Proceedings of the 33rd European Safety and Reliability Conference (ESREL 2023)
Edited by Mário P. Brito, Terje Aven, Piero Baraldi, Marko Čepin and Enrico Zio

doi: 10.3850/978-981-18-8071-1_P080-cd

Stochastic Analysis of RC Rectangular Shear Wall Considering Material Properties Uncertainty

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Abstract

It is widely acknowledged that concrete's mechanical properties exhibit some degree of uncertainty due to many factors (e.g., the mix proportion, placing, curing, etc.). Therefore, it is crucial to consider such uncertainty when assessing the performance of real-world engineering structures. So, the main objective of this study is to perform a series of probabilistic analyses on one of the critical structural components that is widely utilized in high-rise buildings which is a rectangular shear wall. The material properties uncertainty is represented by a random field generator based on covariance matrix decomposition incorporated with the Generalized F-discrepancy-based point selection strategy to efficiently generate the samples. The multi-axial concrete damage plasticity model combined with the multilayered shell element is adopted in the simulation of the shear wall. The stochastic analysis results revealed that the material spatial variability caused a random distribution of initial damages and the subsequent stochastic non-linear evolution. Moreover, the probability density evolution method (PDEM) is employed to perform the reliability assessment for the considered shear wall. The proposed framework could well capture the stochastic inelastic behavior of the considered shear wall and can be further implemented to capture the stochastic response and safety assessment of high-rise buildings.

Keywords: Random Field, Stochastic Analysis, Rectangular Shear Wall, PDEM, Reliability Assessment.

1. Introduction

Rectangular shear walls are an important structural element in building structures, providing resistance to lateral loads due to wind or seismic events. The response of these walls is affected by several factors, including the material properties used in their construction. Random material properties can lead to uncertainties in the structural response of these walls, particularly in the context of seismic design. Research in this area has focused on developing probabilistic models for the material properties of concrete used in shear walls and investigating the impact of these varying material properties on wall response. Studies have also examined the use of reinforced concrete and fiber-reinforced polymer composites to enhance the performance of shear walls with random material properties. In general, there is a growing trend for studying the response of rectangular shear walls with random material properties, highlighting the need for careful consideration of material uncertainties in the design of these critical structural elements.

Concrete is a type of composite material consisting of multiple phases and characterized by non-linear and random mechanical behavior. To design and evaluate reinforced concrete (RC) structures, it is necessary to employ non-linear finite element (FE) analyses due to the nonlinearity of concrete's mechanical behavior when subjected to loading (Castaldo et al. 2020). This is particularly vital during catastrophic events like strong earthquakes where the complete stress-strain process of concrete is essential.

The non-uniformity of concrete material has a significant impact on the structural damage location and the reliability of the structure, as reported by several researchers (Nguyen et al. 2013). The mechanical properties' stochastic nature of concrete leads to the occurrence of damage initiation followed by random non-linear evolution, influenced by the concrete tensile strength uncertainty, as demonstrated in various studies (Xu and Li 2019). The correlation length of the concrete properties affects the reliability and the variability of the dissipated energy of the shear wall (Chen et al. 2018). Further investigation of the same shear wall (He 2018) revealed that the studied wall exhibited bending-shear failure when accounting for the spatial variation of concrete rather than bending failure observed in the deterministic analysis, which could be misguided.

In the current study, spatial variability of the concrete mechanical properties is simulated through a random field generator based on covariance matrix decomposition incorporated with the Generalized F-discrepancy-based point selection strategy to efficiently generate the samples. The FE model of a rectangular shear wall is built up on ABAQUS software, where the multilayered shell element is adopted, and the concrete nonlinearity is reflected by the multi-axial concrete damage plasticity model in the analysis process through the user's defined subroutine (VUMAT). Moreover, the probability density evolution method (PDEM) is employed to conduct the reliability evaluation for the considered shear wall.

2. Material models

The constitutive relations of the adopted material properties (e.g., steel and concrete) are greatly important in the simulation procedure to accurately represent the non-linear structural response under complex loading processes such as seismic excitation.

