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A Novel Random Vibration Accelerated Life Test Model under Non-stationary Non-Gaussian Excitation

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Abstracts: In the field of engineering, accurate reliability assessment and optimization are of paramount importance for ensuring the safety and longevity of structures and components. Traditional Gaussian-based accelerated life test (ALT) models have been widely used but often face limitations in dealing with complex vibration scenarios. To overcome these challenges, this study proposes a novel accelerated life test (ALT) model for random vibrations under non-stationary non-Gaussian excitation. Building upon Gaussian random vibration ALT models and incorporating the previously developed kurtosis transmission model for non-stationary non-Gaussian processes, an acceleration factor for structural fatigue life is derived. The proposed model significantly enhances the acceleration of structural fatigue failure and accurately predicts structural fatigue life. By effectively leveraging the high kurtosis characteristics of non-stationary non-Gaussian excitation, this model addresses the limitations of traditional Gaussian-based methods, offering a novel framework for reliability evaluation and optimization in engineering applications.

Keywords: accelerated life test, random vibration, non-stationary non-Gaussian.

1. Introduction

In modern engineering, products such as aerospace components, automobiles, and electronic devices often operate under complex and random vibration environments. Traditional life testing under normal conditions is time-consuming, making it challenging to meet the demands of rapid product development and iteration. Additionally, these tests often fail to uncover potential issues comprehensively. As product functionalities become increasingly complex, the reliability requirements for these systems have significantly heightened. Against this backdrop, accelerated life testing under random vibration conditions has become a necessity. This approach accelerates the failure process, enabling the rapid acquisition of critical information about product lifespan and reliability. It effectively addresses the urgent need in engineering practices for swift product evaluation and optimization. Therefore, Extensive research has been conducted by scholars in this field.

Allegri et al. (Allegri and Zhang 2008) investigated the inverse power law model for accelerated testing under stationary broadband Gaussian random vibrations to evaluate relative structural damage. Van et al. (Van, Baren, and John 2015) applied the fatigue damage spectrum (FDS) to design and evaluate random vibration accelerated tests. However, their approach considered only the power spectral density (PSD) of random vibration signals, limiting its applicability to stationary Gaussian random excitations. Pothula et al. (Pothula 2009) conducted random vibration accelerated tests on identical structures with varying damping ratios, studying the influence of structural damping on the acceleration factor of random vibrations. Allegri et al. (Angeli, Comelis, and Troncossi 2018) proposed a method for generating accelerated test signals by superimposing sinusoidal vibrations onto random vibrations based on the FDS. They highlighted that this method could serve as an alternative to synthesizing accelerated test signal PSD using fatigue equivalence principles. Steinwolf et al. (Wolfsteiner and Steinwolf 2019) introduced a technique for controlling the fatigue damage spectrum of vibration test signals, analogous to controlling their PSD. This method is applicable to random vibration accelerated testing. Jang et al. (Jiang et al. 2015) accelerates the fatigue failure of the structure by converting the Gaussian excitation signal into a stationary non - Gaussian excitation signal.

The above acceleration models primarily accelerate structural vibration fatigue damage by increasing the magnitude of the excitation signal's PSD or by transforming Gaussian excitation signals into stationary non-Gaussian excitation signals. However, increasing the PSD magnitude may cause the structural failure mode to shift away from fatigue, while the acceleration effect of stationary non-Gaussian excitation signals on vibration fatigue damage is relatively limited. In contrast, non-stationary non-Gaussian random excitation can effectively transmit its high kurtosis to structural responses (Palmieri et al. 2017, Kihm et al. 2013), significantly reducing the vibration fatigue life of structures. Based on this, this study employs non-stationary non-Gaussian random excitation as input to establish a random vibration accelerated life model. The proposed model not only effectively shortens testing durations but also accurately predicts the vibration fatigue life of structures, offering substantial value for engineering applications.

2. Random Vibration Fatigue Model

2.1. *Narrow-Band Model*

Narrowband random loading refers to random loads with frequency components concentrated within a relatively narrow range, resulting in relatively simple time series characteristics. When the random stress follows a narrowband Gaussian distribution, its stress amplitude conforms to a Rayleigh distribution, as shown in Eq. (1).

$$P_{a,NB}(S) = \frac{S}{\sigma^2}\exp(-\frac{S^2}{2\sigma^2}) \tag{1}$$

where S is rain-flow stress amplitude, σ^2 is the variance of the stress response.

