

Reliability analysis of the ancient Nezahualcoyotl's dike: Investigating failure due to overflow using an improved hydrological model.

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Investigating the reliability of ancient hydraulic structures constructed without modern probabilistic criteria allows an understanding of why and how the structure fails. In this paper, we present an extended method, firstly introduced by Torres-Alves and Morales-Nápoles (2020), to perform the reliability analysis of the Nezahualcoyotl's dike that was designed (most likely) without probabilistic criteria. The dike was built around 1450 by the Aztec empire dividing Lake Texcoco from north to south (present-day Mexico City). We estimate the probability of failure due to overflow. By using a discrete time-state Markov chain and bi-variate copulas to generate large synthetic observations of the environmental variables precipitation and evaporation. In addition to the previous methodology, two sources of uncertainty were taken into account (i) the characterization of the environmental conditions during the dry season to estimate initial water levels on the lake and (ii) the influence of surface runoff and subsurface seepage losses on the water levels. The extended method allows for better characterization of the lacustrine system. Therefore an improved extent of the hydrology of the system and a more reliable estimation of the probability of failure of Nezahualcoyotl's dike are presented.

Keywords: Dike, copulas, Markov chains, water budget

1. Introduction

The great city of Tenochtitlan, the capital of the Aztec empire, was situated within a vast lake that covered a large part of the basin in Central Mexico. It was built on artificial platforms supported by the trunks of local trees (Merlo, 2022). In Tenochtitlan, water was managed and controlled by means of aqueducts and dikes. The largest hydraulic structure (apart from land reclamation through *chinampas*) was the Nezahualcoyotl dike which was around 16 km in length protected the city from floods, and also separated brackish and fresh waters (Bravo-Almazán, 2022; Palerm, 1973). The Nezahualcoyotl dike has not been operational since 1519 and stopped existing around the late 16th century. It has been somewhat studied from a historical perspective but only

recently from an engineering point of view by Torres-Alves and Morales-Nápoles (2020); Pouliasis et al. (2021) where the probability of failure due to overflow (P_O) was investigated. Several probabilistic approaches have been used to estimate P_O , the most popular including Monte Carlo Simulation methods utilizing Bivariate distributions and copulas to describe hydraulic and hydrological variables (see Pol et al. (2023); Rajabzadeh et al. (2023) for example). Rongen et al. (2022) used Cooke's method for combining experts' estimates in a structured way to estimate discharges that lead to at least one dike failure. However, as expected, all the studies have been focused on existing structures. In order to assess the P_O of unexistent structure, in this paper, we present an extended and improved methodology based on

Markov Chains, copulas, and a simplified water balance equation.

This paper is organized as follows, the theory behind the method is first discussed in section 2. Then, in section 3, the case study is presented. The application of the methodology is explained in section 4. Next, in section 5, the main findings of the study are discussed. Finally, in section 6 the conclusions of the investigation are drawn.

2. Methods

In this section, we describe the methodology to estimate the probability of failure of the Nezahuacoyotl dike due to overflow. The model consists of three main parts: the statistical model to characterize the hydrology of the study area, the water budget model to estimate the lacustrine system water levels, and the estimation of the probability of failure (P_O). In this study, failure is assumed to occur when an overflow event occurs in the dike.

2.1. Statistical model for wet and dry reason

In order to describe the hydrology of the study area, a statistical model based on Markov chains (MCs) and bi-variate copulas is developed. The model takes into account the dependence between variables. For this study, precipitation and evaporation are the only environmental variables considered. The model proposed by Torres-Alves and Morales-Nápoles (2020), recently used by Pouliasis et al. (2021), consists of three stages. In the first stage, a discrete time-state MC is used to generate a long synthetic series of occurrences of wet days (occurrence of precipitation) and dry days (absence of precipitation). Next, a one-dimensional probability distribution function is correspondingly fitted to consider environmental variables. Finally, the amount of precipitation and evaporation are simulated using bi-variate copulas. In this paper, we have developed an extension to the statistical model by characterizing and simulating the environmental conditions during the dry season.

