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Optimized Variational Mode Decomposition Algorithm Based on Pre-Processing Algorithm

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Existing optimization algorithms for Variational Mode Decomposition (VMD) are all based on multiple iterations, which pose a problem due to their large computational load, making them unsuitable for real-time fault diagnosis of engineering signals. This paper proposes an optimized VMD algorithm based on a preprocessing algorithm, which avoids iterative computation and thus improves computational efficiency. Firstly, the signal is Fourier transformed, and the spectrum is segmented based on the peak frequency of the signal, resulting in the number of decomposition layers after optimization. Secondly, based on the correlation between the quadratic penalty term and the signal noise, the optimal solution under different signal noises and different penalty terms is calculated, and the signal root mean square is used to describe the noise situation of the signal, constructing a penalty term optimization function based on signal root mean square. Finally, the optimization algorithm is validated using a diesel engine cylinder head simulation signal. The results show that the optimization algorithm can significantly reduce the computational load while ensuring the decomposition accuracy of the sine signal and avoiding mode mixing, with the computation time accounting for only 0.01% of the total time, providing a feasible optimization strategy for engineering applications.

Keywords: Signal Processing of Reliability Test, Variational Mode Decomposition, Fault Diagnosis, Fault Feature Extraction, VMD, Preprocessing Algorithm, Spectral Segmentation.

1.Introduction

The Variational Mode Decomposition (VMD) algorithm was proposed based on a variational model, which can decompose components with close frequencies and is highly robust to noise (Dragomiretskiy 2013). The VMD algorithm has been effectively applied in various fields, including rotating machinery fault diagnosis, vibration signal decomposition, and delay analysis(ISHAM M F et al. 2018;Li et al. 2024;Jia et al. 2021). The VMD algorithm requires the user to input two parameters: the number of decomposition layers(k)and the secondary penalty term(α). The choice of these parameters directly affects the final decomposition results, which to some extent limits the application of VMD(BI F et al.2019). Currently, there is more research on determining the number of decomposition layers in the VMD algorithm, while research on determining the secondary penalty term is relatively sparse, with studies focusing mainly on multiple decomposition-based methods and search algorithm-based methods.

For the methods based on multiple decompositions, the basic idea is to perform calculations multiple times within a certain range of decomposition levels, and take the decomposition result that meets a certain criterion as the optimal decomposition level. Bi carried out a power spectrum transformation on the signal, and determined the number of decomposition levels by performing a frequency-domain segmentation on the signal according to the power spectrum, achieving certain results(Bi et al. 2020).Based on the singular value decomposition method, Yang calculated the relationship between the component signals at different decomposition levels and the original signal to determine the optimal decomposition level, and achieved good results for impact signals (Yang et al. 2023). In summary, methods based on multiple decompositions require the establishment of a judgment criterion for determining the optimal decomposition level. However, due to the large differences among different signals and the different perspectives of consideration by various scholars, there is no conclusive conclusion on how to select the corresponding criterion under different circumstances.

For methods based on search algorithms, the basic idea is to use various optimization algorithms in combination with a fitness function to select the optimal combination of the number of decomposition layers and the secondary penalty term. Babiker used the kurtosis index as a fitness parameter and employed the particle swarm optimization algorithm to obtain k and α , achieving certain results in noise reduction for early fault signals of bearings(Babiker et al. 2021). Anjaiah used the minimization of L-kurtosis as the objective function, combined with the whale optimization algorithm to determine k and α , and achieved good results in circuit fault diagnosis by obtaining the optimal components through statistical features(Anjaiah et al. 2021). In summary, methods based on search algorithms also have the problem of choosing a judgment criterion, different fitness functions will lead to obvious differences in the optimization results. However, the methods based on search algorithms can determine both k and α simultaneously, which gives them certain advantages compared with the methods based on multiple decompositions.

The basic approach of the above two optimization methods involves multiple variations of VMD parameters to perform iterative calculations for optimization. For engineering signals with large data volumes, both optimization approaches face the issue of excessive computational cost. Pre-processing algorithms can partially alleviate this problem by determining the parameters k and α before performing VMD operations, thus saving the computational power required for multiple iterative calculations and improving the algorithm's efficiency.

In summary, current research on optimizing the VMD algorithm mainly focuses on iterative calculations for optimization. However, for signals with large data volumes, this approach incurs high computational costs. To address this issue, this paper proposes a pre-processing algorithm to select parameters for the VMD algorithm based on the VMD's characteristic of frequency domain segmentation. In the second chapter of the article, the optimization algorithm for the decomposition layers(k) based on the Fourier spectrum and the quadratic penalty(α) based on the root mean square value are respectively derived. In the third chapter,the construction process of the optimized VMD algorithm is proposed through a flowchart. In the fourth chapter, the results of simulation calculation show that the optimized algorithm can significantly reduce the computational load while ensuring the decomposition accuracy of the sine signal and avoiding mode mixing.

2. Optimized VMD Algorithm Based on Pre-Processing Algorithm

2.1. *Variational Mode Decomposition*

VMD (Variational Mode Decomposition) is an adaptive, non-recursive signal decomposition method based on Wiener filtering and Hilbert transform (YANG et al.2019).It uses a variational model to adaptively select frequency bands for decomposition and derive the corresponding mode functions (ZHONG et al.2021).