For narrowband Gaussian response signals, the vibration fatigue damage can be calculated using Eq. (2).

$$\begin{aligned} D_{NB} &= V_0^+ C^{-1} T\left(\sqrt{2m_0}\right)^b \Gamma(1+\frac{b}{2}) \\ V_0^+ &= \sqrt{\frac{m_2}{m_0}} \\ m_i &= \int_0^\infty f^i G_{yy}(f)df \end{aligned} \tag{2}$$

where V_0^+ is the frequency of the positive slope zero crossing, b is the fatigue exponent, C is the fatigue strength coefficient, m_i is the i-th spectral moment of $G_{yy}(f)$, $G_{yy}(f)$ is the PSD of the response.

2.2. *Broad-Band Model*

Broadband random loading refers to random loads with frequency components distributed over a wider range, such as random loads with flat spectra, bimodal spectra, or multimodal spectra. Compared to narrowband random loading, broadband random loading exhibits significantly more complex time-domain structures. The Dirlik method is one of the most commonly used methods in engineering for broadband random processes (Zorman, Slavič, and Boltežar 2023). The Dirlik model is an approximate model formed by a linear combination of an exponential distribution and two Rayleigh distributions, as shown in Eq. (3).

$$\begin{aligned} P_{a,DK}(Z) &= \frac{\frac{D_1}{Q}\exp(-\frac{Z}{Q}) + \frac{D_2 Z}{R^2}\exp(-\frac{Z^2}{2R^2}) + D_3 Z\exp(-\frac{Z^2}{2})}{\sqrt{m_0}} \\ Z &= \frac{S}{\sqrt{m_0}}, D_1 = \frac{2(\chi_m - \gamma^2)}{1+\gamma^2}, D_2 = \frac{1-\gamma-D_1+D_1^2}{1-R}, D_3 = 1-D_1-D_2, \\ Q &= \frac{1.25(\gamma - D_3 - D_2 R)}{D_1}, R = \frac{\gamma - \chi_m - D_1^2}{1-\gamma-D_1+D_1^2}, \\ \chi_m &= \frac{m_1}{m_0}\sqrt{\frac{m_2}{m_4}}, \gamma = \frac{m_2}{\sqrt{m_0 m_4}} \end{aligned} \tag{3}$$

The fatigue damage of broadband Gaussian response signals can be calculated using Eq. (4).

$$\begin{aligned} D_{DK} &= \frac{V_p T}{C}(\sqrt{m_0})^b \times A \\ A &= \left[D_1 Q^b \Gamma(1+b) + (\sqrt{2})^b \Gamma(1+\frac{b}{2})(D_2 |R|^b + D_3) \right] \\ V_p &= \sqrt{\frac{m_4}{m_2}} \end{aligned} \tag{4}$$

3. Random Vibration Accelerated Life Test Model

3.1. *Gaussian Random Vibration Accelerated Life Test Model*

When the structural response follows a narrowband Gaussian distribution, based on Eq. (2), setting the fatigue damage *D*=1, the narrowband Gaussian vibration life model is:

$$T_{NB} = \frac{C}{V_0^+ \left(\sqrt{2}\sigma\right)^b \Gamma(1+\frac{b}{2})} \tag{5}$$

When the structural response follows a broadband Gaussian distribution, based on Eq. (4), setting the fatigue damage *D*=1, the broadband Gaussian vibration life model is:

$$T_{BB} = \frac{C}{V_p(\sigma)^b A} \tag{6}$$

The two Gaussian vibration life models described above illustrate the relationship between structural vibration fatigue life, structural response parameters, and structural characteristics. In accelerated life testing, the frequency distribution and spectral shape of the input signal are typically not changed; only the overall magnitude of the input signal's power spectral density (PSD) is adjusted. Suppose input signals 1 and 2 have the same spectral shape but different PSD magnitudes. Therefore, for input signals 1 and 2, once the PSD spectral shape is determined, the values of V_0^+ and V_P are already fixed. Similarly, once the structural characteristics are defined, the values of *C*, $\Gamma(1+\frac{b}{2})$ and *A* will not change with variations in the PSD magnitude of the input signal. Hence, the acceleration factor k_{Gaussian} for narrowband and broadband Gaussian random vibration can both be expressed as:

$$k_{Gaussian} = \frac{T1}{T2} = (\frac{\sigma_2}{\sigma_1})^b \tag{7}$$

For linear systems, the root mean square (RMS) response of the system is proportional to the RMS value of the input signal. We define an overall amplification factor K_{psd} for the power spectral magnitude of the input signal, as shown Eq. (8).

$$k_{psd} = \frac{A_1}{A_2} \tag{8}$$

In Eq. (8), A_1 is the power spectral magnitude of input signal 1, and A_2 is the power spectral magnitude of input signal 2. Based on linear system theory, Eq. (7) can be expressed as:

$$k_{Gaussian} = \frac{T1}{T2} = (\frac{\sigma_2}{\sigma_1})^b = (\frac{A_2}{A_1})^{\frac{b}{2}} \tag{9}$$

Eq. (9) establishes the relationship between the power spectral magnitude of the input signal and the structural life.