2.1.1. Markov chains

The Markov chain model's definition in terms of rainfall occurrence is formulated in Gabriel and

Neumann (1962). The Markov chain (MC) algorithm examines data on sequences of occurrences where each event's probability solely depends on the state obtained in the preceding time step. Transition probabilities are used to describe the probability of changing states S . For precipitation data, two states are presented: absence of rain $\Rightarrow 0$ or occurrence of rain $\Rightarrow 1$, hence $S = \{0, 1\}$. Let X denote a random variable representing the precipitation observations. Since we are interested in the time series $\{X_t\}$, $t \in \mathbb{N}$. The first-order discrete state-time Markov process is described by $P(X_t = s|X_{t-1})$. MCs are represented by a state transition matrix \mathbf{P} , which for our case is defined as follows:

$$\mathbf{P} = \begin{bmatrix} P(X_t = 0|X_{t-1} = 0) & P(X_t = 0|X_{t-1} = 1) \\ P(X_t = 0|X_{t-1} = 1) & P(X_t = 1|X_{t-1} = 1) \end{bmatrix} \quad (1)$$

2.1.2. Bi-variate copulas

A bi-variate copula is a joint distribution with uniform margins in $[0, 1]$. According to Sklar (1959), the dependence between the random variables and their corresponding uni-variate marginal distribution functions can be expressed in terms of a copula for any multivariate joint distribution. Looking back to our precipitation observations with probability distribution F_X , the transition distribution is given by:

$$H(x|x_{t-1}) = P(X_t \leq x_t | X_{t-1} = x_{t-1}) = C_{\theta_X}(F(x_t)|F(x_{t-1})) \quad (2)$$

where $C_{\theta_X}(F(x_t)|F(x_{t-1}))$ is the conditional copula with parameters θ_X which would model the order 1 auto-correlation for the time series of interest. For the remainder of this paper, the conditional copula will be referenced using the following notation $C_{X_t, X_{t-1}}$. For specific details regarding copula modelling the reader is referred to Joe (2014) and references therein.

2.2. Water budget

Water balance models are formulated with a simple structure to account for all water entering and leaving the basin or water bodies. Water input may be from precipitation (X), streamflow into the wa-

ter body (Q), surface runoff (Q_r), and subsurface runoff (Q_s). Outflow could be evaporation (Y) streamflow discharge from the water body (Q_o), and subsurface permeability losses (Q_d) McCuen et al. (2005). Therefore, since water balance is based on the conservation of mass, the principle can be used for describing the time rate of change of the volume (dV/dt) in a water body as follows:

$$dV/dt = X + Q + Q_r + Q_s - Y - Q_o - Q_d \quad (3)$$

where dt is a selected time increment for which each increment of volume dV is measured. Q_s is the water balance variable that is hardest to quantify. It requires a variety of measurements and the assumption that the properties of subsurface runoff in the research region are homogeneous. Q_d is dependent on the soil texture and soil structure at the water body bottom which mainly determines the rate of the subsurface seepage losses. Regarding the precipitation, one part runs off the surface (surface runoff), another part infiltrates, recharging aquifers, and the remaining volume evaporates. To know the amount of precipitation corresponds to Q_r , runoff coefficients (c) defined as the ratio of runoff to precipitation are obtained by determining the soil type, gradient, permeability, and land use. Hence, $Q_r = cX$.

When the values of water elevation (E) of the water body are not available, Volume-Elevation (V-E) and Volume-Area (V-A) curves can be employed to obtain a rough estimation of the volume (V) and surface area (A) of the water body at certain water levels. These curves can be constructed by plotting the volume (or area) below selected elevations against the corresponding elevation. Then, a line can be fitted to the data, and the water elevation (H_{Wt}) at the time step t can be estimated from the corresponding value of dV/dt .

