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Stochastic Model Updating and Model Class Selection for Quantification of Different Types of Uncertainties

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Stochastic model updating has been increasingly utilized in various engineering applications to quantify parameter uncertainty from multiple measurement datasets. We have recently developed a stochastic updating framework, in which the parameter distributions are approximated by staircase density functions (SDFs). This framework is applicable without any prior knowledge of the distribution formats; thus, it can be considered as a distribution-free approach. On the other hand, measurement uncertainty should also be considered in model updating since the measurement is typically performed under hard-to-control randomness. However, in model updating, it is difficult to distinguish different types of uncertainties in the measurement datasets, and measurement uncertainty is often embedded in parameter uncertainty. To address this issue, this study employs the Bayesian model class selection framework, in which different types of probabilistic models are used to represent different types of uncertainties and the most appropriate model is determined based on the associated evidence. In this sense, the proposed framework does not require any prior knowledge about the sources of uncertainty in the measurement datasets. Simple numerical examples are used to demonstrate the proposed framework.

Keywords: Uncertainty quantification, stochastic model updating, Bayesian model updating, Bayesian model class selection, staircase density function, distribution-free approach.

1. Introduction

Stochastic model updating has gained increasing attention as a powerful technique to quantify parameter uncertainty by assigning a probabilistic model to the model parameters and calibrating its hyper parameters from multiple measurement datasets. Thus, the updating results depend on the probabilistic model assumption.

We have recently developed a distribution-free approach to stochastic model updating, where the probabilistic model is defined using SDFs (Kitahara et al., 2022). This approach relies only on the bounded set of the probabilistic model, and within this set, a broad range of distributions are arbitrarily approximated using SDFs.

On the other hand, measurement uncertainty should also be considered in model updating since the measurement is typically done under hard-to-control randomness. In stochastic updating, these two types of uncertainties are often not separated and are both characterized using the parameter distribution. However, this can result in the over-estimation of parameter uncertainty.

In this study, the aforementioned framework is combined with Bayesian model class selection (Beck and Yuen, 2004) to quantify different types of uncertainties. This framework uses different probabilistic model classes to represent different types of uncertainties. The optimal model class is then determined based on the evidence that can be computed as the by-product in the model updating through a Bayesian fashion.

2. Outline of the Proposed Framework

In this framework, the following three probabilistic model classes are considered:

$$\mathcal{M}_1(x, \sigma_\varepsilon^2) = h(x) + \mathcal{N}(0, \sigma_\varepsilon^2) \quad (1)$$

$$\mathcal{M}_2(x|\boldsymbol{\theta}) = h(x|\boldsymbol{\theta}), \text{ with } x \sim \mathcal{SD}(\underline{x}, \overline{x}, \boldsymbol{\theta}) \quad (2)$$

$$\mathcal{M}_3(x|\boldsymbol{\theta}, \sigma_\varepsilon^2) = h(x|\boldsymbol{\theta}) + \mathcal{N}(0, \sigma_\varepsilon^2), \\ \text{with } x \sim \mathcal{SD}(\underline{x}, \overline{x}, \boldsymbol{\theta}) \quad (3)$$

where h is the simulator with the model parameter x; $\mathcal{N}(0, \sigma_\varepsilon^2)$ is a random variable that follows the Gaussian distribution with the zero mean and the variance σ_ε^2; $\mathcal{SD}(\underline{x}, \overline{x}, \boldsymbol{\theta})$ is the staircase random variable with the lower and upper bounds $\underline{x}$ and $\overline{x}$ and hyper parameters $\boldsymbol{\theta}$. In these model classes, measurement uncertainty is given as the Gaussian variable $\mathcal{N}(0, \sigma_\varepsilon^2)$ while parameter uncertainty is modeled as the staircase variable $\mathcal{SD}(\underline{x}, \overline{x}, \boldsymbol{\theta})$.

