Proceedings of the 33rd European Safety and Reliability Conference (ESREL 2023)
Edited by Mário P. Brito, Terje Aven, Piero Baraldi, Marko Čepin and Enrico Zio

doi: 10.3850/978-981-18-8071-1_P342-cd

Degradation modeling and RUL estimation of feedback control systems using stochastic diffusion process

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This paper considers a deteriorating feedback control system suffering from stochastic internal damage, and investigates a methodology for modeling its system-level degradation index with predicting its remaining useful life. The design of the controller allows the system output to track the input and, at the same time, hides part of the degradation and makes it difficult to be detected. To avoid the strong control impact, we equip an extra low-intensity controller while preserving the closed-loop structure and stability of the system with the aim to reveal the effects of inner damage on system performance. Then, at each inspection date, we apply only observable input and output from newly controlled system to estimate transfer function and extract the peak value of its step response as a degradation index which is thus less subject to control action. Thereafter, a stochastic diffusion process with nonlinear drift and diffusion is used to model the evolution of this index. To calculate the probability density function of system remaining useful life, Lamperti transform and Ricciardi transform are applied to convert this model to the standard Brownian motion so that an approximation of its first hitting time with a time-varying failure threshold is used to obtain the remaining useful life of our system. A case study from an inertial platform is given to proof the feasibility of the aforementioned methods.

Keywords: Feedback control system, degradation modeling, stochastic diffusion process, RUL prognosis, Lamperti transform, Ricciardi transform.

1. Introduction

Feedback control system (FCS) is a closed-loop system driven by a controller aiming to ensure expected dynamic performances. With time and usage, its inner components suffer from stochastic damages which accumulate and eventually cause system failure, Aggab et al. (2022). Our goal is to model the degradation process at the system-level of a FCS followed by the prognosis of its remaining useful life (RUL).

RUL is the residual lifetime of the degraded FCS from its current deteriorating state until its failure. Approaches for predicting the pRUL of FCS generally falls into two categories, as illustrated in our previous work (Gong et al., 2022). One is to model the component degradation of the system by stochastic processes so that the probability density function (PDF) of the first hitting time (FHT) are then calculated for RUL prognosis via the selected model, where the FHT is the first time when the degradation index exceeding the failure threshold, Do and Söffker (2021). However, when the degradation of the components is not synchronized with the degradation of the whole system, this approach is no longer accurate. To overcome this drawback, an alternative approach is to apply a filter constructed from the physical structure of the FCS. It estimates both system states and the stochastically hidden damages, and then combines them with Monte Carlo simulation to predict the system RUL, e.g., in (Obando et al., 2021; Moulahi and Hmida, 2022). Obviously, when the physical knowledge of the FCS is unknown, the filter cannot be established. In a nutshell, the system-level health index, which is applied to monitor the degradation state of the entire FCS, created neglecting system physical structure, is a more suitable choice for monitoring the degradation of the whole FCS. It can thereafter be applied to obtain a prognostic system RUL. In (Gong et al., 2022), we apply merely the input and output information of the deteriorating FCS to estimate their transfer functions, then extract the pole position changes and maximum gains as two system-level degradation indices. The strong effects from the controller make the output of the FCS track its input but at the same time offsets part of the degradation and makes it difficult to be inspected at the early degrading stage, but increase dramatically when FCS approaches the failure. Such trajectories are hard to successfully model with stochastic processes so that we obtain the RUL of the system by Monte Carlo simulation.