2.3. Limit state function

In order to evaluate the probability of failure a limit state function Z should be previously defined. The condition Z is the point at which a structure or a portion of a structure ceases to meet one of its performance requirements. For overflow failure (O), the limit state function Z_O is assessed

by means of the total height of the structure (H), and the maximum water levels at the lacustrine system ($\max H_W$). Z_O is given by Eq. 4. Thus, P_O due to overflow is $P_O = P(Z_O \leq 0)$. It is considered that the dike fails when the water level at the lake exceeds the height of the dike.

$$Z_O = H - \max H_W \quad (4)$$

With the previous ideas in mind, the simulation algorithm can be summarized as follows:

- (i) Split the data set into the wet season and dry season.
- (ii) Select the observations representing the precipitation X on the wet season
- (iii) Select the largest block of consecutive wet days ($X_t = 1$) and the largest block of consecutive dry days ($X_t = 0$).
- (iv) Fit the appropriate one-dimensional probability distribution functions, $F_X|_{X_t=1}, F_Y|_{X_t=0}$ and $F_Y|_{X_t=1}$ characterizing the observations in the blocks. Where Y denote the random variable representing the evaporation observations.
- (v) Select and fit the suitable copulas $C_{X_t, X_{t-1}}, C_{X_t, Y_t}$ and $C_{Y_t, Y_{t-1}}$.
- (vi) Compute \mathbf{P} of the Markov chain for the intermittent process X_t .
- (vii) Simulate the desired length of the Markov sequence and split the time series into dry and wet blocks.
- (viii) Generate one random sample of the precipitation fitted distribution function
- (ix) For wet blocks use the copula $C_{X_t, X_{t-1}}$ to obtain values of precipitation. Then use C_{X_t, Y_t} for evaporation values. For dry blocks, use $C_{Y_t, Y_{t-1}}$ to obtain synthetic observations of evaporation. Notice that the first value of precipitation is obtained as a random realization of the fitted one-dimensional probability function.
- (x) Repeat steps (ii) to (ix) for the dry season.

Once the model has been fully explained, the case study is presented in the next section.

3. Case study

In order to demonstrate how the suggested approach can be used to flood defences where knowledge is limited, this research gives the estimation of the Nezahualcoyotl dike’s reliability in the Valley of Mexico basin. This case study focuses on modern-day Mexico City at the height of the Aztec empire.

3.1. *Nezahualcoyotl dike and the lacustrine system*

Mexico-Tenochtitlán was, at the time of the arrival of the Spaniards to central Mexico in 1519, the capital city-state of a large empire. At that time, the lakes of Chalco, Xochimilco, Texcoco (saltwater lake), Xaltocan, Zumpango, and Mexico (where Tenochtitlan was settled) formed the lacustrine system of interest. The Nezahualcoyotl dike, built around 1450 (Palerm, 1973), was one of the most remarkable hydraulic structures of the time. It had a length of approximately 16 km, a total height (H) of 8 m, and a width of 3.5 m. The dike divided Lake Texcoco from north to south, forming Lake Mexico on its west side, and it prevented saline water from Lake Texcoco from flowing into Lake Mexico, where the city of Tenochtitlan was located (see Figure 1). The reader is directed to Torres-Alves and Morales-Nápoles (2020) and any references therein for more specific information regarding the characterization of the dike.

3.2. *Data*

The Valley of Mexico basin is a topographically closed basin, with an area over 9600 km², nowadays it is the most populated region in the country. The National Water Commission (CONAGUA, for its acronyms in Spanish) divides the Valley of Mexico basin into seven zones (sub-basins). Table 1 and Figure 1 show the name and locations of the sub-basins.

Several data sources were analyzed. We use the national meteorological service (SMN, for its acronyms in Spanish) database SMN (2023) to obtain measurements of X and Y in mm/day. The data were filtered with the open-source Geographic Information System (GIS) software QGIS

v3.10 (QGIS Development Team, 2022) to account for stations in the Valley of Mexico basin. The nearest stations to the geometrical centroid of each one of the seven sub-basins were selected. The measurement date was used to match the two data sets per study station. To quantify our models we only utilize days where there were measurements available for both X and Y .

Digital elevation models (DEMs) of the study area with a resolution of 15 metres, available on the National Institute of Statistics and Geography (INEGI, for its acronyms in Spanish) web page (INEGI, 2023), were employed to characterize the topography of the lacustrine zone.

