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Prognostic and Energy Management for Multi-Stack Fuel Cell Systems with Stochastic Non-Homogeneous Degradation

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This paper presents a load-dependent degradation model for Proton Exchange Membrane Fuel Cells using a non-homogeneous stochastic gamma process to predict degradation under varying load conditions. The model effectively captures uncertainties, variability, and non-linearities in the degradation process. Building on this foundation, a novel energy management strategy is developed, specifically designed for multi-stack fuel cell systems, combining degradation prognosis with health-conscious energy management for piecewise-static power applications. The methodology is demonstrated on a two-stack PEMFC system, achieving significant improvements in system lifespan and reliability compared to traditional energy management approaches. These results underscore the proposed model's potential to enhance the durability and operational efficiency of multi-stack systems.

Keywords: Proton Exchange Membrane Fuel Cells, Stochastic Degradation Modeling, Prognostics and Health Management, Energy Management, Multi-stack Systems.

1. Introduction

The intensification of the greenhouse effect has underscored the critical need for sustainable energy sources. Proton Exchange Membrane Fuel Cells (PEMFCs) present a promising carbon-free solution by converting chemical energy directly into electrical energy. Multi-stack PEMFC systems enhance power capability and provide redundancy, making them well-suited for high-demand applications. However, their commercialization faces significant challenges, particularly regarding durability, reliability, and operational costs. For commercial viability, automotive PEMFCs are expected to achieve a lifetime of approximately 5,000 hours under typical operating conditions. In practice, however, their lifetimes often range between 2,500 and 3,000 hours Arrigoni et al. (2022).

To address the degradation issues in PEMFCs, Prognostics and Health Management (PHM) offers a pathway to enhance the durability of multi-stack systems by optimizing operational parameters and maintenance strategies. However, accurately modeling PEMFC degradation and integrating these characteristics into decision-making processes—particularly energy management strategies (EMS)—remain significant challenges. Effective EMS implementation necessitates a load-dependent degradation model. Unfortunately, existing studies have rarely established explicit links between degradation models and power demand profiles, thereby limiting their practical applicability.

Current EMS approaches primarily focus on hybrid systems that integrate fuel cells and batteries, often optimizing fuel consumption and durability. For example, aging-aware EMS approaches have been proposed for hybrid vehicles De Pascali et al. (2020), and real-time optimization methods have been developed for hybrid fuel cell vehicles using speed predictions Zhang et al. (2021). More recently, a deterioration-aware EMS was developed for pure multi-stack PEMFC systems under random load profiles, leveraging a Gamma process stochastic degradation model Zuo et al. (2024). However, this approach typically assumes a homogeneous degradation process, which does not accurately reflect the varying degradation rates observed in PEMFC systems as they age.

This work addresses these limitations by developing a degradation model based on a non-homogeneous Gamma process, effectively capturing both age- and load-dependent characteristics of PEMFC degradation. Building on this enhanced prognostic model, a novel EMS formulation is proposed to improve the durability of multi-stack PEMFC systems.

2. Fuel Cell Degradation Modeling

2.1. *Fuel Cell Health Index*

Parameters such as output voltage, output power, and internal resistance are commonly used to evaluate the degradation state of Proton Exchange Membrane Fuel Cells (PEMFCs). Furthermore, the electrochemical surface area (ECSA) of platinum has been identified as a key indicator of catalyst performance.

In this work, the internal resistance R is proposed as the state-of-health (SOH) indicator, as it captures critical degradation characteristics and remains unaffected by load variations, unlike voltage and power output. The value of R can be estimated from the measured polarization curve of the fuel cell using curve-fitting algorithms. A non-linear least squares method is applied to fit the polarization equation described in Dicks and Rand (2018):

$$V_{\text{fc}} = E - V_{\text{act}} - V_{\text{ohm}} - V_{\text{conc}} \tag{1}$$

where V_{fc} is the fuel cell voltage, and E represents the open-circuit voltage. The components of the voltage losses are defined as follows:

$$V_{\text{act}} = A \ln\left(\frac{i}{i_0}\right) \tag{2}$$

$$V_{\text{ohm}} = i \cdot R \tag{3}$$

$$V_{\text{conc}} = m e^{n \cdot i} \tag{4}$$

Here, V_{act} represents the activation loss, V_{ohm} denotes the ohmic loss, and V_{conc} is the concentration loss. The parameter A is a constant determined by the kinetics of the electrochemical reaction, i_0 is the exchange current density, m and n are empirical coefficients, and R is the internal resistance.