3.2. *Non-stationary Non-Gaussian Random Vibration Accelerated Life Test Model*

The Gaussian random vibration life model does not account for the effect of response kurtosis on structural vibration fatigue life, assuming that the response follows a Gaussian distribution. However, related literature indicates that response kurtosis significantly impacts structural vibration fatigue life (Kihm et al. 2013, Lei et al. 2022a, Jiang et al. 2022, Capponi et al. 2017, Palmieri et al. 2017). Therefore, when developing a non-stationary non-Gaussian random vibration life model, it is essential to consider the influence of response kurtosis on vibration fatigue life. Based on the author's previous work on kurtosis transmission (Lei et al. 2022b), the response kurtosis can be quantitatively predicted by the kurtosis of the frequency - domain decomposed signals of the excitation at the structural natural. The predicted response kurtosis is given by Eq. (10).

$$Kur_{response} = \frac{E[(d(t)-u_d)^4]}{\sigma_d^{\ 4}} = \frac{\frac{1}{N}\sum_{t=1}^{N}(d(t)-u_d)^4}{(\frac{1}{N}\sum_{t=1}^{N}(d(t)-u_d)^2)^2} \tag{10}$$

where *d(t)* is the frequency-domain decomposed signal of the non-stationary non-Gaussian excitation at the structural natural frequency. u_d represents the mean of the decomposed signal, and σ_d is the variance of the decomposed signal.

Considering that non-Gaussian responses significantly accelerate structural random vibration fatigue, the acceleration factor for non-stationary non-Gaussian random vibration life is defined based on the Gaussian random vibration acceleration factor, as shown in Eq. (11):

$$k_{NSNG} = \frac{T1}{T2} = k_{Gaussian} \left(\frac{Kur_{response2}}{Kur_{response1}}\right)^{\frac{b}{4}} = \left(\frac{A_2}{A_1}\right)^{\frac{b}{2}} \left(\frac{Kur_{response2}}{Kur_{response1}}\right)^{\frac{b}{4}} \quad (11)$$

In Eq. (11), once the excitation signal is determined, A_1, A_2, $Kur_{\mathrm{response1}}$ and $Kur_{\mathrm{response2}}$ can be determined; the only unknown parameter is b, which needs to be determined through preliminary testing before conducting the non-stationary non-Gaussian random vibration accelerated life test. For instance, b can be determined by applying Gaussian excitation signals with different power spectral magnitudes and comparing the structural vibration fatigue failure times.

4. Experimental Verification

The experimental system is shown in Figure 1. The specimen is a cantilever beam with a notch, and the material of the specimen is AL-6061. Accelerometers are used to record both the excitation signal from the shaker and the response signal from the specimen. The preliminary test profile and the vibration fatigue life of the specimen are shown in Table 1.

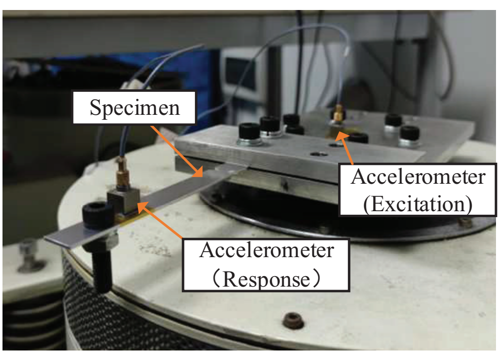

Fig. 1. Experimental system

Table 1. Preliminary test profile and specimen vibration fatigue life

Test Profile	Frequency Range	PSD Magnitude/(g^2/Hz)	Vibration Fatigue Life /(min)
1	10-110	0.01	378
2	10-110	0.02	86

By substituting the results from Table 1 into Eq. (9), the parameter b is obtained as 4.27. Subsequently, a non-stationary non-Gaussian random vibration accelerated life test is conducted, with the test profile shown in Table 2. The test results are shown in Table 3. The predicted vibration fatigue life for the test profiles in Table 2 can be calculated separately based on Test Profile 1 and Test Profile 2. The final predicted result is obtained by averaging these two predicted values. The results of the non-stationary non-Gaussian random vibration accelerated life test are shown in Table 3. The time histories of the excitation signal and the response signal are presented in Figure 2.