Regarding water balance variables, the stream flow Q is neglected due to the topographic characteristics of the basin. To calculate Q_r we use the runoff coefficients given by the studies carried out by CONAGUA (Peña Díaz, 2019; Secretaría de Gobernación, 2011). The surface runoff coefficients for each sub-basins into which the Valley of Mexico Basin is divided are presented in the last column of Table 1. It is clear that the present-day Valley of Mexico basin presents a higher degree of urbanization. This implies that the runoff coefficient reported by CONAGUA is higher than the value that could be estimated around the year 1519 since urbanized areas experience significantly more runoff volume than undeveloped areas (Chow, 1964).

Table 1. Valley of Mexico sub-basins. Total basin area of 9611.6 km².

Sub-basin no.	Name	Area [km ²]	c
I	Xochimilco	506.8	0.120
II	Río la Compañía	1154.2	0.097
III	Tochac-Tecocomulco	1309.7	0.089
IV	Río de las avenidas	2628.5	0.085
V	Texcoco	1386.5	0.100
VI	Ciudad de México	1804.2	0.140
VII	Río Cuautitlán	821.7	0.120

Subsurface runoff (Q_s) is neglected. According to Secretaría de Gobernación (2016) the groundwater levels are located at an elevation of 2190 meters above sea level, i.e. 40 m below the bottom of the lacustrine system. In order to estimate

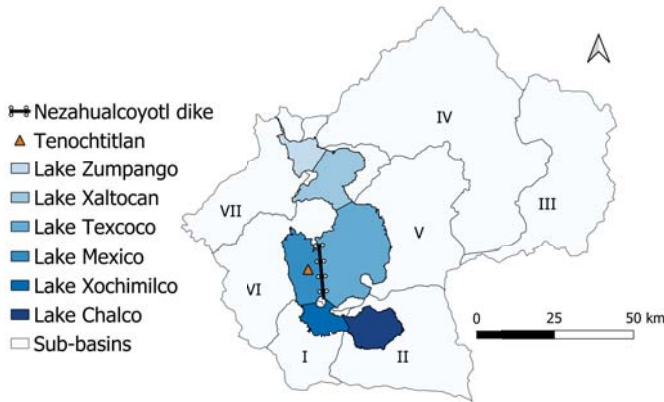


Fig. 1. Valley of Mexico basin. Sub-basins notation according to Table 1

the Q_d , we take into account information on the vertical permeability (vertical hydraulic conductivity) k_v of the Upper Clay Formation (UCF, from 2 to 30 m depth) of the lacustrine zone. It has been shown by several authors (López-Acosta et al. (2019); Lucero Rivera (2018); Herrera et al. (1974) for example) that there are large method-to-method discrepancies in the estimated permeability that range from 4.21×10^{-10} m/s to 1.29×10^{-7} m/s. Finally, Q_o is not considered in the analysis since the first conduit to drain the valley's water was constructed in the 17th century. Therefore, the time rate of change of the volume for the LS can be described by modifying Eq. 3:

$$\begin{aligned}
 dV/dt = & X_t A_{LS,t-1} + 0.95 X_t (A_{LS} - A_{LS,t-1}) \\
 & + \sum_{j=1}^7 c_j X_t A_{sbj} - Y_t A_{LS,t-1} \\
 & - k_v A_{LS,t-1}
 \end{aligned} \quad (5)$$

where A_{LS} is the area of the lacustrine system at the elevation of 2250 m above sea level and A_{sb} is the dry area of the sub-basin j , i.e the total area of the sub-basin minus the proportion of A_{LS} presented at the sub-basin. $A_{LS} - A_{LS,t-1}$ represents the dry area of the LS in the time frame $t - 1$. Hence $A_{LS,t-1}$ is the LS flooded area for the same time frame. Notice that a runoff coefficient of 0.95 is applied to the second term of the equation. After detailing the data used, in the successive section,

the application of the model is described.

4. Applying the model

To implement the proposed model, the following assumptions and simplifications were made:

- The interaction between the lakes does not influence the growth of the water level in Lake Texcoco.
- Present-day data were used in order to characterize the fluxes in and out of the basin in a hydrological balance.
- As mentioned in Section 3.2, subsurface runoff and stream flow are neglected.
- The minimal runoff coefficient ($c = 0.085$) reported by CONAGUA is assumed for the five sub-basins because urban expansion, which did not exist 500 years ago, has an impact on the runoff coefficient values reported.
- Due to the large difference of reported permeabilities (up to 3 orders of magnitude) a value of $k_v = 2.6$ mm/day is assumed.
- Only overflow is considered in the reliability analysis.
- One station is used for the quantification of the model since the variables at all seven stations can be modeled by a very similar one-dimensional marginal probability distribution function and have a

similar transition probability matrix.