2.2. *Empirical Modeling of Load-Induced Degradation*

The degradation of PEMFCs is influenced by load-dependent factors, with both load amplitude and load variation playing critical roles in the deterioration process Pei et al. (2008). Additionally, degradation typically progresses more rapidly during the early stages of the cell's life due to the accelerated deterioration of the catalyst layer Ao et al. (2020). Under the assumption of no start-stop effects, the total resistance increment of the fuel cell is expressed as Zuo et al. (2022):

$$\Delta R = \Delta R_L + K \Delta L \tag{5}$$

where ΔR represents the overall resistance increment, ΔR_L accounts for the load-dependent contribution, K is a proportional constant, and ΔL denotes the magnitude of the load variation.

ΔR_L is modeled using a non-homogeneous Gamma process with a power-law shape function $A(t) = \alpha_L t^\beta$ and a scale parameter b. The resistance increase due to load L over a time interval δt for a fuel cell of age t_0 is given by:

$$\begin{aligned} \Delta R_L(t_0, \delta t) &= R_L(t_0 + \delta t) - R_L(t_0) \\ &\sim \text{Gamma}\left(\alpha_L (t_0 + \delta t)^\beta - \alpha_L t_0^\beta, b\right) \end{aligned} \tag{6}$$

This formulation extends the previous degradation model by incorporating both load and age dependencies. The load dependency is inherited through α_L, as formulated in Eq. (7) Zuo et al.

(2024).

$$\alpha_L = \begin{cases} K_1(L - L_{\text{nom}})^2 + \alpha_{\text{nom}}, & \text{if } L > L_{\text{nom}} \\ K_2(L - L_{\text{nom}})^2 + \alpha_{\text{nom}}, & \text{if } L < L_{\text{nom}} \end{cases} \tag{7}$$

While the inclusion of the parameter β introduces age dependency, enabling the model to account for time-varying degradation rates. This feature is novel and adds significant depth to the modeling framework.

The function α_L, shown in Figure 1, exhibits a parabolic shape, assigning the lowest deterioration rate to the nominal load, while minimal and maximal loads result in higher rates.

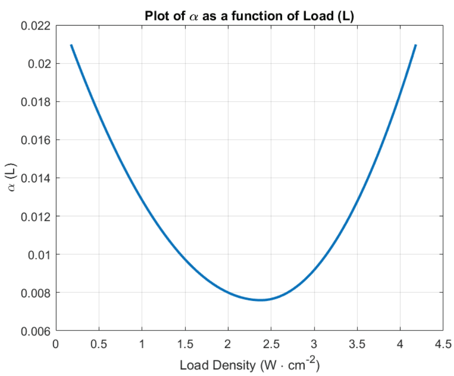

Fig. 1. α_L as a function of load density ($W \cdot \text{cm}^{-2}$).

2.3. *Parameter Estimation*

The parameters α_{nom}, β, and b are estimated from data using the maximum likelihood method outlined in Majeed (2020). The values are determined as $b = 0.0517$, $\alpha_{\text{nom}} = 0.0076$, and $\beta = 0.94$.

The parameter β, with a value less than one, reflects the effect of early, rapid degradation. It is anticipated that β would decrease further under varying load conditions. However, due to insufficient data, β is fixed at 0.7 and treated as constant throughout this study. Future research could explore dynamic modeling of β to account for load variations.

To estimate the parameters K_1 and K_2 in Eq. (7), a relationship can be established using the following properties developed by Paroissin and Salami (2014):

$$\mathbb{E}[T] \approx A^{-1}(\mathbb{E}[T']) \tag{8}$$

Here, $\mathbb{E}[T]$ denotes the expected lifetime for a non-homogeneous Gamma process with a shape function $A(t)$ and a scale parameter b, $A^{-1}(x)$ represents the inverse function of $A(t)$, and $\mathbb{E}[T']$ is the mean lifetime of a homogeneous Gamma process with a shape function t and a scale parameter b. For the specific case where $A(t) = \alpha_L t^{\beta}$, we have:

$$A^{-1}(x) = \left(\frac{x}{\alpha_L}\right)^{1/\beta} \tag{9}$$

Furthermore, Bérenguer et al. (2003) provide the following approximation for $\mathbb{E}[T']$:

$$\mathbb{E}[T'] \approx \frac{FT}{b} + \frac{1}{2} \tag{10}$$

By substituting Eq. (10) into Eq. (8), the expected lifetime becomes:

$$\begin{aligned} \mathbb{E}[T] &\approx A^{-1}\left(\frac{FT}{b} + \frac{1}{2}\right) \\ &\approx \left(\frac{FT}{\alpha_L b} + \frac{1}{2\alpha_L}\right)^{1/\beta} \end{aligned} \tag{11}$$

Let $\mathbb{E}[T_1]$ be the expected lifetime for a cell working under maximal load and $\mathbb{E}[T_2]$ be the expected lifetime for a cell working under minimal load. Using Eq. (11) and Eq. (7), the parameters K_1 and K_2 are estimated as:

$$K_1 = \frac{\left(\frac{FT}{b} + \frac{1}{2}\right) - \alpha_{\text{nom}}\mathbb{E}[T_1]^{\beta}}{\mathbb{E}[T_1]^{\beta}(L_{\max} - L_{\text{nom}})^2} \tag{12}$$

$$K_2 = \frac{\left(\frac{FT}{b} + \frac{1}{2}\right) - \alpha_{\text{nom}}\mathbb{E}[T_2]^{\beta}}{\mathbb{E}[T_2]^{\beta}(L_{\min} - L_{\text{nom}})^2} \tag{13}$$

For this work, FT corresponds to a resistance increment of $0.097\,\Omega \cdot \text{cm}^2$. Using K_1 and K_2, α_L can be calculated for any load using Eq. (7). Table 1 summarizes the parameters values under different loads.

Table 1. Estimated Model Parameters for Different Loads

Load ($W \cdot \text{cm}^{-2}$)	E[Lifetime] (h)	α_L
$L_{\text{max}} = 4.181$	$\mathbb{E}[T_1] = 600$	0.0214
$L_{\text{nom}} = 2.381$	–	$\alpha_{\text{nom}} = 0.0076$
$L_{\text{min}} = 0.175$	$\mathbb{E}[T_2] = 600$	0.0214

3. Prognostic and Remaining Useful Life Prediction

Given the future load profile, current age, and state of health (SOH) characterized by the resistance R, the prognostic tool estimates the probability distribution of the remaining useful life (RUL) of the PEMFC. The stack is considered to have failed when its degradation (resistance) exceeds a predefined failure threshold FT.

The random time of failure, T_f, is defined as:

$$T_f = \inf \{t \geq 0 : R_t \geq FT\}$$

where R_t represents the resistance of the stack at time t.

Consider a stack with an age t_c and a current resistance R_c. The RUL is defined as the time elapsed from the current time t_c until the failure time T_f, given that the stack has not failed during the interval $[0, t_c]$. For $t \geq t_c$ it is expressed as:

$$RUL(t_c) = \inf \{(t - t_c) : R_t - R_c \geq FT - R_c\}$$

Let $t' = t - t_c \geq 0$, and consider the case of a fuel cell operating under a static load L. The cumulative distribution function (CDF) of $RUL(t_c)$, denoted as $F_{\text{RUL}(t_c)}(t')$, and the probability density function (PDF) of $\text{RUL}(t_c)$ are given by Eqs. (14) and (16), respectively:

$$\begin{aligned} F_{\text{RUL}(t_c)}(t') &= P(\text{RUL}(t_c) \leq t' \mid R_c < FT) \\ &= \frac{\Gamma(A(t'+t_c) - A(t_c), \frac{FT-R_c}{b})}{\Gamma(A(t'+t_c) - A(t_c))} \\ &= \frac{\Gamma\left(\alpha_L\left((t'+t_c)^\beta - t_c^\beta\right), \frac{FT-R_c}{b}\right)}{\Gamma\left(\alpha_L\left((t'+t_c)^\beta - t_c^\beta\right)\right)} \end{aligned} \quad (14)$$

