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Importance Measures from Complex Reliability Simulations

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We consider the problem of estimating the importance of components and parameters in the dynamic simulations of complex systems. We aim at a conceptualization that is capable of retaining the meaning of traditional (static) importance measures in a dynamic concept. We also approach the problem by defining the importance measures in such a way that they draw from the rich output of a dynamic simulator exploiting information that is typically hidden when focusing solely on the probabilistic-level data (e.g., component failure probabilities, system-level failure probability). By incorporating detailed observable information such as failure times and component operational characteristics, additional dimensions of decision making are made available to system designers and operators that allow a focus on the margin to failure. The goal of our work is to create a scalable and flexible approach to enrich the insights from a time- and physics-informed simulation with explanations as to what are the drivers of the system behavior at a fundamental level of system behavior.

Keywords: Importance Measures, Reliability Analysis, Complex Systems Simulation.

1. Introduction

Importance measures play a relevant role in aiding reliability engineers in understanding the elements (component, parameters) that matter the most for the system or application under scrutiny. Knowing which component is important helps analysts prioritize maintenance activities or quality assurance programs.

Over the years, a variety of reliability importance measures have been developed. Some of them are designed for specific applications, some of them are of a more general nature. In the first class, we find indicators such as the ones proposed in Dui et al. (2017) for preventive maintenance, or Li et al. (2017) for AC power systems, in the second we find indices such as the Birnbaum importance measure, the Fussell-Vesely importance, the Risk Achievement worth, the Risk Reduction Worth and several others. In this work, we consider a general setting, in which a reliability metric of interest is computed by a complex dynamic simulator that captures both the logical configuration and physical characteristics of the system. The logical configuration aims to model how the components are connected. At the basis of the logical configuration is a structure function from which one determines the minimal path and cut sets. The physics portion of the system increases the realism of the analysis, making the simulation more powerful.

The failure and physics modeling aspect of our work is addressed through the use of the open source EMRALD simulation tool. This tool is under development at the Idaho National Laboratory (INL), and released through GitHub, with the goal of representing complex systems using risk and reliability analysis approaches INL (2024). EMRALD focuses on a couple of important characteristics, including: (a) providing a simplifying modeling approach using states and a modern browser-based editor; (b) allowing the user to couple other software tools including physics simulations; (c) calculating detailed time-based sequence of system and component state-space evolution, and (d) allowing traditional aspects of reliability analysis such as fault trees. The analysis of EMRALD models takes place through a separate module that performs the probabilistic simulation. EMRALD adopts standard approaches such as the use of JSON for storing all modeling information.

2. Importance Measures

We review the classical importance measure setup, as originating from the works of Birnbaum (1969) and Barlow and Proschan (1975). One considers a binary system with n components, each of which can be in two states, working (0) or failed (1), with indicator variable $\varphi_i = \{0, 1\}$. We denote by Ω the collection of the 2^n configurations and collect the indicator variables in vector $\varphi = [\varphi_1, \varphi_2, ..., \varphi_n]$. The final state of the system is the top event and $\Psi = \{0, 1\}$. The relationship that links the top event to the components states is called structure function, which one denotes via $\Psi = \Psi(\varphi)$, with $\Psi : \Omega \mapsto [0, 1]$. A minimal cut set is a system state

To define the the system reliability, we need two additional notions: a mission time T and the probabilities of component failures within T. We let $\mathbb{P}$ be the probability measure on $(\Omega, \mathcal{B}(\Omega))$ that reflects the engineer/analyst/decision-maker's degree of belief about the likelihood of the component failures. We then let p_i be the probability that φ_i switches from 0 to 1 within the time horizon T, that is $p_i = \Pr(\varphi_i = 1, T)$. For simplicity, because we consider a fixed time horizon, we drop the reference to T in the reminder. The reliability of the system is then defined as

$$R = \Pr(\Psi = 0), \tag{1}$$

and correspondingly the system failure probability is

$$F = 1 - R = \Pr(\Psi = 1). \tag{2}$$

We consider a coherent system for simplicity. A central notion in the definition of reliability importance measures is played by criticality. A component is critical if it is in such a state that its failure causes the system to fail. Let φ_i the indicator variable of component i and $\varphi_{\sim i}$ the vector collecting the indicato variables of all other components. Using the notation in Birnbaum, we define the criticality indicator variable at $\varphi_{\sim i}$ as

$$\delta_i(\varphi_{\sim i}) = \Psi(1_i, \varphi_{\sim i}) - \Psi(0_i, \varphi_{\sim i}) \tag{3}$$

Component i then critical in state $\varphi_{\sim i}$ if $\delta_i(\varphi_{\sim i}) = 1$, that is, if the component switches from working to failed when the other components are in state $\varphi_{\sim i}$ the system also switches from working to failed.