The hyper parameters, i.e., $\boldsymbol{\theta}$ and σ_ε^2, can be calibrated using the well-known Bayes' theorem:

$$P(\boldsymbol{\vartheta}|\mathcal{M}, \mathbf{Y}_D) = \frac{\mathcal{L}(\mathbf{Y}_D|\boldsymbol{\vartheta}, \mathcal{M})P(\boldsymbol{\vartheta}|\mathcal{M})}{P(\mathbf{Y}_D|\mathcal{M})} \quad (4)$$

where $P(\boldsymbol{\vartheta}|\mathcal{M})$ is the prior distribution of $\boldsymbol{\vartheta}$ that consists of $\boldsymbol{\theta}$ or/and σ_ε^2; $P(\boldsymbol{\vartheta}|\mathcal{M}, \mathbf{Y}_D)$ denotes the posterior distribution of $\boldsymbol{\vartheta}$; $\mathcal{L}(\mathbf{Y}_D|\boldsymbol{\vartheta}, \mathcal{M})$ denotes the likelihood function based on the measurement datasets $\mathbf{Y}_D$; $P(\mathbf{Y}_D|\mathcal{M})$ indicates the evidence for the model class $\mathcal{M}$. The posterior distribution is obtained via the transitional Markov chain Monte Carlo (TMCMC) algorithm. In its procedure, the evidence is obtained as a by-product, which serves as the plausibility measure of the model class given the measurement datasets. Thus, the optimal model class can be determined as the one that provides the largest evidence value.

3. Numerical Example

The framework is demonstrated using examples of a polynomial function $h(x) = 0.1(x+1)^2 + 0.5$. We consider three synthetic datasets with the same sample size of 1000; (i) datasets with measurement uncertainty, where the true system response $h(0)$ is contaminated by Gaussian noise $\mathcal{N}(0, \sigma_\varepsilon^2)$ with σ_ε chosen as 10 % of the system response; (ii) datasets with parameter uncertainty which are generated by assigning the parameter distribution $x \sim \mathcal{N}(0, 1)$; (iii) datasets with both measurement and parameter uncertainty, in which the multiple system responses are contaminated by the Gaussian noise.

For each datasets, the proposed framework is carried out using the three model classes above; hence, a total of 9 TMCMC procedures are in use. The evidence values obtained are summarized in Table 1. It can be seen that, for each datasets, the appropriate model class is chosen as the optimal one as it provides the largest evidence value.

Table 1. Evidence for each model class.

Datasets	Model class	Evidence
(i) Measurement uncertainty	$\mathcal{M}_1$	2.0×10^{-3}
	$\mathcal{M}_2$	2.3×10^{-5}
	$\mathcal{M}_3$	2.8×10^{-5}
(ii) Parameter uncertainty	$\mathcal{M}_1$	5.9×10^{-14}
	$\mathcal{M}_2$	1.6×10^{-3}
	$\mathcal{M}_3$	6.4×10^{-4}
(iii) Both uncertainties	$\mathcal{M}_1$	8.0×10^{-21}
	$\mathcal{M}_2$	1.2×10^{-5}
	$\mathcal{M}_3$	5.1×10^{-4}

For datasets (iii), the calibrated distribution of x is obtained for $\mathcal{M}_2$ and $\mathcal{M}_3$ as SDF assigning the posterior estimates of $\boldsymbol{\theta}$ and shown in Fig. 1. As can be seen, $\mathcal{M}_2$ results in the overestimation of parameter uncertainty whereas $\mathcal{M}_3$ provides an appropriate estimation.

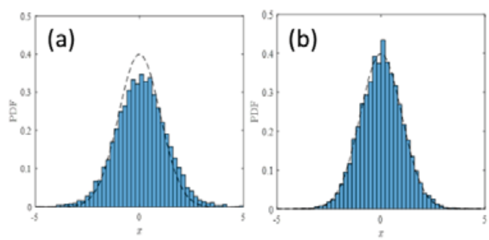

Fig. 1. Calibrated PDF of x for (a): $\mathcal{M}_2$ and (b): $\mathcal{M}_3$.

4. Conclusion

An uncertainty quantification approach combining Bayesian model updating and model class selection is developed. Simple numerical examples indicate that the approach can distinguish different sources of uncertainties and quantify parameter uncertainty as appropriate.

References

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