It is well known that calculating the RUL distribution based on stochastic degradation models can save a significant amount of time and expense compared to only using Monte Carlo simulation. Therefore, our work is carried out on the basis of establishing a system-level degradation index using a similar approach of (Gong et al., 2022) which are then modeled using stochastic processes for the purpose of obtaining analytical PDF of RUL by means of probability calculation. For reducing controller effects on the health index, we set up an extra controller for obtaining the degradation information from the FCS. To further explain, the system works normally with original controller except at each inspection date we switch to a low-intensity controller for deriving the system input and output. Their estimated transfer function thus contains less control affected degradation information. The peak value extracted from its step response, can thus reveal more precisely the system deterioration caused by stochastic hidden damage and is regarded as a degradation index which increases as the system degrades. Thereafter, a stochastic diffusion process (SDP), whose drift and diffusion are the combination of two polynomials with respect to time and degradation state respectively, is applied to model the degradation evolution of the aforementioned peak value while the maximum likelihood estimation (MLE) from (Särkkä and Solin, 2019) is employed to estimate all the parameters. To derive the PDF of its FHT, we attempt to convert the above SDP into standard Brownian motion (SBM) via the Ricciardi transform approach proposed in (Ricciardi, 1976) in order to easily apply the method developed in (Durbin, 1985), which aims to approximate the PDF of the FHT about a Markovian process given a time-varying failure threshold. However, considering the drift and diffusion of our SDP is too complex to apply Ricciardi transform directly, we first apply the Lamperti transform (Egorov et al., 2003) to convert the previously given SDP into an SDP with unit diffusion and drift depending on both time and degradation state. Then, we replace the state item of the above drift by a fixed degradation level which is the current degradation state at each inspection time interval. The approximated drift is thus only time-related making its corresponding SDP easy to transform into a SBM by Ricciardi transform. The RUL prognosis of the degraded FCS can be thus approximated by analytical PDF obtained above.

In summary, this paper attempts to obtain analytical PDF for predicting the RUL of FCS by applying SDP modeling on peak value, the system-level degradation index. The highlights of this paper are: Firstly, at each inspection date, we switch to the low-intensity controller to observe system input and output used to obtain the peaks. This aims to avoid the controller trying to offset excessive FCS degradation in order to achieve its control objectives, thus making it difficult to model the degradation index with little inner damage revealed in the early stage of degradation.

Secondly, in order to estimate the PDF of the RUL, an available method is to transform the modeled SDP into an SBM with time-varying failure threshold and approximate the PDF of its FHT via the method in (Durbin, 1985). However, for the SDP where both drift and diffusion are a combination of two polynomials related by time and degradation state, a direct application of the Ricciardi transform is not feasible. Therefore, we apply the Lamperti transform to convert the SDP to the one with unit diffusion, where the state of its drift is then approximated by a fixed degradation state at each inspection time interval. Thus, the only time-dependent drift and unit diffusion allow further conversion to SBM by the Ricciardi transform.

The rest of this paper is as follows. Section 2 introduces the deteriorating FCS and builds peak value as its degradation index. Section 3 models the evolution of peak using SDP and obtains the prognosis for the system RUL after a series of transformations and FHT approximation. Numerical validations of the above method are supplied in Section 4 through a case study. Section 5 gives the conclusion and perspectives.

2. Degradation Index Construction

To elaborate on our research issues, a deteriorating single-input single-output degradation-dependent FCS is given as

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(D(t,\mathbf{x}(t)))\mathbf{x}(t) + \mathbf{B}(D(t,\mathbf{x}(t)))u(t), \\ y_o(t) = \mathbf{C}(D(t,\mathbf{x}(t)))\mathbf{x}(t), \end{cases} \tag{1}$$

where $\mathbf{x}(t)$ and $y_o(t)$ are system state and output respectively. $\mathbf{A}(D(t,\mathbf{x}(t))) \in \mathbb{R}^{n\times n}$, $\mathbf{B}(D(t,\mathbf{x}(t))) \in \mathbb{R}^{n\times 1}$ and $\mathbf{C}(D(t,\mathbf{x}(t))) \in \mathbb{R}^{1\times n}$ are system matrices with unknown physical knowledge and dependent on stochastic inner damage $D(t,\mathbf{x}(t))$ defined as

$$D(t,\mathbf{x}(t)) = D(0,\mathbf{x}(0)) + \int_0^t \mu(r,\mathbf{x}(r))dr + \int_0^t \sigma(r,\mathbf{x}(r))dB(r). \tag{2}$$