We then define the Birnbaum importance of component i as the probability that component i is critical for system failure:

$$B_i = \Pr(\delta_i(\varphi_{\sim i}) = 1). \tag{4}$$

Under the hypothesis of independent component failure probabilities, it turns out that the Birnbaum importance is the partial derivative of the system reliability function

Other well known importance measures are the Risk Achievement Worth and the Fussell-Vesely importance

The Risk Achievement Worth (RAW) suggests the potential increase in risk level in case component i failed (as stated in Vesely et al. (1983)). We can write the RAW of a component on a ratio scale as:

$$RAW_i = \frac{\Pr[\Psi(\varphi_i = 1, \varphi_{\sim i}) = 1]}{\Pr(\Psi = 1)}, \tag{5}$$

or alternatively, as in Vesely et al. (1983), on an interval scale:

$$RAW_i = \Pr[\Psi(\varphi_i = 1, \varphi_{\sim i}) = 1] - \Pr[\Psi = 1]. \tag{6}$$

The Risk Reduction Worth Vesely et al. (1990) quantifies the change in risk when component i is assumed to be perfectly reliable. Hence, we have:

$$RRW_i = \frac{\Pr(\Psi = 1)}{\Pr[\Psi(\varphi_i = 0, \varphi_{\sim i}) = 1]}, \tag{7}$$

or alternatively, as in Vesely et al. (1983), on an interval scale:

$$RRW_i = \Pr(\Psi = 1) - \Pr[\Psi(\varphi_i = 0, \varphi_{\sim i}) = 1]. \tag{8}$$

The Fussell-Vesely (FV) importance measure Fussell (1975) is defined as the conditional probability that a specific basic event has occurred given that the system has failed. The FV importance measure of component i can then be written as

$$FV_i = \Pr(\varphi_i = 1 | \Psi = 1) = \frac{\Pr(\varphi_i = 1 \wedge \Psi = 1)}{\Pr[\Psi = 1]}. \tag{9}$$

Finally, we have the Barlow-Proshan importance

$$BP_i = \int_0^{\infty} BI_i(t) f_i(t) dt \qquad (10)$$

which is the probability that failure of component i coincides with the system failure.

3. The Problem

The problem we address in our work is to calculate these importance measures from a complex simultation. In particular, we aim at extracting the importance measures from a computer program that mixes not only the logical behavior of the system, but also relevant simulation parts related to the system thermodynamics and more directly from the simulation data, without performing additional calculations.

The challenge we face is that under consideration is a system with an intrinsecally stochastic response. Also, when components are highly reliable, failures are rare and we need to simulate thousands of scenarios to obtain a significant number of top even realizations. This aspect increases computational burden. The cost would increase even more if the importance measures were calculated via sensitivities. For instance, suppose we fix the number of stochastic replicates (scenarios) to N. Then, computing RAW_i would imply running the simulation $C = nN$ times, exacerbating the computational difficulties.

We tackle the estimation problem of calculating the importance measures at a cost $C = N$, that is post-processing the output of the stochastic simulator, without actually performing any additional algorithmic operation.

To do so, we need to define estimators for the importance measures we presented before. Some observations. Let S^T be the subset of scenarios in which the system has failed and denote with s its cardinality. Then, $\frac{s}{N}$ is an estimate of the system failure probability. By the law of large numbers, as N increases, this estimate tends to the probability of system failure. Let n_i be the number of scenarios in which component i fails. Then, $\frac{n_i}{N}$ is an estimate of the component failure probability. Let us now study estimators for the importance measures.

For the Birbaum importance we need to proceed as follows. Consider first estimating $\Pr(\Psi(1_i, \varphi_{-i}))$. This is the conditional system failure probability given that component i has failed. Consider the subset of scenarios in which component i has failed. Let n_i be the number of these scenarios. Then, let f_i the number of times in which the system fails in this set. Then, the ratio $\frac{f_i}{n_i}$ is an estimate for the conditional system failure probability $\Pr(\Psi(1_i, \varphi_{-i}))$. Formally,

$$\frac{f_i(N)}{n_i(N)} \underset{N\to\infty}{\to} \Pr(\Psi(1_i, \varphi_{-i})), \qquad (11)$$

where we have evidenced that the ratio $\frac{f_i(N)}{n_i(N)}$ depends on N. An estimate of $\Pr(\Psi(0_i, \varphi_{-i}))$ is found in a similar way. Consider the subset of scenarios in which component i has not failed, whose cardinality is $N - n_i$. Then let $\overline{f}_i$ the number of scenarios in which the system fails, given that component i has not failed. Then, the ratio $\frac{\overline{f}_i}{N - n_i}$ is an estimate of $\Pr(\Psi(0_i, \varphi_{-i}))$. Then, a difference between these two numbers is an estimate of B_i, which we denote by $\widehat{B}_i$.

