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Expanding the uncertainty toolkit for risk analysis

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Uncertainty is an inherent part of risk: where there is no uncertainty, there is no risk. However, most international standards for risk analysis provide minimal guidance on the consideration of uncertainty. In particular, the relationship between uncertainty and risk remains disputed, resulting in the lack of an agreed definition and the absence of comprehensive methodologies for uncertainty analysis. To address this issue, a conceptual framework for uncertainty analysis in risk science is presented. This framework describes the intrinsic uncertainty of risk and is supplemented by a three-layered approach of first, second and third order uncertainty analysis. Somewhat surprisingly, first order uncertainty analysis is risk assessment itself. The definition of risk, according to ISO 31000:2018 is "the effect of uncertainty on objectives". By implication, risk assessment requires analysis of that uncertainty. Typically, the uncertainty in question relates to knowledge of the probability and consequences of risk. Second order uncertainty analysis within this context is thereby viewed as evaluation of the residual uncertainty that remains from the risk assessment but also includes consideration of other components of risk analysis, such as risk communication, risk management and risk governance. Finally, third order uncertainty analysis involves evaluation of higher order uncertainty of the risk paradigm and is described as metauncertainty. This comprehensive conceptual framework for uncertainty can assist in the identification and treatment of different types of uncertainty. Consideration of metauncertainty further enhances our uncertainty toolkit to advance risk science.

Keywords: Uncertainty, Risk, Risk analysis, Risk paradigm, Metauncertainty, Safety, Ambiguity, Ignorance

1. Introduction

Uncertainty is an inherent part of risk: where there is no uncertainty, there is no risk (Wilson and Crouch 1987). Indeed, uncertainty has been explicitly acknowledged in some definitions of risk e.g. "objectified uncertainty regarding the occurrence of an undesirable event" (Willett 1901); "measurable uncertainty" (Knight 1921); "a state of uncertainty where some possible outcomes have an undesired effect or significant loss" (Hubbard 2010); "the effect of uncertainty on objectives" (International Organization for Standardization 2018).

While there are some purpose specific toolkits for handling uncertainty (Skinner et al. 2014; EFSA Scientific Committee 2018; Bevan 2022), they are yet to achieve widespread acceptance (Aven and Renn 2014). Most recommendations for uncertainty analysis focus on empirical uncertainty (Li et al. 2013; EFSA et al. 2018; Stackelberg and Williams 2021; Cruz et al. 2022; Hamilton et al. 2024) or residual uncertainty of risk assessment (Bevan 2022). Nevertheless, the relationship between uncertainty and risk remains disputed, resulting in the lack of an agreed definition and the absence of comprehensive methodologies for analysing uncertainty. Consequently, most international standards for risk analysis provide minimal guidance on the consideration of uncertainty. To address this issue, a conceptual framework for uncertainty analysis in risk science is presented.

The terminology and concepts used here are based on the SRA Glossary (SRA 2018a) and SRA Fundamental Principles (SRA 2018b). The risk notation and the main risk concepts are derived from Ylönen and Aven (2023).

2. The nature of uncertainty

One qualitative definition of uncertainty is "imperfect or incomplete information/knowledge about a hypothesis, a quantity, or the occurrence of an event" (SRA 2018a). This form of analytical uncertainty is considered in the broadest sense to include incomplete knowledge, ignorance, variability, vagueness, indeterminacy, ambiguity, undecidability, model error, or the like.

Types of uncertainty that refer to an absence of form or substance, hesitancy, or unsteadiness, such as the 'uncertain' light of a candle or an 'uncertain' smile are outside the scope of this discussion.

Uncertainty is used here as 'imperfect knowledge', where knowledge is a form of 'justified belief'. The belief may be justified by different forms of evidence (Thekdi and Aven 2024). In many cases, the justification is restricted to empirical means based on direct or indirect inferences of the senses usually depicted as epistemic or aleatory knowledge (EFSA et al. 2018). However, important knowledge can be justified by other means. For example, mathematical knowledge is gained from logical deductions. Therefore, two types of knowledge are introduced here.

Empirical knowledge – facts derived directly or indirectly from the senses

- fixed – epistemic
- variable – aleatory

Conceptual knowledge – ideas derived from the mind

- intuitive reasoning (perception)
- mental reasoning

The nature of empirical knowledge is well known and understood in its application to risk science. Less clear is the nature and role of conceptual knowledge that uses ideas derived from the mind. and includes intuitive reasoning (perception) and mental reasoning.