$D(0,\mathbf{x}(0)) = 0$ represents that the system (1) is healthy initially, $\mu(r,\mathbf{x}(r))$ and $\sigma(r,\mathbf{x}(r))$ are

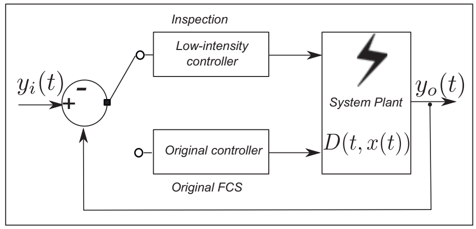


Fig. 1.: Inspection scheme.

both time- and degradation state-related items. $\{B(t)\}_{t\geq 0}$ stands for a SBM i.e. $B(t) \sim \mathcal{N}(0,t)$. $u(t)$ is the control signal designed to narrow the gap between the system output $y_o(t)$ and input $y_i(t)$. Then, we attempt to reveal the system-level degradation evolution of (1) caused by hidden stochastic damage $\{D(t,\mathbf{x}(t))\}_{t\geqq 0}$. Our research problem is started from building a system-level degradation index of (1) for the monitoring of its degrading process. As explained in (Gong et al., 2022), the system matrices are unknown so that they propose to estimate the transfer function at each inspection date of the original FCS (1) via a couple of observable input and output, then extracts alternative system-level indices that contain the information of $D(t,\mathbf{x}(t))$. However, directly observing the transfer function of original FCS cannot reveal more about the degradation occurring in the system plant, as the original well-designed controller offsets part of the inner damage on system for the reason of achieving the control goal. The degradation indices thus obtained fail to show significant system degradation in the early stage of degradation but increase dramatically in the later stage as the controller fail to meet the control requirements. This makes it difficult to be modeled by suitable and simple stochastic processes. To bypass it, we set up a new low-intensity controller (as it shows in Fig. 1) at each inspection date while preserving this FCS stability, with the aim of observing $y_o(t)$ and $y_i(t)$ that revealing more the effects of inner damage on system performance, to estimate its transfer function $H(s)$ via the method in (Ljung, 1998). Out of inspection, the FCS operates with original controller. The peak value $P(t)$ of the system step response observed via $H(s)$ is thus selected

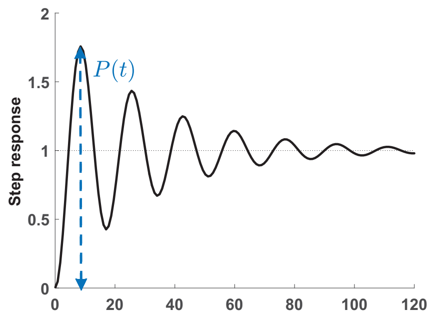


Fig. 2.: $P(t)$ from the step response of the transfer function $H(s)$.

as a degradation index to monitor the degrading evolution of (1) (See Fig. 2). As it shows in Fig. 3, $P(t)$ is increasing with the accumulation of hidden damage. We define $P(t)$ a fixed failure threshold L_f over which the FCS is failed, and the FHT T_f is denoted as

$$T_f = \inf\{t \geq 0, P(t) \geq L_f\}. \tag{3}$$

Degradation modeling and RUL prognosis are then developed via the degradation data inspected from $P(t)$.

3. Degradation Modeling & RUL Estimation

In this section, we model $P(t)$ by an SDP in Section 3.1, and then transform it to SBM via the methods given in Section 3.2 so as to approximate its FHT using the PDF.