An estimate of RAW_i is given by $\frac{f_i}{n_i}$ and an estimate of RRW_i is given by $\frac{N - n_i}{\overline{f}_i}$.

An estimate of FV_i is found as follows. Consider all scenarios in which component i fails and also the system fails and denote by fv_i their number. Then, an estimate of FV$_i$ is given by

$$\widehat{FV}_i = \frac{fv_i}{n_i}. \qquad (12)$$

Finally, a stochastic simulation approach also allows us to easily estimate the Barlow-Proschan importance of component i. The count consists in taking all scenarios in which the system fails simultaneously to component i (let this number be bp_i) and normalize by n_i. We then define the estimate as

$$\widehat{BP}_i = \frac{bp_i}{n_i}. \qquad (13)$$

This number gives us the probability that component i causes the system to fail.

4. Case Study

As an illustrative example, we use a four component system designed to work as two redundant trains with a valve and pump in each. We report the system logic structure in Figure 1. Given the logical connections in Figure 1, if we let $\phi_1, \phi_2, \phi_3, \phi_4$ denote the Boolean variables of pumps 1 and 2, and valves 1 and 2, respectively, we obtain the structure function:

$$\Psi = (1-(1-\phi_1)(1-\phi_3))(1-(1-\phi_2)(1-\phi_4)) \tag{14}$$

Simplifying, we obtain:

$$\begin{aligned}\Psi =& \phi_1\phi_2 + \phi_1\phi_3 + \phi_2\phi_3 + \phi_3\phi_4 \\ & - \phi_1\phi_2\phi_3 - \phi_1\phi_2\phi_4 - \phi_1\phi_3\phi_4 \\ & - \phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4\end{aligned} \tag{15}$$

Assuming independent failures, we get the system failure probability:

$$\begin{aligned}F = & P_1P_2 + P_1V_2 + P_2V_1 + V_1V_2 \\ & - P_1P_2V_1 - P_1P_2V_2 - P_1V_1V_2 - P_2V_1V_2 \\ & + P_1P_2V_1V_2\end{aligned} \tag{16}$$

where $P_1 = \Pr(\phi_1 = 1)$, $P_2 = \Pr(\phi_2 = 1)$ denote the pump failure probabilities and $V_1 = \Pr(\phi_3 = 1)$, $V_2 = \Pr(\phi_4 = 1)$ the valve failure probabilities.

The importance measures can then be obtained analytically. By taking the derivative of F with respect to the failure probability of the first pump, we find the Birnbaum importance:

$$\begin{aligned}B[\text{Pump}_1] =& P_2 + V_2 - P_2V_1 - P_2V_2 \\ & - V_1V_2 + P_2V_1V_2.\end{aligned} \tag{17}$$

We also find the following expressions for the RAW and Fussell-Vesely:

$$RAW[\text{Pump}_1] = \frac{P_2 + V_2 - P_2V_2}{F}, \tag{18}$$

and

$$FV[\text{Pump}_1] = \frac{P_1V_2 + P_1P_2 - P_1V_2P_2}{F}, \tag{19}$$

Assuming exponential failure probabilities,

$$\begin{aligned}&P_1(t) = P_2(t) = 1 - e^{-\lambda_p t} \quad \text{and} \\ &V_1(t) = V_2(t) = 1 - e^{-\lambda_v t},\end{aligned} \tag{20}$$

we find the Barlow-Proschan importance of the first pump from

$$BP[\text{Pump}_1] = \int_0^\infty B[\text{Pump}_1](t) \cdot \lambda_p e^{-\lambda_p t}\, \mathrm{d}\, t. \tag{21}$$

Assigning the failure rates $\lambda_p = 0.003$[1/h] and $\lambda_v = 0.002$[1/h] for the pumps and valves, respectively, at $t = 24$, the component failure probabilities are $P_1 = P_2 = 0.069$ and $V_1 = V_2 = 0.047$. The system unreliability is $F = 0.0128$. Substituting these values is Equations 17-21, we find the numerical values of the importance measures in Table 1.

Table 1. Analytical values of the importance measures.