2.1 Constitutive relation of concrete

The multi-axial plasticity damage constitutive model (Wu, Li, and Faria 2006; J. Li and Wu 2005; Jie Li and Ren 2009) is employed to precisely capture the non-linear response of the studied shear wall, which is briefly discussed herein. On the basis of the continuum damage mechanics, the second-order Cauchy stress tensor σ can be represented as:

$$\boldsymbol{\sigma} = (\mathbb{I} - \mathbb{D}):\bar{\sigma} = (\mathbb{I} - \mathbb{D}):\mathbb{E}_0:(\varepsilon - \varepsilon^{p}) \qquad (1)$$

where σ stands for the stress tensor and $\varepsilon, \varepsilon^{p}$ refer to the total and plastic strain, respectively. $\mathbb{E}_0$ represents the elastic stiffness tensor, $\mathbb{I}$ refers to the identity tensor, and $\mathbb{D}$ denotes the damage tensor, which can be decomposed as:

$$\mathbb{D} = d^{+}\mathbb{P}^{+} + d^{-}\mathbb{P}^{-} \qquad (2)$$

where $\mathbb{P}^{-}$ and $\mathbb{P}^{+}$ represent the projective tensors that describe the anisotropies of compressive and tensile degradation, respectively. d^{-} and d^{+} denote the compressive and tensile damage scalars ϵ [0, 1], respectively, which can be expressed as:

$$d^{\pm} = g^{\pm}(Y^{\pm}) \qquad (3)$$

where $g^{\pm}(\cdot)$ denote the monotonic damage evolution functions, $Y^{\pm}$ refer to the tensile and compressive damage energy release rate by which the energy-equivalent strain that describes the multi-axial strain states can be identified.

2.2 Constitutive relation of steel reinforcement

To reproduce the behavior of steel reinforcement in the current study, the bilinear elastic-plastic model is considered where the total strain ε and plastic strain ε_p can be expressed as:

$$\varepsilon = \begin{cases} \dfrac{\sigma_y}{E} & when \quad \sigma \le \sigma_y \\ \dfrac{\sigma_y}{E} + \dfrac{1}{E_t}(\sigma - \sigma_y) & when \ \sigma > \sigma_y \end{cases} \tag{4}$$

$$\varepsilon_p = \varepsilon - \frac{\sigma_0}{E} \tag{5}$$

where σ_y and ε_y represent the yield strength of steel reinforcement and the associated strain, respectively. σ and ε denote the stress and the associated strain at a specific point, respectively. E refers to the modulus of elasticity, and α denotes the hardening ratio within the range of 0.05 to 0.10. The stress-strain curves for concrete and steel materials are depicted in Fig.1.

3. Random field simulation and reliability assessment

3.1 Random field simulation

Random field simulation is a powerful tool used in many fields, including geostatistics, environmental science, and engineering. It involves generating a set of random values that are spatially correlated based on a given covariance matrix. Covariance matrix decomposition is a popular method for simulating random fields. This method decomposes the covariance matrix into a product of square roots, which allows for the efficient generation of the random values. The Cholesky decomposition is a common technique used for this purpose.

To simulate a random field based on the decomposed covariance matrix, a set of uncorrelated random values, usually generated from a normal distribution, are multiplied by the square roots of the decomposed matrix. The resulting values are then combined to create a simulated random field with spatial correlation. This technique can be used to model the material properties uncertainty. It can also be used for uncertainty analysis and risk assessment in complex systems. Generally, covariance matrix decomposition-based random field simulation is a powerful and flexible tool for simulating spatially correlated random values and has numerous applications in research and industry. The algorithm used in the current study can be summarized as follows:

Algorithm-1: Random Field Generator
1- Define the domain of discretization Ω
2- Establish the Covariance matrix $C_{zz}(\xi, \sigma)$
3- Obtain the autocorrelation function
$\rho_{zz}(\xi) = f(\xi, l_{corr})$
4- Construct the autocorrelation matrix $[C]$
5- Obtain the eigenvalue and the corresponding eigenvector through the Singular Value Decomposition (SVD)
6- Generate a set of Gaussian random variables $(\zeta_i(\theta))$
7- Generate the approximated Random Field
$\hat{H}(x, \theta) = \mu + \sum_{i=1}^{N} U_i\, D_i\, \zeta_i(\theta)$

A simple example of generating a 2D random field for a 1mx1m plate is shown in Fig.2.