where $\Gamma(x) = \int_0^\infty z^{x-1} e^{-z}\, dz$ is the Euler Gamma function, and $\Gamma(s, x)$ is the upper incomplete Gamma function, defined as:

$$\Gamma(s, x) = \int_x^\infty t^{s-1} e^{-t}\, dt \quad (15)$$

The probability density function (PDF) of $\text{RUL}(t_c)$ is:

$$\begin{aligned} &f_{[FT-R_c|A(t'+t_c)-A(t_c),b]}(t') = \\ &\frac{(1/b)^{A(t'+t_c)-A(t_c)}}{\Gamma(A(t'+t_c) - A(t_c))}(FT - R_c)^{A(t'+t_c)-A(t_c)-1} \\ &\times \exp\left(-\frac{FT - R_c}{b}\right) \end{aligned} \quad (16)$$

The mean remaining useful life (RUL) can be expressed as:

$$\mathbb{E}[\text{RUL}(t_c)] = \int_0^\infty t'\, f_{[FT-R_c|A(t'+t_c)-A(t_c),b]}(t')\, dt' \quad (17)$$

Consider an aged fuel cell with a current age of $t_c = 1500\,\text{h}$ and resistance $R_c = 0.04\,\Omega \cdot \text{cm}^2$, operating under a static nominal load. The simulated degradation paths for this scenario are shown in Figure 2.

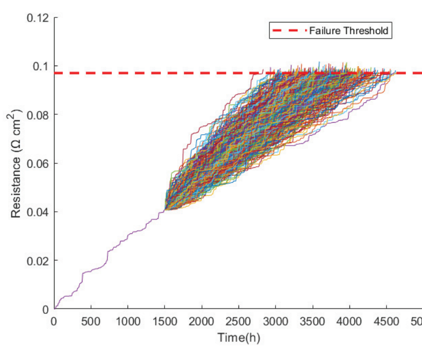

Fig. 2. Simulated degradation paths for an aged fuel cell with $t_c = 1500\,\text{h}$ and $R_c = 0.04\,\Omega \cdot \text{cm}^2$ under static nominal load.

The exact mean RUL, calculated using Eq. (17), is found to be $2185.4\,\text{h}$. In comparison, the simulated mean RUL using 500 degradation paths is 2178.5 h. To gain deeper insights into the RUL distribution and its accuracy, Figure 3 shows the exact probability density function (PDF) vs. the kernel density estimate of the PDF from simulation, while Figure 4 shows the exact and simulated cumulative distribution function (CDF) of the RUL for this aged fuel cell. It is concluded that 500 paths are sufficient to estimate the RUL with accuracy since the difference between the simulation and analytical results is small enough.

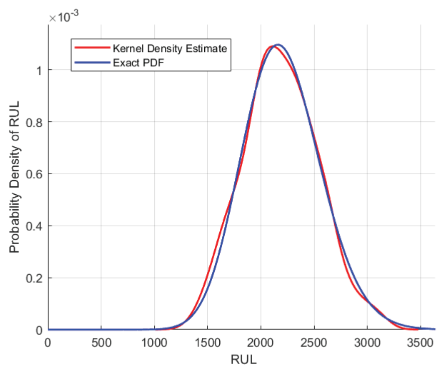

Fig. 3. Comparison of the simulated and exact PDF of the RUL for $t_c = 1500$ h and $R_c = 0.04\,\Omega\cdot\text{cm}^2$ under static nominal load.

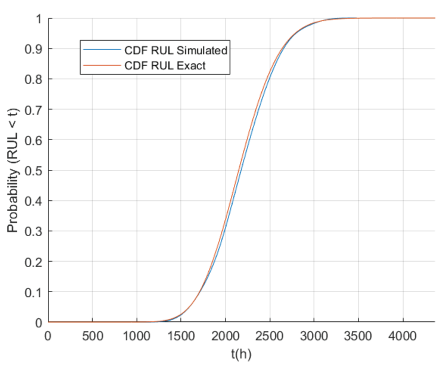

Fig. 4. Comparison of the simulated and exact CDF of the RUL for $t_c = 1500$ h and $R_c = 0.04\,\Omega\cdot\text{cm}^2$ under static nominal load..

For a fuel cell operating under a variable load, deriving analytical expressions for the RUL distribution becomes more challenging. In such cases, Monte Carlo simulations can still be employed to perform prognostics, estimate reliability metrics, and predict failure.