Component	B	RAW	FV	BP
P1	0.116	8.84	0.61	0.3
P2	0.116	8.84	0.61	0.3
V1	0.116	8.84	0.41	0.2
V2	0.116	8.84	0.41	0.2

The Fussell-Vesely for the pumps is approximately 33% larger than that of the valves. The reason for this is twofold: (1) the system is “symmetric” meaning the pumps and valves appear in the cut sets in the same manner and (2) the failure rate of the pumps is 33% larger than the valves. Since the failure models of these components are assumed to be exponential, the mean-time-to-failure (MTTF) are 333 hours and 500 hours for the pump and the valve, respectively.

5. Numerical Experiments: Nominal time-based simulation

The EMRALD diagrams for the valve and pump include a single transition state given by a single failure rate for each component (valve failure rate = 0.002/hr, pump failure rate = 0.003/hr) and are shown in Figure 2. The corresponding fault tree for this example is shown in Figure 1. From the EMRALD dynamic simulation model (`https://github.com/idaholab/EMRALD`), we can run this example to determine overall system failure insights. For example, running 50,000 samples (with a compu-

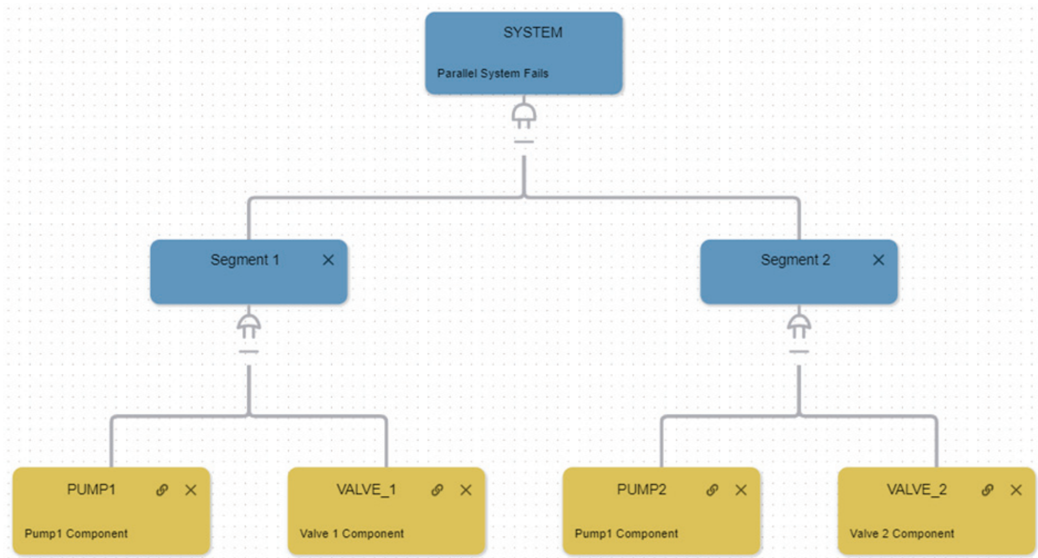

Fig. 1. The logic structure representing the parallel system under investigation.

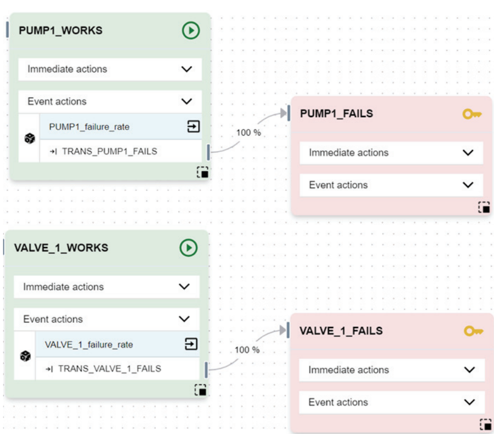

Fig. 2. Valve and pump EMRALD simulation diagram.

tation time of less than six seconds) for this example provides the following detailed information related to the components and system state (Table 2): While at first glance this type of information may seem daunting, we can quickly appreciate its worth. Further, behind this information is even a more detailed layer of information that includes the time of failures for specific components and for the overall system. If we focus on one component, say Valve 1, we see detailed information such as those listed in Table 2.

In addition to the component failure information, the system state is computed using the fault tree logic. Essentially, the logic becomes an internal state-diagram for the simulation that is solved (very rapidly) during each simulation iteration. The overall, or baseline, results for all of the components and the system are shown in Table 3. Comparing the values in Table 3 to the failure rates assigned at the beginning of the simulation by the analyst provides an indication about how close the simulation is to the reproducing the failure-related features of the system. In our case, Table 3 shows very close values.