3.1. *Degradation modeling*

Since the hidden damage (2) has both positive and negative increments, we propose an SDP

$$dX(t) = \mu(t, X(t))dt + \sigma(t, X(t))dB(t) \tag{4}$$

to model $P(t)$, where $\mu(t, X(t)) = a_{t_m}t^m + \cdots + a_{t_1}t + a_{X_m}X^m + \cdots + a_{X_1}X + a_c$ and $\sigma(t, X(t)) = b_{t_m}t^m + \cdots + b_{t_1}t + b_{X_m}X^m + \cdots + b_{X_1}X + b_c$. $\{a_{t_m}, \cdots, a_{t_1}, a_{X_m}, \cdots, a_{X_1}, a_c\}$ and $\{b_{t_m}, \cdots, b_{t_1}, b_{X_m}, \cdots, b_{X_1}, b_c\}$ are the parameters. $m, n \in R$ are the orders. According to the Euler–Maruyama equation, the solution of Eq. (4) can be approximated by

$$\begin{aligned}\hat{X}(t_{c+1}) &= \hat{X}(t_c) + \mu(\hat{X}(t_c), t_c; \theta)\Delta t \\ &\quad + \sigma(\hat{X}(t_c), t_c, \theta)\Delta B(t_c),\end{aligned} \tag{5}$$

where θ represents the unknown parameters vector and $\Delta B(t_c) \sim N(0, \Delta t)$. Assuming $\{t_c,\ c = 1, 2, \cdots, T\}$ as the equally-spaced inspection moments, We thus apply the MLE method given in (Särkkä and Solin, 2019) with the log-likelihood function $l(\theta)$ formulated as

$$\begin{aligned}&l(\theta) = \\ &\sum_{c=0}^{T-1}\Big[\frac{1}{2}\log|2\pi\sigma(t_c, X(t_c);\theta)\sigma^T(t_c, X(t_c);\theta)\Delta t| \\ &+ \frac{1}{2}(X(t_{c+1}) - X(t_c) - \mu(t_c, X(t_c);\theta)\Delta t)^T \\ &\times [\sigma(t_c, X(t_c);\theta)\sigma^T(t_c, X(t_c);\theta)]^{-1} \\ &\times (X(t_{c+1}) - X(t_c) - \mu(t_c, X(t_c);\theta)\Delta t)\Big]\end{aligned} \tag{6}$$

to estimate all parameters. The mean and variance of the degradation paths $P(t)$ and its increment $\Delta P \triangleq P(t_c) - P(t_{c-1})$ are close to $X(t)$ and $\Delta X \triangleq X(t_c) - X(t_{c-1})$ under the estimated parameters, respectively. $(\cdot)^T$ is the transpose.

3.2. *RUL estimation process*

Once we have modeled $P(t)$ with the stochastic process $X(t)$, given the failure threshold L_f, the PDF of the RUL R_f, which is defined as

$$R_f = \inf\{r > 0; P(t_c + r) \geq L_f | P(t_c) = P_c\} \tag{7}$$

at the current inspection date t_c and degradation level P_c, should be obtained through probability calculation. Since it is difficult to directly calculate the RUL of an SDP, we propose to first transform the general SDP into SBM and then apply the estimated PDF of FHT for SBM under time-varying failure thresholds in order to obtain the distribution of FHT and RUL with respect to SDP.

3.2.1. *Lamperti transform*

A Lamperti transform

$$Y(t) := \varphi(t, X(t)) := \int_{\xi}^{x} \frac{1}{\sigma(t, u)} du \Big|_{x=X(t)} \tag{8}$$

exists to transform the general SDP (4) into

$$dY(t) = \mu_Y(t, Y(t))dt + dB(t) \tag{9}$$

with unit diffusion via a one-to-one mapping, where ξ is any value. The drift function $\mu_Y(t, Y(t))$ is obtained by *Itô* formula as

$$\mu_Y(t, Y(t)) = \frac{\partial \varphi(t, \varphi^{-1}(t, Y(t)))}{\partial t} + \frac{\mu(t, \varphi^{-1}(t, Y(t)))}{\sigma(t, \varphi^{-1}(t, Y(t)))} - \frac{1}{2}\frac{\partial \sigma(t, \varphi^{-1}(t, Y(t)))}{\partial X(t)}. \tag{10}$$

The failure threshold L_f is thereafter transformed as a only time-varying

$$L_{f_L}(t) = \int_{\xi}^{x} \frac{1}{\sigma(t, u)} du \bigg|_{x=L_f}. \tag{11}$$