According to the SRA Fundamental Principles, risk perception is "considered and incorporated into risk management" (SRA 2018b). However, intuitive reasoning in the form of perception is also used informally to estimate the level of risk (Slovic 1987). Intuitive reasoning can arise from different sources, e.g. psychological (Slovic 1987; Tversky and Kahneman 1974), social/cultural (Douglas and Wildavsky 1982; Kasperson et al. 1988), or as the result of evolutionary adaptation (Tucker and Ferson 2008).

Conceptual knowledge gained through mental reasoning is made manifest through some form of description, which may include scientific models, language, pictograms/graphs, formal logic or mathematics. Counterintuitively, these apparently disparate descriptive forms overlap significantly in the forms of uncertainty associated with them, such as vagueness or under specificity (see below).

Conceptual knowledge can be supported by empirical information such as the parameters from scientific models. Nevertheless, scientific models are not material objects but abstractions; mental constructs useful for understanding and explaining physical phenomena. Furthermore, empirical knowledge and conceptual knowledge differ in the forms of uncertainty they are prone to.

Measures to estimate the level of risk use both empirical and conceptual knowledge in formal and informal risk assessments. Formal risk assessments typically rely on the use of empirical knowledge.

In addition, another possible form of conceptual knowledge can be described as collective beliefs that are justified through authority or received wisdom. This is not compatible with the concept of risk science and therefore falls outside of the discussion here.

In conclusion, it is suggested that uncertainty in risk science considers all forms of imperfect knowledge, both empirical and conceptual, to satisfy the widest range of risk analysis concepts and risk measures that are used formally and informally.

3. Relationship between risk and uncertainty

Risk commonly encompasses the following set of elements $(E, C, P, K, U)_A$.

E, Events, is used to describe or characterize an individual risk, often expressed in some form of the relational model of risk described by Boholm and Corvellec (2011). Namely, a risk consists of a risk source, a risk recipient (an object of value) and a causal pathway or set of circumstances where a risk source has the potential to cause harm to the risk recipient.

C, Consequences, represents harm (undesirable outcome) to an object of value. The harm can vary in degree of seriousness and is a key measure to estimate the level of risk. Harm is a subjective value judgement, which is amaterial. C is also the main criterion that distinguishes risk from opportunity. With risk there must be at least one possible outcome that is undesirable, although other outcomes may be neutral or desirable. In contrast, opportunity occurs where at least one possible outcome is considered desirable and is typically part of a cost/benefit analysis.

P, Probability, used here in the broadest sense to include not only probability theory and its close relatives, but also qualitative measures commonly expressed linguistically as degrees of likelihood and non-probabilistic measures such as info gap theory, possibility, chance.

K, Knowledge, is the basis for examining measures of risk used to estimate the level of risk.

U, Uncertainty, is considered from an analytical perspective as a form of imperfect knowledge.

A, Activity, provides the setting in which risk is considered, such as such as riding a bike, climate change or regulatory decision-making. The activity forms part of the context, scope and criteria (ISO 31000 2018) established for conducting risk analysis.

The relationship of uncertainty to risk in this risk set is unclear. There are several different risk concepts that can be depicted as different subsets of the general set (E, C, P, K, U) (Ylönen and Aven 2023). Some risk concepts explicitly incorporate uncertainty (Ylönen and Aven 2023). Others, however, substitute probability as the measure of uncertainty or exclude uncertainty (Ylönen and Aven 2023; Rosa 1998; IRGC 2005). References to uncertainty in measures of risk are often restricted to probabilities or empirical knowledge uncertainty due to factors such as incomplete knowledge, variability, model error, measurement error, sample size, surrogate data and the like.

The traditional perspective is that risk and uncertainty are viewed as overlapping, but distinct concepts (Fig. 1), but there is no consensus on what constitutes the area of overlap between risk and uncertainty.

An alternative perspective is proposed where risk can be represented as a subset of uncertainty (Fig. 1). From this latter perspective, all elements of risk should be subject to some form of uncertainty.

A conceptual framework for uncertainty analysis is presented in the next section according to the proposed representation in Figure 1.