4. Energy Management Formulation

Consider a multi-stack fuel cell system with n stacks. The energy management problem involves distributing the load demand across the n stacks to minimize the expected degradation over the decision horizon h. In this context, it is assumed that the load demand is known for the upcoming decision horizon and remains constant until the next decision point. Additionally, the resistance $R_{\text{obs}i}$, the age t_i of stack i, and the failure threshold FT are known at the decision time, as illustrated in Figure 5.

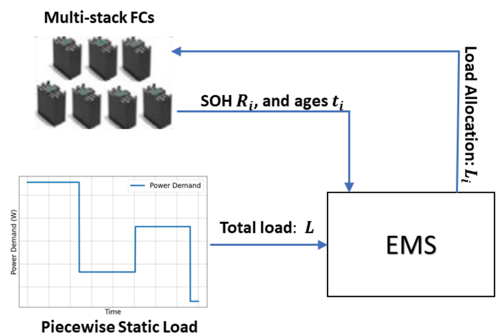

Fig. 5. Load allocation diagram.

Let $D(L, t_1, t_2)$ represent the degradation rate of a cell of age t_1 subjected to load L over the time interval $[t_1, t_2]$, defined as:

$$D(L, t_1, t_2) = \frac{\alpha_L b(t_2^\beta - t_1^\beta)}{(t_2 - t_1)} \tag{18}$$

The expected resistance increment due to the load amplitude L for a fuel cell stack with an initial age t_0 over a future time horizon h, denoted as $\Delta R_L(L, t_0, h)$, is expressed as:

$$\begin{aligned}\Delta R_L(L, t_0, h) &= D(L, t_0, t_0 + h) \cdot h \\ &= \alpha_L b[(t_0 + h)^\beta - (t_0)^\beta]\end{aligned} \quad (19)$$

The objective function of the load allocation problem can be formulated as the sum of all expected increments across all stacks, considering both load level and load variation:

Minimize

$$\begin{aligned}J(L_i, t_i, h) &= \sum_{i=1}^{n} \Delta R_L(L_i, t_i, h) + K\Delta L_i \\ &= \sum_{i=1}^{n} \alpha_{L_i} b((t_i + h)^\beta - (t_i)^\beta) \\ &\quad + K\Delta L_i\end{aligned} \quad (20)$$

subject to:

$$\sum_{i=1}^{n} L_i = L$$

$$L_{\text{min}} \le L_i \le L_{\text{max}} \quad \text{for all Stacks}$$

$$\Delta L_i = L_i - L_i^{\text{previous}}$$

where L_i^{previous} is the load supplied by stack i prior to the decision-making step.

The optimization problem is solved using sequential quadratic programming (SQP) in MATLAB.

5. Simulations and Results

5.1. *Simulation Setup*

The simulation setup assumes a system comprising two fuel cell stacks with different initial ages and degradation levels, operating under piecewise static load. The load demand is considered sufficient to ensure both stacks operate at least at their nominal load levels while not exceeding the maximum load capacity of the two stacks. These loads remain constant between decision times but are updated at decision intervals. At each decision point, the load is allocated between the two stacks to minimize degradation.

To evaluate the effect of load allocation, the system is simulated over an extended period using the proposed strategy and the average load split method. A comparison of the resulting lifetime distributions is then provided. During the simulation, stacks may fail and require replacement. A simple replacement policy is employed to ensure this does not affect the performance of the load allocation strategy. Specifically, if a stack fails, it is replaced immediately, and the load distribution is recalculated based on the updated system configuration.

This setup ensures uninterrupted operation and isolates the effectiveness of the Energy Management Strategy in prolonging stack life.

The primary parameters used in the simulation are summarized in Table 2.

Table 2. Simulation Parameters for EMS

Parameter	**Value**
Total Run Time	10^6 hours
Simulation Time Step	1 hour
Decision Horizon	300 hours
Fuel Cell 1 Initial Age	0 hours
Fuel Cell 2 Initial Age	600 hours
Failure Threshold	$0.097\,\Omega \cdot \text{cm}^2$
Fuel Cell 1 Initial Resistance	0
Fuel Cell 2 Initial Resistance	$0.05\,\Omega \cdot \text{cm}^2$
Load	$2L_{nom} \le L \le 2L_{max}$

5.2. *Simulation Results*

EMS showed advantages compared to the average load split method in terms of extending the lifetime of stacks. Figure 6 shows the probability density of lifetimes collected during the simulation for both strategies. The EMS strategy shifts the lifetime distribution to the right, indicating a clear improvement in fuel cell lifetime while delivering the same load.