Table 2. Non-stationary Non-Gaussian Random Vibration Accelerated Life Test Profile

Test Profile	Frequency Range	Kurtosis of Excitation	PSD Magnitude/ (g^2/Hz)	Kur_{response}
3	10-110	6	0.01	5.49
4	10-110	8	0.01	7.52
5	10-110	6	0.02	4.27
6	10-110	8	0.02	7.70

Table 3. Non-stationary Non-Gaussian Random Vibration Accelerated Life Test Results

Test Profile	RMS of the response/ g	Predicted Life (min)	Vibration Fatigue Life /(min)	Relative Error/ %
3	2.93	198	237	16.46
4	3.02	142	121	17.36
5	4.24	39	48	18.75
6	4.11	31	24	29.17

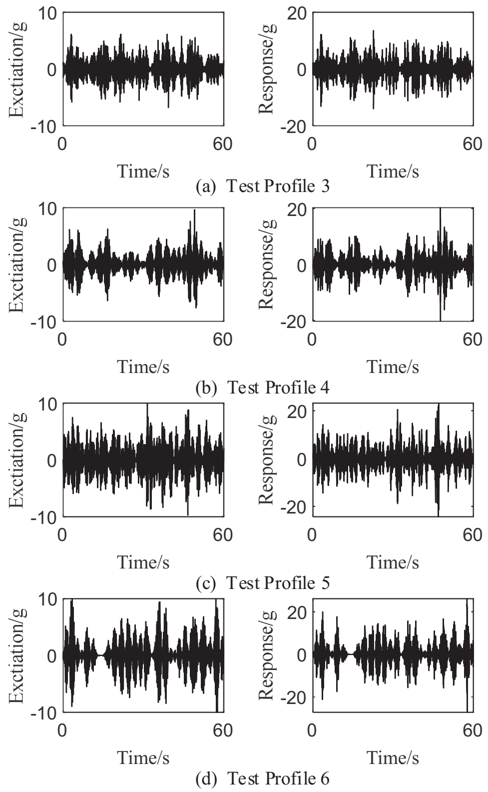

Fig. 1. Non-stationary non-Gaussian excitation signal and specimen response

As can be seen from Table 3, (1) The root - mean - square value of the response signal is proportional to the *b/2* power of the power spectral level, which verifies the correctness of Eq. (9) in this paper. (2) The non - stationary non - Gaussian random accelerated life model established in this paper can effectively accelerate the failure of the structure. When the power spectral level is 0.01 g^2/Hz, the structural fatigue life decreases from 378 minutes to 121 minutes, and the acceleration factor is 3.12. When the power spectral level is 0.02 g^2/Hz, the maximum acceleration factor is 3.58. (3) The non - stationary non - Gaussian random accelerated life model established in this paper can effectively predict the vibration fatigue life of the specimen under the accelerated excitation signal, and the prediction error of the fatigue life is within 30%, which meets the requirements of engineering practice.

6. Conclusion

This study presents a novel accelerated life test (ALT) model for random vibrations under non-stationary non-Gaussian excitation, addressing key limitations of traditional Gaussian-based ALT models in engineering applications. The main findings and contributions are summarized as follows:

(1) Validation of Non-Gaussian Effects on Fatigue Life

By incorporating the kurtosis transmission characteristics of non-stationary non-Gaussian processes, the study confirms that response kurtosis plays a significant role in structural vibration fatigue. The proposed model demonstrates that higher kurtosis in excitation signals effectively accelerates fatigue failure, extending the applicability of ALT models to more complex and realistic vibration environments.

(2) Accurate Prediction and Acceleration of Fatigue Life

The developed model accurately predicts structural vibration fatigue life under non-stationary non-Gaussian excitation, with prediction errors within 30%, meeting engineering accuracy requirements. Moreover, under the condition of the same input signal magnitude, the model significantly shortens the fatigue life compared to Gaussian excitation, achieving acceleration factors of up to 3.44. This highlights its potential to substantially reduce test durations while maintaining reliability and precision in fatigue life assessment.

(3) Novel Contributions to Reliability Evaluation

By bridging the gap between Gaussian and non-Gaussian excitation in ALT modeling, this research offers a new method for random vibration reliability evaluation. The insights gained from this study advance the understanding of vibration-induced fatigue and provide a practical tool for optimizing structural reliability and safety in modern engineering systems.

This work establishes a foundation for further exploration of non-stationary and non-Gaussian excitation effects in fatigue analysis. Future research may focus on extending the model to multi-degree-of-freedom systems and validating its performance across a broader range of material properties and excitation conditions.

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