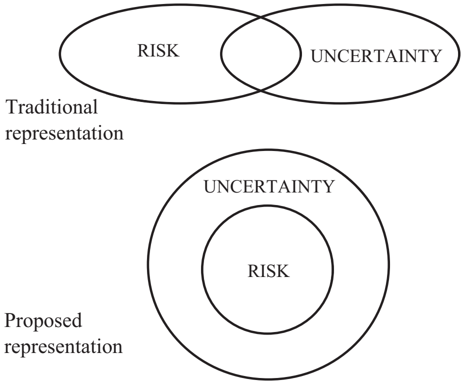

Fig. 1. Relationship of risk and uncertainty

4. Conceptual framework for uncertainty

Uncertainty in the context of risk is described as the set (U_i, $U_{1,2,3}$) (Fig. 2). This proposed conceptual framework places uncertainty as an integral component of risk at all levels of analysis.

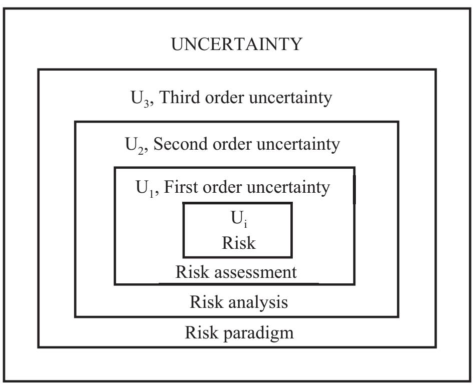

Fig. 2. Conceptual framework depicting the relationship between uncertainty and risk.

U_i, Intrinsic uncertainty, is a feature of all risks that consider a future outcome where the probability of an outcome is, $0 < P < 1$. For example, even in the case where all parameters are known with certitude, such as a gamble on the toss of a fair coin, the outcome of the next toss is uncertain. Namely, if U_i is not present, then there is no risk.

$U_{1,2,3}$, corresponds to first order, second order and third order uncertainty associated with risk.

U_1, First order uncertainty, is uncertainty associated with all parts of a risk assessment, including risk characterization to identify and describe risks, risk measures to estimate the level of risk, and risk evaluation to determine the significance of risk.

U_2, Second order uncertainty, is uncertainty associated with risk analysis, including risk communication, risk management, risk governance and residual uncertainty of risk assessment.

U_3, Third order uncertainty, is higher order uncertainty associated with the risk paradigm, including the risk concept, fundamentals of risk analysis, nature and formulation of risk science, and the relationship of risk to overlapping concepts. It is described here as metauncertainty, ‘uncertainty of uncertainty’.

4.1. *First order uncertainty analysis*

In addition to intrinsic uncertainty, risk assessment typically involves some degree of imperfect or incomplete knowledge associated with E, C, P or K. Therefore, the focus of a risk assessment is understanding the nature and degree of uncertainty associated with each parameter of the risk assessment, including knowledge uncertainty.

Event (E) uncertainty may include: the identity of the risk source and risk recipient; the reasoning for the risk recipient being an object of value; or identifying a causal chain whereby a risk source potentially causes harm to the risk recipient.

The risks to the European honeybee from climate change provides one example of event uncertainty. On the one hand, the honeybee is widely considered an object of value due to its importance in pollinating crops. On the other hand, bee stings induce anaphylactic shock in vulnerable people and outside of Europe the honeybee has the potential to reduce biodiversity by outcompeting native bee species.

Consequences (C) uncertainty includes determining the nature and seriousness of harm to the risk recipient. Empirical evidence can be used to determine if those values are affected (e.g. injury requiring hospitalization as a measure of major health consequences). The empirical evidence used to measure proxies for values, often in the form of a scientific model, is subject to knowledge uncertainty.

Uncertainty of probabilities (P) is dependent on the available knowledge and uncertainty is often increased by the presence of multiple steps in a causal chain. In addition, in the case of pathogens, probability uncertainty can be affected by population variation in the risk source and vulnerability of the risk recipient.

Knowledge (K) forms the basis of measures to estimate the level of risk. Empirical knowledge uncertainty includes the types and accuracy of those measures (ISO 5725-1 2023).

Conceptual knowledge uncertainty based on intuitive reasoning include examples such as heuristics and biases associated with psychological forms (Tversky and Kahneman 1974).