3.2 Generalized F-discrepancy-based point selection strategy

(Chen, Yang, and Li 2016) proposed a point set rearrangement approach to effectively produce a representative point set with the reduced GF discrepancy. There are two basic steps to this procedure. In the first stage of this technique, a starting point set is created, and it is then reordered to minimize the GF discrepancy. For more details about this strategy, it is referred to (Chen, Yang, and Li 2016), which can be summarized as follows:

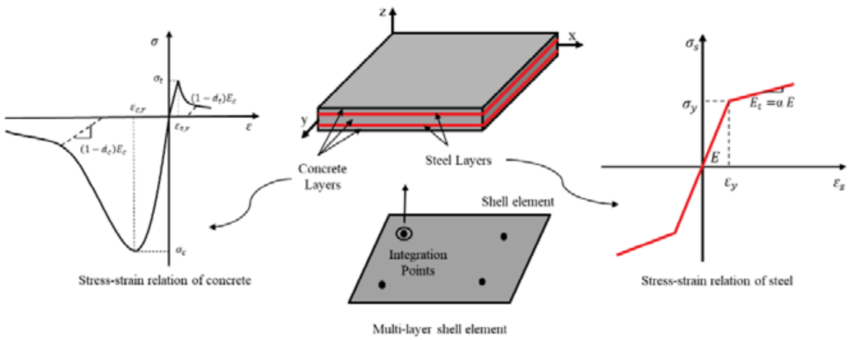


Fig.1. Stress-strain curves and multilayered shell element

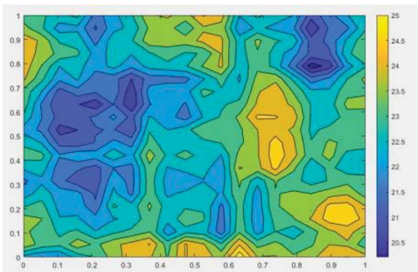


Fig.2. 2D random field of a simple plate

Algorithm-2: point selection strategy

1- *Generating n initial point set adopting Sobol sequence $x_{m,i}$*
2- *Perform the first transformation $x'_{m,i}$ of the n initial points to make the assigned probabilities near to each other.*
3- *Evaluate the assigned probabilities p_k of the transformed n points.*
4- *Establish the second transformation $x''_{m,i}$ to reduce the GF discrepancy.*
5- *Adopt the final transformed points with minimum GF discrepancy as a representative point set.*

3.3 Reliability assessment based on PDEM

In the engineering field, the reliability assessment of structures is a crucial task that ensures the safety and durability of buildings, bridges, and other infrastructure. The PDEM developed by (J. Li and Chen 2008; Chen and Li 2005; J. Li, Chen, and Fan 2007; Chen and Li 2007), is a mathematical framework that has been applied to this task. The PDEM method is a statistical technique that provides a framework for analyzing the evolution of the probability distribution function (PDF) of a system over time. The PDF describes the likelihood of different outcomes for a given variable, and PDEM tracks how this distribution changes as the system operates and experiences various stresses and loads.

The generalized density evolution equation (GDEE) is formulated as follows when one response Y is considered:

$$\frac{\partial p_{Y\Theta}(y,\boldsymbol{\theta},t)}{\partial t} + \dot{Y}(\theta,t)\frac{\partial p_{Y\Theta}(y,\boldsymbol{\theta},t)}{\partial y} = 0 \quad (6)$$

where $y(t)$ is a vector relevant to the response of the structure (e.g., displacement) at a certain point. Θ ($\Theta_1, \Theta_2, \ldots, \Theta_n$) refers to the basic random vector that could influence the structural response. $\partial p_{Y\Theta}(y,\theta,t)$ refers to the joint PDF of $y(t)$ and Θ. For a certain θ of the random vector Θ, $\dot{Y}(\theta,t)$ stands for the velocity variable. The total variation dimensioning (TVD) approach is utilized herein to deduce a solution for the partial differential equation and get the overall joint PDF which can be represented as follows:

$$p_Y(y,t) = \sum_{k=1}^{N} p_{Y\Theta}(y,\theta_k,t) \quad (7)$$

Based on the limit state functions, the reliability of a structure can be assessed. For more details about PDEM, it is referred to (J. Li and Chen 2008; Chen and Li 2005; J. Li, Chen, and Fan 2007; Chen and Li 2007), which can be summarized as indicated in Fig.3.

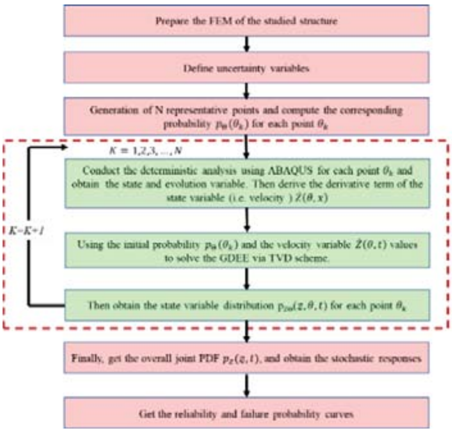


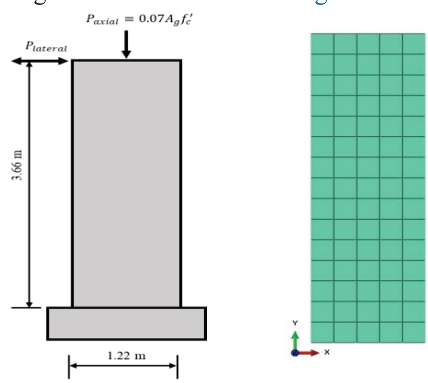
Fig.3. Stochastic analysis and failure probability via PDEM

4. Numerical simulation and stochastic analysis of RC rectangular shear wall

4.1. Finite element modeling

In this section, the simulation of a rectangular shear wall that was tested in literature by (Thomsen and Wallace 2004) is developed, where the specimen named RW2 is adopted herein. In this test, a vertical axial compression force ($0.07 f_c A_s$) was applied at the specimen's top in the first step. Then, a reversible lateral displacement was implemented in the second loading procedure. A schematic representation of the test setup and the finite element (FE) model is presented in Fig.4, and more details about the material properties, reinforcement detailing, and applied loading process it is referred to (Thomsen and Wallace 2004). The FE model is developed using multilayered shell elements in ABAQUS software which is a common approach for modeling and analyzing the behavior of reinforced concrete structures under lateral loads.

ABAQUS offers a range of shell elements suitable for modeling reinforced concrete structures, including solid and thin-shell elements such as S4R and S3R. The 4-nodes doubly curved shell element (S4R) is adopted for element discretization. The multilayered shell element approach involves dividing the wall into several layers, with each layer modeled using different thicknesses, and various material properties can be appointed depending on the specific characteristics of the wall being modeled. The multi-axial concrete damage plasticity model is embedded in the analysis procedure through the VUMAT subroutine to well capture the concrete non-linear behavior and the performance of the considered wall. A comparison between the results of numerical simulation and experimental is depicted in Fig.5, which was found to be in good agreement. The compressive and tensile damage contours are shown in Fig.6.