In addition to these simulation, we performed a direct calculation of the components RAW. To this end, we have increased the failure rate to approximate a failure probability equal to 1 of the components, and recomputed the simulation. This amounts to calculate $4 \times N$ simulations (where N=50000 and 500,000) in our case. Table 4 reports the results. The calculated values are very close to the analytical values in Table 1. We could also have proceeded with a direct calculation of the Birnbaum importance measure, setting the components first to always failed and then to always working and taking the difference of the results.

Table 2. Baseline Results for 50000 simulations

Item	Results
Valve 1 failed	2330 times out of the 50000 iterations
P(Valve 1 fails)	= 2330/50000 = 0.047
Mean failure time	11 hours, 52 minutes
Valve 1 fails by itself	1895 times
P(Valve 1 fails — no other failures)	= 0.038
Valve 1 failed with another component	435 times
P(Valve 1 fails — one other component failure)	= 0.0087

This would have mean additional model simulations. This calculation strategy can become extremely expensive if either the number of components is large or the simulation takes a long time, as it is likely for real systems. Instead, we then consider a direct estimation of all the importance measures directly from the simulation results. To do so, we consider the output of the simulation (Table 5). The data offer full details about the story of the system and its components iteration by iteration. To illustrate, the last row in Table 5 shows that the system has failed in iteration 95 with Valve 2 failed after 2.76 hours and pump one failing after 5.43 hours. Thus, in this case Pump 1 caused the system to fail. We can then process these data with the estimators proposed in Section 3, obtaining the values in Table 6. For instance, to calculate the Barlow-Proshan importance measure, we need to count all instances in which the time of system failure was equal to the time of a given component failure and normalize by the number of times in which the system failed. The comparison shows an error at most at the second decimal place for all importance measures. At N=50000, the minimum error is of 0.004% for the RAW of Pump 1, the maximum of 10% for the Barlow-Proschan importance of Valve 1. Increasing the iterations to $N = 500,000$ further improves estimation accuracy, with a maximum error now decreasing at 3% for the Birbaum importance.

6. Conclusions

We have discussed the calculation of classical reliability importance measures for complex reliability simulations. We have proposed new estimators for the Birnbaum, Risk Achievement Worth, Fussell-Vesely and Barlow-Proschan importance measures. These estimators eliminate the need of performing an actual sensitivity, that is to run the model with components set specifically to failed or to working, notably reducing computational burden.

Numerical experiments on a simple model demonstrate that the estimators yield results that closely align with the analytical values, providing a proof of concept of the design.

The research associated with this work is in progress. Next steps following this preliminary investigation are the application to a more sophisticated model, which includes also the simulation of the system physics. Extensions are also to noncoherent systems and to simulations that include repairs and common cause failure modes.

Acknowledgement

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Table 3. Summary results for 50000 simulations

Item	Results
Pump1 and 2 failure rate	0.003/hr
Valve 1 and 2 failure rate	0.002/hr
P(Pump 1 fails)	= 0.0690
P(Pump 2 fails)	= 0.0695
P(Valve 1 fails)	= 0.0468
P(Valve 2 fails)	= 0.0466
P(System fails)	= 0.0127

Table 4. Sensitivity Results 50,000 and 500,000 iterations

	N=50,000	N=500,000
RAW(Pump 1)	0.113/0.0127=8.9	0.113/0.0127=8.9
RAW(Pump 2)	0.111/0.0127=8.7	0.113/0.0127=8.9
RAW(Valve 1)	0.114/0.0127=9.0	0.113/0.0127=8.9
RAW(Valve 2)	0.113/0.0127=8.9	0.113/0.0127=8.9

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Table 5. Output Data. Excerpt out of 50,000 scenarios

Iteration	Pump 1	Pump 2	Valve 1	Valve 2	System
1	0.00	0.00	0.00	0.00	0.00
4	19.45	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	19.89	0.00
94	0.00	0.00	0.00	0.00	0.00
95	5.43	0.00	0.00	2.76	5.43

Table 6. Estimates with 50,000 (first four columns) and 500,000 (last four columns) iterations

	B	RAW	FV	BP	B	RAW	FV	BP
P1	0.115	8.883	0.618	0.292	0.105	8.747	0.609	0.305322
P2	0.119	9.178	0.634	0.328	0.105	8.780	0.608	0.293249
V1	0.111	8.565	0.403	0.178	0.105	8.925	0.418	0.205242
V2	0.113	8.680	0.403	0.202	0.106	9.011	0.421	0.196187