Conceptual knowledge based on mental reasoning can take many distinct forms of description (e.g. language, scientific models or formal logic). Nevertheless, they are subject to many similar types of uncertainty. Linguistic uncertainty has been described as an important consideration in qualitative risk assessments (Regan et al. 2002; Carey and Burgman 2008) where measures of risk are represented by qualitative terms such as moderate or severe consequences, or likelihoods described as unlikely, likely or highly likely. Typically, linguistic uncertainty includes vagueness, ambiguity, context dependence, under specificity, and indeterminacy of theoretical terms (Regan et al. 2002). These types of uncertainty are applicable to other types of mental reasoning, particularly scientific models. For example, contextual uncertainty can be an important form of uncertainty for many types of descriptive knowledge (Humpherson 2024).

If we view risk as a subset of uncertainty, then risk assessment can be defined as a structured, rational approach to address uncertainty based on the plausibility and strength of knowledge about the characterization, calculation and evaluation of risk, which can be depicted as (E_U, C_U, P_U, K_U). In this form, first order uncertainty is addressed through the risk assessment (Fig. 2).

4.2. *Second order uncertainty analysis*

Second order uncertainty considers uncertainty associated with risk analysis. This includes consideration of the residual uncertainty from the risk assessment and may require the use of tools such as confidence levels, sensitivity analysis (Frey and Patil 2002), or worst-case scenario mapping.

Second order uncertainty of other components of risk analysis includes uncertainty analysis of risk communication, risk management and risk governance. For example, there can be risk communication uncertainty in the identification of critical stakeholders, level of engagement, types of communication channels to be used, and perception off the stakeholders involved.

Uncertainty in risk management may include consideration of the effectiveness and efficiency of proposed risk treatment measures, acceptability of risk treatment measures, and the potential for risk management to introduce new risks, or increased level of previously identified risks.

Uncertainty in risk governance may include risk governance strategies, trustworthiness and legitimacy of an authority for risk governance, decision-making processes and the basis of stakeholder involvement (Klinke and Renn 2021).

4.3. *Third order uncertainty analysis*

Third order uncertainty addresses uncertainty of the risk paradigm. It is described as metauncertainty, analogous to Gödel's incompleteness theorems. Namely, if we treat the risk analysis as a consistent formal system (SRA 2018b), then there are statements that cannot be proven or disproved within that system. Furthermore, risk analysis cannot prove its own consistency. Higher order concepts are required to address fundamental concepts that underpin the risk paradigm and unresolved uncertainties arising from first and second order uncertainty analysis.

The discussion provided below is not an attempt to provide comprehensive coverage of metauncertainty, but to serve as some examples of higher order uncertainty that might merit further consideration.

4.3.1. *Third order knowledge uncertainty*

Knowledge uncertainty in risk assessment is commonly addressed by obtaining additional empirical information. However, there is also higher order knowledge uncertainty that is not typically covered by risk assessment or by analysis of residual uncertainty from the risk assessment. Higher order knowledge uncertainty can be inherent in both the type (Levins 1966, 1993) and choice (Lyytimäki et al. 2011) of knowledge used in risk assessment.

For example, measures of risk in formal risk assessment often rely on scientific model building. However, all models are inherently inadequate (Levins 1966, 1993). Models generally sacrifice one of the key desirable features, namely, generality, realism or precision. Statistical models emphasize generality and precision but may fail the test of realism as they examine correlation rather than causality. Mechanistic studies conducted in the laboratory or under highly controlled field conditions may meet the requirements of realism and precision but fail in the degree of generality. While the use of biological principles may satisfy the test of generality and realism but fail the need for precision. Other desirable features of models, such as, manageability and understandability may also be compromised by certain models (Levins 1966).

In addition, models are simplifications of reality, and therefore, always subject to under specificity, as is common with linguistic uncertainty.

Increasing the accuracy of a model through additional information comes at a cost that increases exponentially as the uncertainty decreases.

Likewise, the choice of empirical knowledge used in risk assessment is selective. Not all information is necessarily included. Missing or excluded information may affect the measures of risk (Lyytimäki et al. 2011). As well, the included information can vary considerably in quality. Some form of weight of evidence test is often used to address knowledge uncertainty but rarely examines the broader issue of what counts as quality of information, or the inherent knowledge uncertainty associated with the use of scientific models.