3.2.2. *Ricciardi transform*

The SBM transform (Ricciardi, 1976)

$$\begin{aligned} \tilde{X}(\tilde{t}) &\triangleq \psi(t, y) = e^{-\frac{1}{2}\int_0^t c_2(\tau)d\tau} y \\ &- \frac{1}{2}\int_0^t c_1(\tau) e^{-\frac{1}{2}\int_0^\tau c_2(r)dr} d\tau \\ \tilde{t} &\triangleq \phi(t) = \int_0^t e^{-\int_0^\tau c_2(r)dr} d\tau, \end{aligned} \tag{12}$$

is then given to transform (9) into a SBM $\tilde{X}(\tilde{t})$, where $c_1(t)$ and $c_2(t)$ are only time-related functions satisfying

$$\mu_Y(t, Y(t)) = \frac{1}{2}\left(c_1(t) + \int_z^y c_2(t)dy\right) \tag{13}$$

with $z \in I$. Accordingly, the failure threshold $L_{f_L}(t)$ is transformed into

$$L_{f_R}(t) = \psi(t, L_{f_L}(t)). \tag{14}$$

Obviously, $\mu_Y(t, Y(t))$ is a complex $t-$ and $Y(t)-$ related formula. Thus, there may not be $c_1(t)$ and $c_2(t)$ satisfying Eq. (13). To bypass it, at each inspection date t_c, we substitute the time-varying $Y(t)$ of $\mu_Y(t, Y(t))$ by its current value $Y_c := Y(t_c) = \int_\xi^x \frac{1}{\sigma(t,u)} du \Big|_{x=X_c:=X(t_c)}$ so that $\forall t \in [t_c, t_{c+1})$, we derive the approximation of (9) as

$$dY_t(t) = \mu_{Y_t}(t, Y_c)dt + dB(t). \tag{15}$$

Thus, $c_1(t)$ and $c_2(t)$ can be selected as

$$\begin{aligned} c_1(t) &= 2\mu_Y(t, Y_c) \\ c_2(t) &= 0 \end{aligned} \tag{16}$$

3.2.3. *FHT approximation*

After the above transforms, we thus apply the method in (Durbin, 1985) on the transformed SBM (12) to approximate the PDF of FHT under time-varying failure threshold $L_{f_R}(t)$ by formula

$$\begin{aligned} f_{T_f}(t) &= \frac{1}{\sqrt{(2\pi\tilde{t})}}\left(\frac{L_{f_R}(t)}{\tilde{t}} - \frac{\partial \tilde{X}(\tilde{t})}{\partial \tilde{t}}\right) e^{-\frac{L_{f_R}^2(t)}{2\tilde{t}}} \\ &= \frac{1}{\sqrt{(2\pi t)}}\left(\frac{L_{f_R}(t)}{t} - \frac{\partial \tilde{X}(t)}{\partial t}\right) e^{-\frac{L_{f_R}^2(t)}{2t}}. \end{aligned} \tag{17}$$

Given current inspection date t_c and RUL l, the PDF of RUL is obtained as

$$\begin{aligned} & f_{T_f}(l|t_c) = \frac{1}{\sqrt{(2\pi l)}} \cdot \\ & \left(\frac{L_{f_R}(t_c + l) - \tilde{X}(t_c)}{l} - \frac{\partial \tilde{X}(t)}{\partial t}\bigg|_{t=t_c+l}\right) \cdot \\ & e^{-\frac{\left(L_{f_R}(t_c+l) - \tilde{X}(t_c)\right)^2}{2l}}, \end{aligned} \tag{18}$$

which can thus be used to predict system RUL at each inspection date.