4.1. Synthetic data series

Following the algorithm described in Section 2, the observations of precipitation (X) and evaporation (Y) of the selected station named Atenco during the wet and dry seasons were split into blocks (Steps i to ii). Next, each X and each Y were fitted to probability distributions F_X and F_Y correspondingly (Step iv). The results are shown in Table 2. The probability distributions were obtained based on maximum likelihood estimation and by visual inspection. The dependence and auto-correlation that characterize the data were calculated and later modelled with bi-variate copulas $C_{X_t, X_{t-1}}$, C_{X_t, Y_t} and $C_{Y_t, Y_{t-1}}$ (Step v). The parameters of the copulas can be found in Table 3. Bayesian information criterion (Neath and Cavanaugh, 2012) was used to select the best-fit copula. The transition probability matrices were quantified for both, wet and dry, seasons (Step vi, see Eq. 6).

$$\mathbf{P}_{\text{wet}} = \begin{bmatrix} 0.68 & 0.32 \\ 0.36 & 0.64 \end{bmatrix} \quad \mathbf{P}_{\text{dry}} = \begin{bmatrix} 0.92 & 0.08 \\ 0.62 & 0.38 \end{bmatrix} \quad (6)$$

Table 2. Fitted distributions.

Season	Case	Name	Parameters		
			μ	σ	ξ
Wet season	$F_X _{X_t=1}$	Gumbel	13.13	11.37	-
	$F_Y _{X_t=0}$	Weibull	-	5.63	5.63
	$F_Y _{X_t=1}$	Weibull	-	2.91	4.60
Dry season	$F_X _{X_t=1}$	Normal	3.01	2.35	-
	$F_Y _{X_t=0}$	Weibull	-	2.74	7.05
	$F_Y _{X_t=1}$	Gumbel	4.44	0.83	-

Table 3. Copula fit for all stations and pairs of variables.

Season	Case	Name	Parameter θ
Wet season	$C_{X_t, X_{t-1}}$	Joe 180° rotated	1.01
	C_{X_t, Y_t}	Gumbel	1.55
	$C_{Y_t, Y_{t-1}}$	Frank	4.73
Dry season	$C_{X_t, X_{t-1}}$	Joe	1.61
	C_{X_t, Y_t}	Gumbel	1.22
	$C_{Y_t, Y_{t-1}}$	Gumbel	2.34

Next, an MC sequence is generated. Note that the MC will not provide precipitation amounts, only its occurrence. To generate synthetic observations series for precipitation and evaporation the fitted copulas were utilized (Steps vii to ix). For illustration purposes, Figure 2 shows a comparison of twenty random runs of 750 consecutive days (2 years) of synthetic observations against the measurements of the Atenco station. Notice that the model can simulate the trend of wet and dry seasons.

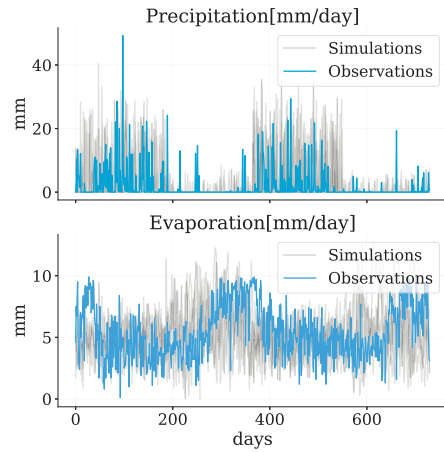


Fig. 2. Comparison of twenty runs of 750 consecutive days simulated against observed data during 1970 and 1971.