Finally, there is the issue of the additional information paradox, where additional information used to reduce knowledge uncertainty may instead increase uncertainty (Ben-Haim 2024). Scientific endeavour has greatly increased our collective knowledge over time, but the universe of unknowns is now greater than ever before. For example, the microscope introduced scientists to microorganisms, but this generated many new questions regarding unknown types, functions and interactions of this expanded universe of known organisms.

4.3.2. *Third order uncertainty of risk measures*

The set $(E, C, P, K, U)_A$ contains all the elements that comprise different concepts of risk. This set also contains elements used as measures to estimate the level of risk. In most cases, P and C are designated as the two risk measures applicable to risk assessment.

However, other risk measures also appear to be relevant to formal and informal risk assessment. Therefore, risk measures could be represented by the set (P, C, X), where X is a parameter used as a risk measure.

For example, utility is a core measure of economic risks, where costs(risks) and benefits (utility) are always considered together. Measures based on perception, such as affect (e.g. dread), availability or familiarity may be used informally as risk measures (Slovic 1987). Even uncertainty can be a risk measure, where higher uncertainty is seen as riskier and can be influenced by the type and scale of uncertainty (e.g. incertitude, stochastic, indeterminacy or wickedness). Other possible risk measures might include complexity, manageability, historical contingencies or biological adaptations.

One approach is to view these measures as either risk management or risk communication issues (SRA 2018b). Alternatively, they could be treated as additional conditional (subjective) probabilities, thereby restoring (P, C) as the only set of risk measures. However, it can also be considered an area of metauncertainty that requires further examination at a fundamental level.

4.3.3. *Risk landscape*

Another area of metauncertainty is the scope of the risk concept. Both Knight (1921) and Keynes (1921) established risk as restricted to 'measurable uncertainty', which has been reinforced by use of probability theory as the dominant tool for risk assessment.

In addition, this viewpoint is formalized by Stirling and Gee (2002) where risk is assigned when the primary measures of risk, probability (P) and Consequences (C), are both determinate (low uncertainty) (Fig. 3). If either P or C, or both are indeterminate (highly uncertain), then some form of uncertainty is assigned as ambiguity, uncertainty or ignorance. This designation has been adapted by others (Aven 2011).

However, P and C are measures of risk, regardless of the degree of uncertainty. This should expand the risk landscape to encompass all four quadrants as various forms of risk labelled as Determinate Risk, Hazardous Risk, Strategic Risk and Indeterminate Risk (Fig. 3).

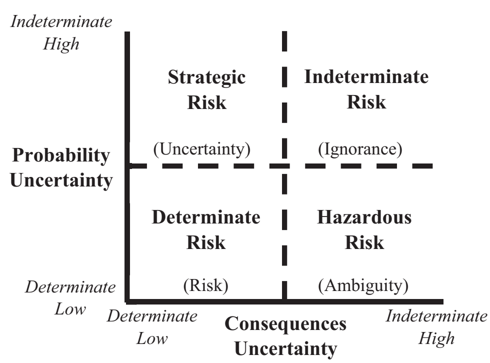

Fig.3. Risk landscape relative to the degree of uncertainty for Probability and Consequences. Designations made according to Stirling and Gee (2002) are in brackets.

These four forms of risk have different properties and favor the use of different tools to assess risk. For example, strategic risk, where C is known, but P is indeterminate or arbitrary (e.g. the Prisoner's Dilemma or the classic rock-paper-scissors game) tends to use Game Theory or the Nash equilibrium for assessing risk.

Consequently, the application of probability theory to certain risks such as indeterminate risks associated with highly complex, dynamic risks (Palmer 2012) (e.g. climate change, warfare or exchange rate fluctuations) can be problematic or provide a misleading impression of greater reliability than is warranted (Ord et al. 2010). Other approaches for estimating the levels of indeterminate risks may be required (Colyvan 2008; Cox 2012; Flage et al. 2014).

Furthermore, some activities, such as regulatory decision-making, may not be well suited for handling risks with indeterminant probabilities, which may bias the treatment of certain risks. Therefore, it may be of value to decide on the type of risk under investigation before assigning the risk assessment methodology.