4. Numerical Experiments

To validate the developed RUL prognosis method in Eq. (18), we consider a deteriorating FCS which is a inertial platform thoroughly introduced in (Gong et al., 2022) whose structures is

$$\begin{cases} L_m x_1'(t) + R_m x_1(t) + K_e x_2(t) = u(t), \\ J x_2'(t) = K_m(t) x_1(t) + \omega(t), \\ x_3'(t) = x_2(t), \\ y_o(t) = x_3(t) + \nu(t), \end{cases} \tag{19}$$

where $x_k(t)$, with $k = 1, 2, 3$, denotes the system state, and $x_k'(t)$ is its time derivative; L_m, R_m, K_e and J are constant parameters; $\omega(t) \sim \mathcal{N}(0, \sigma_\omega)$ and $\nu(t) \sim \mathcal{N}(0, \sigma_\nu)$ are zero-mean Gaussian noises. A proportional-integral-derivative controller in bound $-80 \le u(t) \le 80$

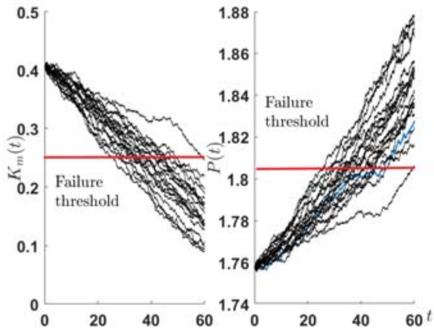


Fig. 3.: $K_m(t)$ and $P(t)$ in $[0, 60]$.

is configured and formulated as

$$u(t) = K_p \left(e(t) + \frac{1}{T_i} \int_0^t e(r)dr + T_d \frac{de(t)}{dt} \right), \tag{20}$$

where $e(t) = y_i(t) - y_o(t)$ is the control error and $y_i(t) = (\pi/10)\sin(0.4t)$ is a sinusoidal reference input. K_p, T_i and T_d are respectively the proportional, integral and derivative coefficients designed to achieve control objective. A Wiener hidden damage $D(t, \mathbf{x}(t)) \sim \mathrm{WP}\left(\lambda x_1^2(r), \sigma\right)$ is assumed to exist within the system and defined as

$$K_m(t) = d_0 + \lambda \int_0^t x_1^2(r)\, dr + \sigma B(t). \tag{21}$$

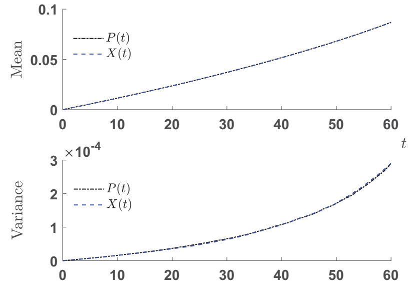
All the parameters are given by Table 1 of (Gong et al., 2022).

4.1. *Degradation process modeling*

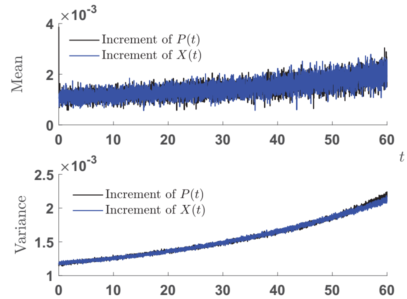
In order to clearly understand the relationship between inner damage $K_m(t)$ and degradation index $P(t)$. We simulate $K_m(t)$ in the time interval $[0, 60]$ and obtain the corresponding $P(t)$ from 1000 similar systems within the same inspection date via the method mentioned in Section 3 and in (Ljung, 1998). Some of the degradation paths of $K_m(t)$ and $P(t)$ are shown in Fig. 3. According to the above degradation data of $P(t)$, we select alternative expressions of drift and diffusion as

$$\mu(t, X) = a_{t_2}t^2 + a_{t_1}t + a_{X_2}X^2 + a_{X_1}X + a_c,$$
$$\sigma(t, X) = b_{t_2}t^2 + b_{t_1}t + b_{X_2}X^2 + b_{X_1}X + b_c. \tag{22}$$

of the SDP model (4) to model $P(t) \triangleq P(t) - P(0)$, i.e. zeroing out the initial value. Applying MLE, $\{a_{t_2}, a_{t_1}, a_{x_2}, a_{x_1}, a_c\} = \{-5.10e-08, 2.95e-06, 0.0806, 0.0034, 0.0011\}$, $\{b_{t_2}, b_{t_1}, b_{x_2}, b_{x_1}, b_c\} = \{9.33e-08, -7.95e-06, -1.31e-08, 0.0122, 0.0012\}$ are estimated. The fitting results are given in Fig. 4 which are



(a) The fitting of $P(t)$.