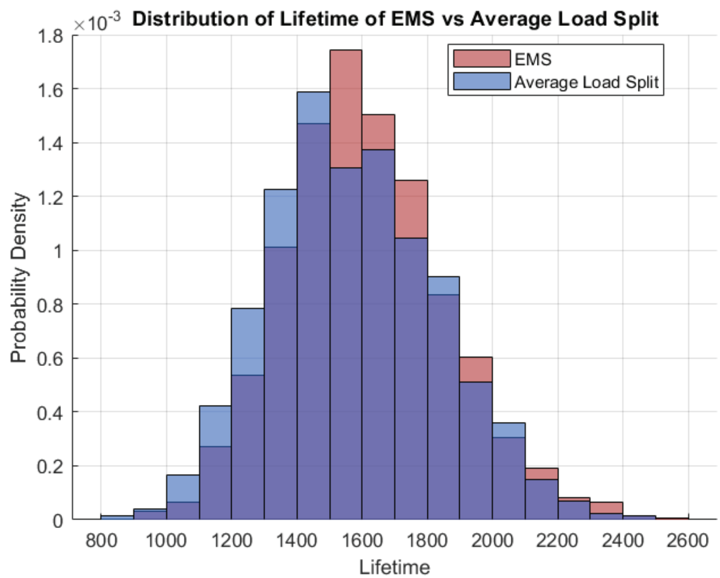

Fig. 6. Lifetime probability density: EMS vs. average load split.

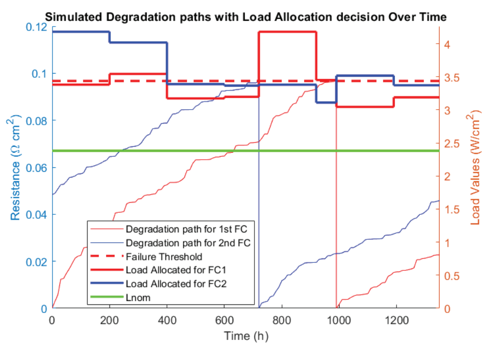

Fig. 7. Simulated degradation paths and load allocation decisions over time.

Further analysis reveals that the EMS allocates loads more effectively between the stacks, as illustrated in Figure 7. Specifically, the newer stack receives loads closer to the nominal load (L_{nom}), while the older stack experiences more extreme loads. This behavior is consistent with expectations, as newer stacks typically exhibit higher degradation rates. Over time, as both stacks age, the load allocation becomes more balanced, optimizing the system's overall durability.

The numerical results underscore the benefits of the EMS strategy. Compared to the average load allocation, the EMS strategy reduced the number of replacements from 1277 to 1235. Additionally, it increased the mean fuel cell lifetime by 3.4%, as detailed in Table 3. These improvements highlight the EMS's effectiveness in extending system life and reducing maintenance demands.

Table 3. Comparison of simulation results of EMS vs average load split.

Parameter	**With EMS**	**AVG Load**
Replacements	1235	1277
Mean Lifetime (h)	1619	1566
95% CI (h)	[1605, 1632]	[1551, 1580]

Note: EMS: Energy Management Strategy. CI: Confidence Interval. AVG Load: Average load split.

6. Conclusion

This work addresses the limitations in fuel cell degradation modeling by developing a non-homogeneous Gamma process-based model that effectively captures both age- and load-dependent characteristics of PEMFC degradation. Building on this enhanced prognostic model, a novel Energy Management Strategy (EMS) formulation is proposed to improve the durability of multi-stack PEMFC systems. Extensive simulations demonstrated that the EMS strategy, compared to equal load distribution, resulted in fewer replacements and a 3.4% increase in the mean fuel cell lifetime. This work can be further extended to develop EMS for dynamic loads with potential rapid variations and to enhance the model by investigating the relationship between its assumed fixed parameters and the load.

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