4.3.4. *Risk related concepts*

The concept of risk overlaps with other terms such as vulnerability, resilience, uncertainty and danger. Understanding the relationship between risk and other concepts can be used to probe the nature and application of the risk paradigm. One relationship that may warrant further examination at a fundamental level is between risk and safety.

In general risk has been viewed as the inverse of safety, namely, high risk equates to low safety and low risk indicates high safety. This basic view does not seem to accord with common usage, where risk and safety appear to operate as distinct concepts (Boholm et al. 2016).

The context in which risk and safety are commonly used was evaluated by tabulating the frequency of word pairs 'x risk' and 'x safety' (Table 1). Some notable differences were observed such as far greater reference is made to 'gun safety' than 'gun risk'. In contrast, 'cancer safety' is far less frequently cited than 'cancer risk'. The context for word pairings was also examined individually and pairings with significant context dependency, ambiguity or misleading association were excluded.

Table 1. Number of citations for paired words, either 'x risk' or 'x safety'. The number of citations were accessed from https://www.nytimes.com on December 8, 2024.

Paired word	Risk	Safety
-	382,410	300,492
Gun	0	1,957
Vehicle	3	545
Food	4	6,303
Nuclear	117	1,229
Chemical	14	298
Electrical	0	61
Cancer	1,467	2
Disease	475	2
Flood	252	8
Injury	280	12
Financial	1,755	289
Political	2,415	66
High	17,959	149
Low	3,927	24

In the cases of gun, vehicle, food, nuclear, chemical, vaccine and electrical, the use of 'x safety' far exceeds the use of 'x risk'. In most of these cases, 'x' refers to a risk source that may result in serious harm to human health, including the possibility of death.

Whereas, for cancer, disease, flood, injury, financial, political, high and low, the use of 'x risk' exceeds the use of 'x safety'. Some of these terms, namely, cancer, disease, flood and injury all refer to the occurrence of harm, namely, the consequences of a risk event.

Some cases, such as the preferred word pairs 'financial risk' and 'political risk', highlight activities that may result in non-health related harm, but also provide opportunities for a windfall, which is not considered in a safety context (Bryce et al. 2024).

Preference for use of 'high risk' and 'low risk' is consistent with the viewpoint that risk is a matter of degrees (probabilistic), whereas safety concerns absolutes regarding the presence or absence of harm (possibilities).

One expectation from these differences in everyday usage is that the starting assumption for safety is 'guilty until proven innocent', whereas risk might favor 'innocent until proven guilty'.

These differences in the everyday understanding of risk and safety can lead to ambiguity in risk perception and risk communication when putting risk analysis into practice. One example is the regulation of genetically modified organisms (GMOs) in many jurisdictions worldwide. For example, the object of the Australian Gene Technology Act 2000 (Australian Government 2000) is "to protect the health and safety of people, and to protect the environment, by identifying risks posed by or as a result of gene technology, and by managing those risks through regulating certain dealings with GMOs". Namely, the regulatory objective includes both safety concepts 'to protect' and risk concepts 'identifying risks' and 'managing risks'. Other safety measures include reference to the Precautionary Principle and employment of prohibition as the default status. Namely, all uses of a GMO are prohibited unless granted specific permission, typically in the form of a licence. Nevertheless, a risk assessment and risk management plan is a key plank for regulatory decision-making.

This overlapping use of risk and safety concepts can be seen as promulgating divisions in regulatory expectations. Certain activist groups can point to protection (safety) references in the legislation as justification for greater restrictions on the use of GMOs, whereas certain commercial interests can use the outcomes of risk assessments as justification for easing restrictions.

5. Conclusions

A comprehensive conceptual framework for uncertainty analysis is provided. The main features of this framework include:

- risk is a subset of uncertainty, such that all aspects of risk are subject to uncertainty analysis
- the four components of the conceptual framework are intrinsic uncertainty of risk, first order uncertainty (risk assessment), second order uncertainty (risk analysis), and third order uncertainty (risk paradigm)
- the conceptual framework encompasses all current interpretations of uncertainty associated with risk
- each component of uncertainty requires a distinct toolkit for analysis of uncertainty.

One component of the conceptual framework, metauncertainty, associated with uncertainty of the risk paradigm, is under-explored. Closer scrutiny of metauncertainty may increase our understanding of risk science and its application to real world problems.

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