a) Test setup b) FE model

Fig.4. Schematic representation of the test and FE model

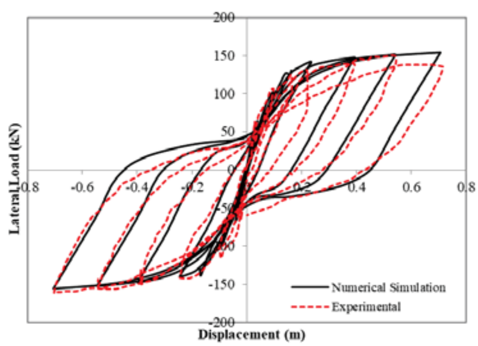
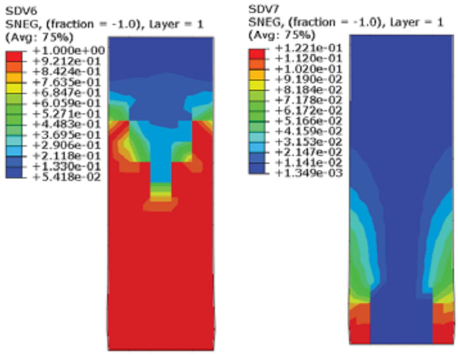


Fig.5. Comparison between experimental and numerical simulation (load-displacement curve)



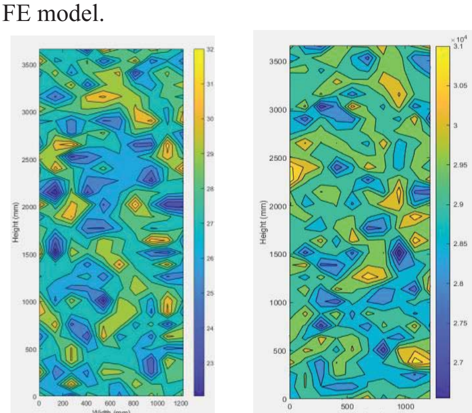
a) Tensile damage b) Compressive damage

Fig.6. Deterministic analysis results

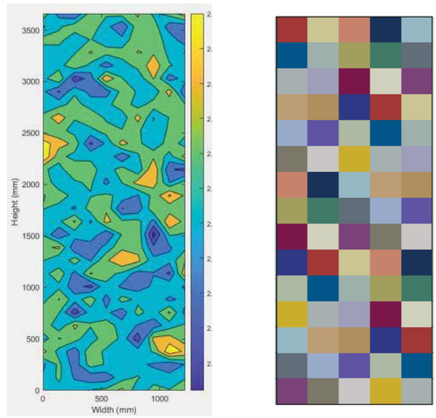
Note: SDV6 and SDV7 refer to the tensile and compressive damage, respectively.

4.2. Generating random field of the material properties

The random field algorithm based on covariance matrix decomposition introduced in section 3.1 is utilized to generate samples with random material properties. The random field of the concrete compressive strength f_c is first generated, and then the random field of the other parameters (i.e., modulus of elasticity, tensile strength, etc.), which is calculated based on the empirical formulae in the Chinese code (*GB50010-2010. Code for Design of Concrete Structures. China Build Ind Press Beijing.* 2015), is generated based on the random field of f_c. Fig.7 shows the generated random field of concrete material properties for a sample and section assignment in the FE model.



a) Compressive strength (f_c) b) Modulus of elasticity (E_c)



c) Tensile Strength (f_t) d) Section assignment

Fig.7. Example for the generated random field of material properties for a sample and section assignment in the FE model

4.3 Stochastic analysis and reliability assessment

The stochastic response of the generated samples in terms of the load-displacement curve is presented in Fig.8, which shows the significant effect of considering the spatial variability of material properties on the response of the studied shear wall. Moreover, the variation of the compressive damage location and value is well captured, as depicted in Fig.9. For the stochastic analysis results; it is observed that in samples C and D, the concrete compressive damage reached the unit value, which refers to the crushing of concrete. It is worth mentioning that these results were observed in the test where the tested shear wall was crushed at the bottom (left and right sides), which could not be revealed through the deterministic analysis. The obtained results of this study highlight the need for careful consideration of material uncertainties in the design and analysis of these critical structural elements.

Moreover, the PDEM is utilized herein to assess the failure probability of the studied shear wall. At the same time, the lateral displacement and time are considered the state and evolution variables, respectively.