(b) The fitting of $\Delta P(t)$.

Fig. 4.: Model $P(t)$ by $X(t)$, $t \in [0, 60]$.

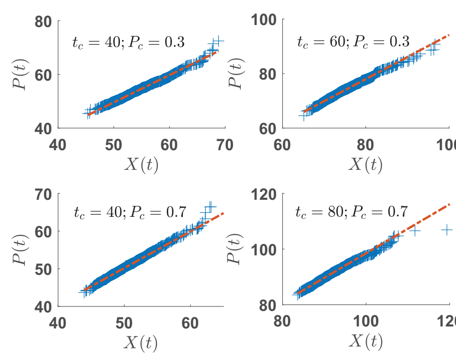


Fig. 5.: Quantile-Quantile plots between $P(t)$ and $X(t)$ under $L_f = P_c + 0.02$ at different groups of inspection dates.

assessed by the coincidence of mean and variance of $P(t)$, $X(t)$ and of $\Delta P(t)$, $\Delta X(t)$. For further assessing the fitting goodness, we compare the RUL of $P(t)$ and $X(t)$ at four different groups of inspection dates $t_c = [40, 40, 60, 80]$ with corresponding $P_c = [0.03, 0.07, 0.03, 0.07]$ given failure threshold $L_f = P_c + 0.02$. The Quantile-Quantile plots are thus given in Fig. 5 to proof the feasibility of the selected model.

4.2. *RUL comparison*

Fig. 8 shows the transforms on the modeled degradation paths and the corresponding changes on the mean and variance of their increments. A general $X(t)$ in Fig. 8(a) with $L_f = 0.02$ is transformed into a unit diffusion SDP $Y(t)$ with corresponding $L_{f_L}(t)$ in Fig. 8(b) by Lamperti transform; $Y(t)$ is further transformed to a SBM $\tilde{X}(t)$ with corresponding $L_{f_R}(t)$ in Fig. 8(c). As we explained in Section 3.2.2, for applying SBM transform (12), Eq. (13) must be satisfied. Thus we apply (15) to approximate (9) and then apply the SBM transform on it to obtain Fig. 8(c). To validate the approximation accuracy, Fig. 6 is given to compare the $\mu_Y(t, Y_c)$ with the $\mu_Y(t, Y(t))$ at different inspection states. From this figure, we could find the approximated $\mu_Y(t, Y_c)$ is functional except the initial degradation state $\{t_c = 0, X_c = 0\}$ which is not a matter since the inspection dates are usually not the initial date. Thus, we apply the PDF of RUL given in (18) based on the transformed $\tilde{X}(t)$, the comparison between the analytical PDF

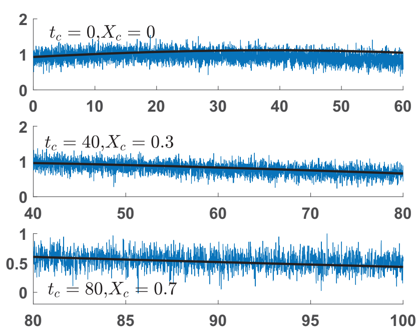
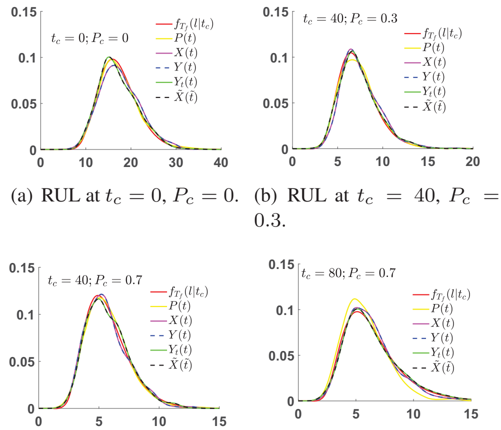


Fig. 6.: Approximate $\mu_Y(t, Y(t))$ by $\mu_Y(t, Y_c)$ at different $\{t_c, X_c\}_3$.



