could be of interest. Some examples of different initial condition will be shown in the next section.

6. Examples

In this section, we show some examples of fire evolution. All examples are set in different locations around the city of Corte, France.

Figure 10 shows the evolution of all three fuzzy sets in a series of iterations, as well as in their final state. Each column represents an iteration while the last column shown the final state of both burnt and unburnt fuzzy sets. As evoked in Section 5.4,

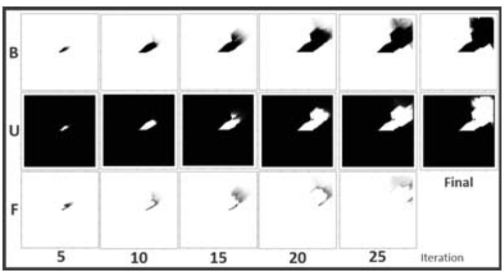

Fig. 10. Various steps of the computation of fire propagation from initial condition presented in section 5.4, with a wind at 60 degrees and a wind speed of 10 km/h (black area is maximum score, white is minimum)

many cases of initial condition are possible, and it is possible to shape it as wanted (for instance to reflect a state in a fire evolution).

Similarly with Figure 10, the evolution of wildfires taking as initial condition two different configurations is shown in Figure 11, with the two separate ignition points and in Figure 12, with a chevron-shaped fire front. In each example, the terrain varies, as well as the wind direction and speed.

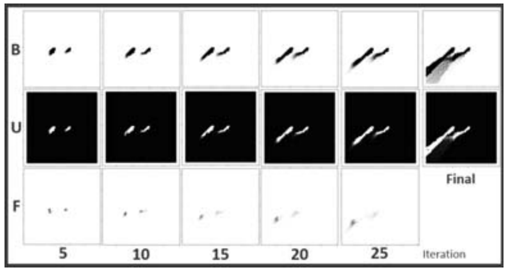

Fig. 11. Various steps of the computation of fire propagation, with a wind at 200 degrees and a wind speed of 20 km/h

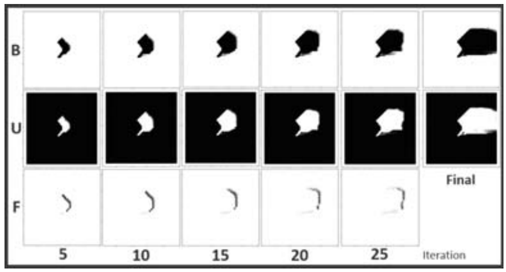

Fig. 12. Various steps of the computation of fire propagation, with a wind at 45 degrees and a wind speed of 40 km/h

7. Conclusion

In this paper, we use fuzzy logic to model the propagation of a wildfire on a given terrain. Indeed, uncertainty is inherent in the spread of a fire, which depends firstly on wind speed and direction, and on the topography. This latter can be deduced from a Digital Terrain Model of the area.

Our approach consists in an iterative process that computes the three possible states of the land subject to a fire, namely unburnt, burnt and burning areas, as 2D fuzzy sets. Their membership values represent the possible spread of a wildfire considering its natural evolution and allow considering the uncertainty of the propagation. To propagate the fire, we selected carefully from the literature the possible parameters for modeling the individual effects of wind and topography and compute their combined effect at each iteration.

In this work, we selected the minimal data needed, *i.e.* the wind parameters and a DTM. This gives an approximation of the fire propagation that can be used worldwide even when historic data are not available.This simulation will be used in crisis management systems to recommend public buildings that should be protected and possibly evacuated regarding the spread of the fire at given times.

Future work includes considering the simulation of other concerns for first responders in order to provide them with a versatile crisis management piece of software.

Acknowledgement

This work is part of the ResponDrone project, funded by the European Union's H2020 Research and Innovation Programme under Grant Agreement No. 833717.

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