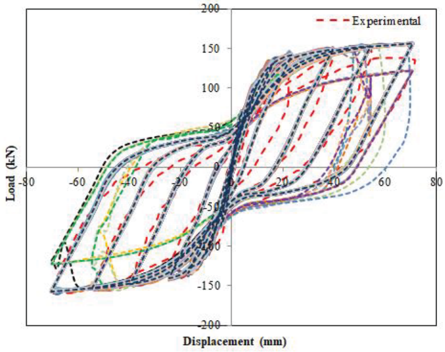


Fig.8. Load-displacement curves for the generated samples

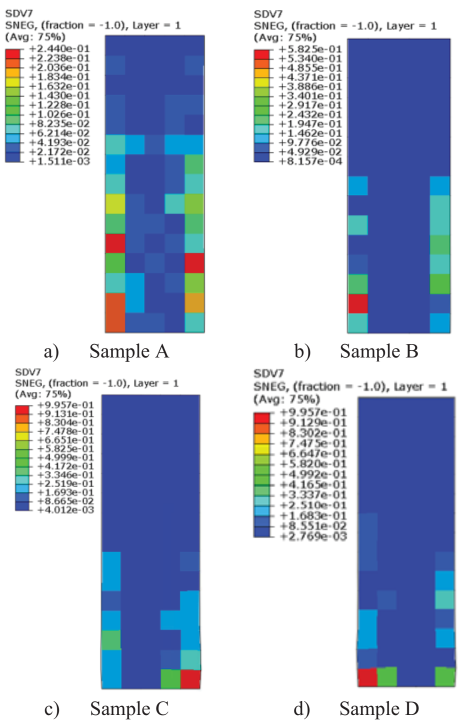


Fig.9. Compressive damage contours for some samples

After performing the deterministic analysis for all the generated samples with material uncertainty, the stochastic analysis is then performed as illustrated in Fig.3. The limit state is considered as the maximum displacement in the experimental test. Furthermore, the stochastic analysis through the PDEM is conducted, and a failure probability of nearly 23% is obtained when considering the material uncertainty, as shown in Fig.10. Where the sudden change in the reliability of the studied wall is observed due to the weak material properties of some samples obtained from the random field, excessive inelastic deformation occurred and exceeded the defined limit state at an early stage of the loading process; therefore, a failure probability is recorded and caused a sudden change in the failure probability curve. Here, it is worth pointing out that, according to the PDEM principles, the failure probability is computed at a certain critical time when the higher response is recorded, not at all the loading time, which caused a sudden change in the curve. Such a result is observed in the literature (Chen and Li 2005). It is worth pointing out that the recorded failure probability is high for such structures.

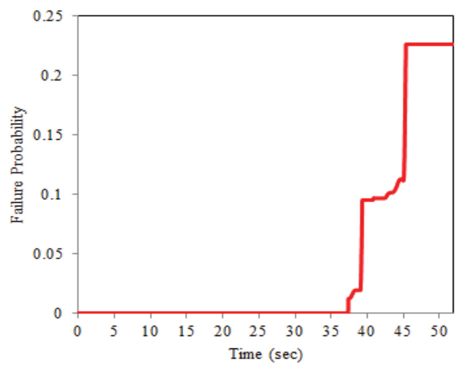


Fig.10. Failure probability curve of the studied shear wall

5. Conclusion

This study introduces a framework to investigate the stochastic response and assess the reliability of RC structures where the random field generator based on covariance matrix decomposition is developed to represent the concrete material spatial variability for an RC rectangular shear wall as an example for a critical member in high-rise buildings. The multi-axial concrete damage plasticity model is incorporated in the analysis procedure using ABAQUS software through the VUMAT subroutine to accurately represent the non-linear behavior of concrete. The stochastic response and inelastic behavior for the generated samples of the studied shear wall are well captured through the proposed framework. The results demonstrated that considering the spatial variability of concrete material properties has a significant effect on the structural response where the variation of the compressive damage location and value is well captured. Also, the maximum load-carrying capacity disparity through the load-displacement curves is recorded. The stochastic analysis through the PDEM revealed a failure probability of nearly 23% when considering the material uncertainty, which could not be captured through the deterministic analysis. This study's results highlight the necessity for considering material uncertainties in designing and analyzing these critical structural elements.

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