5. Results

Once the model has been tested, we generate 365x5000 days (5000 years) of synthetic observations. The time rate of change of the volume was computed for every simulated day using Eq.5. Then, the computed volume was used to obtain the adjusted water level and surface area at the lake based on the second-degree fitted polynomials according to Eq. 7 and Eq.8. (V-E and V-A curves correspondingly). The new value of the lake's surface area is used to update Eq. 5 and the process repeats until the time series of precipitation and evaporation are over. In this way, the time variation of water levels is obtained. As an example, Figure 3 shows 100 years of simulations,

as can be seen, the water level almost hit the mark of 8 at around 8000 days and 19 000 days of this particular realization.

$$E = -1.45 \times 10^{-23} V^2 - 6.04 \times 10^{-11} V + 2.19 \times 10^3 \quad (7)$$

$$A = -5.41 \times 10^{-14} V^2 + 2.10 \times 10^{-1} V - 1.25 \times 10^{11} \quad (8)$$

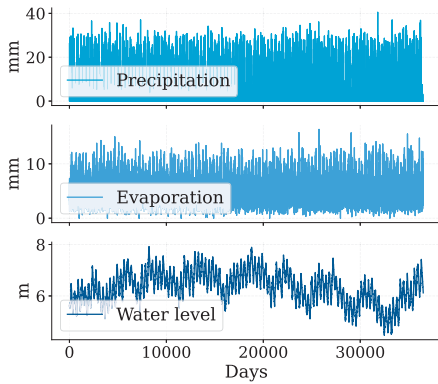


Fig. 3. Simulations for daily fluctuation of precipitation, evaporation and water level at Lake Texcoco (100 years).

5.1. Reliability analysis and discussion

The probability of failure of the dike was tested for overflow. According to the limit state function (Eq. 4), when the lake level rises higher than the total height of the dike, it is said that the structure has failed. Several assumptions were made to simplify the model presented in this paper (Section 4). Given these assumptions, the estimated probability of failure of the dike Nezahualcoyotl is $P_O \approx 0.012015$ which corresponds to a return period T of 83 years. Consequently, it can be inferred that, on average, a singular failure is anticipated to occur over every 83 years of operation. This outcome does not contradict the historical reference that the dike never failed during at least 69 years (from 1450 to 1519). However, this outcome is heavily influenced by c and k_v . Given a large number of possible combinations of c and k_v , 30 scenarios (combinations) have been chosen for

further analysis. These scenarios provide realistic return periods. Figure 4 shows the estimated return periods dependent on the runoff coefficient c and the vertical hydraulic conductivity k_v . As can be seen, both c and k_v have almost the same sensitivity for determining the return period of interest. As can be seen, with values of c of 0.1 and 0.093 larger values of k_v are needed for the dike to be reliable.

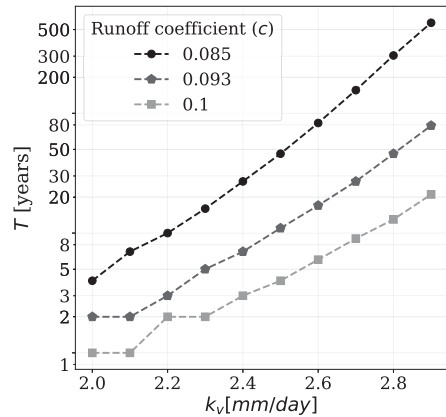


Fig. 4. Return periods given runoff coefficient

6. Conclusions

This paper extends the methodology presented in Torres-Alves and Morales-Nápoles (2020) to investigate the reliability of the Nezahualcoyotl's dike constructed around 1450 without formal probabilistic criteria. The improved methodology takes into account the characterization of the environmental conditions during the dry season and the influence of surface runoff and sub-surface permeability losses on the water levels. The methodology uses a simplified water balance methodology, using hydrogeological conditions as described in the literature, historical records and present-day data. Markov Chains and copulas are used to simulate the stochastic processes that characterize the hydrology of the lacustrine system.

The runoff coefficients and the vertical hydraulic conductivity are the major sources of uncertainty in this study. Regardless of the unknowns, the probabilistic model suggested in this

study gives accurate estimates for the values of precipitation, evaporation, and water levels. We estimate a probability of failure around 0.012 which is consistent with the lack of historical evidence that the dike failed during the course of its roughly 70-year lifespan.

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