(a) RUL at $t_c = 0$, $P_c = 0$.

(b) RUL at $t_c = 40$, $P_c = 0.3$.

(c) RUL at $t_c = 40$, $P_c = 0.7$.

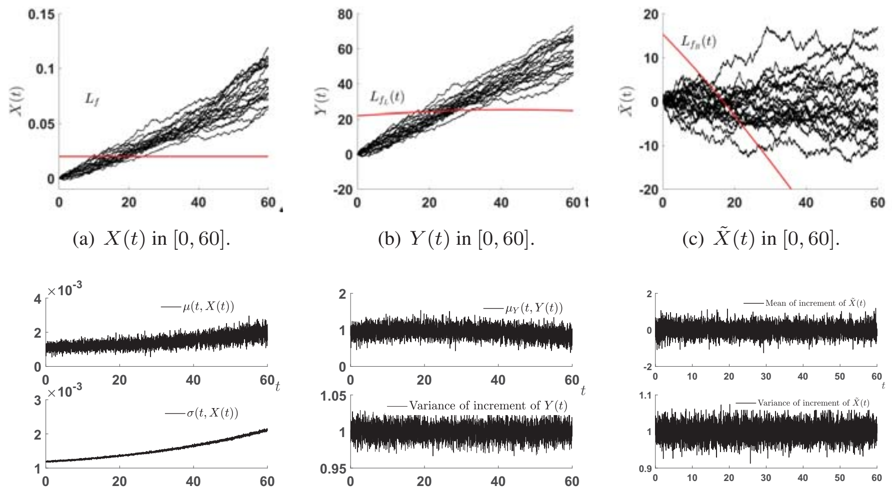
(d) RUL at $t_c = 80$, $P_c = 0.7$.

Fig. 7.: RUL comparison at different inspection dates $\{t_c, P_c\}_4$.

by Eq. (18) with the RUL from the original $P(t)$, modeled $X(t)$, transformed $Y(t)$, approximated $Y_t(t)$, transformed $\tilde{X}(t)$ are given in Fig. 7 where the initial failure threshold of $P(t)$ is chosen as $L_f = P_c + 0.02$. In this figure, we could see the RUL from $X(t)$ and $Y(t)$ are the same while the RUL from $Y_t(t)$ and $\tilde{X}(t)$ are also the same. This is due to the one-to-one relationship of Lamperti and Ricciardi transform. Moreover, we could find the red curve from $f_{T_f}(l|t_c)$ is far from the yellow curve $P(t)$ in the case when $t_c = 80; P_c = 0$. This is coming from the degradation modeling error since we only apply the degradation data in $[0, 60]$.

5. Conclusion and Perspective

In this paper, we study the RUL prediction of deteriorating FCS via SDP where the system suffers from stochastic hidden damage. To bypass strong control effects, a low-intensity controller is set up in the FCS for generating only observable input an output at each inspection date to estimate the transfer function between them. Thus, we extract peak value from its step response as a degradation index which can reveal more the inner damage and model it by an SDP. To study the PDF of the system RUL, an SDP is used to model the



(a) $X(t)$ in $[0, 60]$. (b) $Y(t)$ in $[0, 60]$. (c) $\tilde{X}(t)$ in $[0, 60]$.

(d) Mean and Variance of $X(t)$. (e) Mean and Variance of $Y(t)$. (f) Mean and Variance of $\tilde{X}(t)$.

Fig. 8.: SDP transforms to SBM first by Lamperti transform, then by SBM transform.

degradation index. This SDP is then transformed into SBM by two transformations to allow an easy application of the FHT approximation under time-varying failure threshold. Then, a PDF of the RUL is obtained, which can be applied to the controller reconfiguration decision to extend the system RUL.

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