VMD defines mode functions as amplitude-frequency modulated signals $u_k(t)$. These mode functions are also referred to as Intrinsic Mode Functions (IMFs). It is assumed that each mode has a finite bandwidth and a central frequency. The variational problem involves finding k mode functions $u_k(t)$, with the objective of minimizing the sum of the estimated mode bandwidths(Jin et al.2022; Sharma V. et al.2020). This bandwidth is evaluated by the constrained variational problem shown in Eq.(1).

$$\begin{cases} \min\limits_{\{u_k\},\{w_k\}} \sum\limits_{k=1}^{K} \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-jwk^t} \right\|_2^2 \\ s.t. \sum\limits_{k=1}^{K} u_k = f \end{cases} \quad (1)$$

In the equation, f is the input signal, δ repre sents the Dirac distribution, $\{u_k\} := \{u_1, u_2, \cdots u_k\}$ denotes the k-th Intrinsic Mode Function (IMF) , and $\{\omega_k\} := \{\omega_1, \omega_2, \cdots \omega_k\}$ indicates the central frequency corresponding to the k-th mode functi on.

To solve the variational problem, a secondary penalty factor α and Lagrange operator λ are introduced, transforming the constrained variational problem into an unconstrained variational problem. The augmented lagrange function is shown in Eq.(2).

$$L(\{u_k\},\{\omega_k\},\lambda) = \alpha \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) \times u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 + \left\| f(t) - \sum_k u_k(t) \right\|_2^2 + \left\langle \lambda(t), f(t) - \sum_k u_k(t) \right\rangle \quad (2)$$

In the equation, the optimal solution to Eq. (2) is obtained using the Alternate Direction Method of Multipliers (ADMM). The mode functions u_k and central frequencies ω_k can be iteratively determined using Eq.(3)and Eq.(4)

$$u_k^{n+1} \xleftarrow{\infty} \arg\min_{u_k} L\left(\{u_{i<k}^{n+1}\},\{u_{i\geq k}^{n+1}\},\{\omega_i^{n+1}\},\lambda^n\right) \quad (3)$$

$$\omega_k^{n+1} \xleftarrow{\infty} \arg\min_{u_k} L\left(\{u_i^{n+1}\},\{u_{i<k}^{n+1}\},\{\omega_{i\geq k}^{n+1}\},\lambda^n\right) \quad (4)$$

$$\hat{u}_k^{n+1}(\omega) = \frac{\hat{f}(\omega) - \sum_{i<k} \hat{u}_i^{n+1}(\omega) + \frac{\hat{\lambda}^n(\omega)}{2}}{1 + 2\alpha\left(\omega - \omega_k^n\right)^2} \quad (5)$$

According to Eq.(5), all IMF components can be obtained through contiuous iteration.

2.2. *Derivation of the Optimization Algorithm for Decomposition Layers(k) Based on Fourier Spectrum*

The principle of VMD is to decompose the original signal into multiple IMFs based on the frequency spectrum information of the input signal. According to the VMD principle, the number of decomposition layers required for VMD can be determined by performing frequency spectrum segmentation on the signal and using the number of key frequencies obtained from the segmentation as the parameter.

The Empirical Wavelet Transform (EWT) method provides a feasible approach for frequency spectrum segmentation (GILLES J. et al.2013). After performing a Fourier transform on the input signal, it is expected that the amplitudes of the key frequencies will be significantly larger than that of the noise. By ranking the amplitudes in the frequency domain and using the number of frequency points with amplitudes above a threshold as the number of decomposition layers, the segmentation process can be determined.

The specific steps for implementing the optimization algorithm are as follows:

First, the input time-domain signal needs to be transformed into the frequency domain. Perform a Fourier transform on the input signal s to obtain the signal $\hat{s}$, where S represents the amplitude corresponding to each signal frequency.

Next, extract the amplitude information of key frequencies to determine the number of key frequencies.

Then, construct the minimum amplitude threshold q in the frequency domain, as shown in Eq.(6).

$$q = S_{\min} + a \times \left(S_{\max} - S_{\min} \right) \qquad (6)$$

In the equation, α is the deformation parameter, which is typically set between 0.2 and 0.4. The deformation parameter α has a significant impact on the choice of the number of decomposition layers k, as α increases, the number of decomposition layers obtained decreases.

Finally, the number of decomposition layers for the VMD algorithm is determined based on the amplitude information of the key frequencies. The number of values in the frequency domain amplitude that are greater than the minimum amplitude threshold q is counted, and the segmentation parameter is then obtained, as shown in Eq.(7).

$$k = R(S > q) + 1 \qquad (7)$$

In the expression, $R(S > q)$ is the number of values in the amplitude S of $\hat{s}$ that are greater than q.

2.3. *Derivation of the Optimization Algorithm for Quadratic Penalty(α) based on Root Mean Square Error (RMSE)*

The size of the quadratic penalty term(α)is related to the noise content of the signal. The greater the signal noise, the higher the value of the quadratic penalty term. By comparing the performance of VMD (Variational Mode Decomposition) under various noise levels and different values of the quadratic penalty term, a functional relationship between the noise characteristics and the quadratic penalty term can be established.

Signal-to-noise ratio (SNR) is a direct indicator of signal noise. However, since the SNR is often not available for real-world engineering signals, in this paper, the Root Mean Square Error (RMSE) is employed to describe the level of signal noise. The calculation of RMSE S_{rms} is shown in Eq.(8).

$$S_{rms} = \sqrt{\frac{1}{N} \sum_{n=1}^{N} |x_n|^2} \qquad (8)$$

Each VMD decomposition yields three Intrinsic Mode Function (IMF) components: IMF1 represents the noise component, while IMF2 and IMF3 represent the signal components.

Generate a set of signals with increasing signal-to-noise ratios (SNR), and observe the relationship between Root Mean Square Error (RMSE) and SNR, as shown in Fig. 1. It is evident that as the SNR increases, the RMSE gradually decreases, indicating a linear relationship between the two. Thus, the RMSE of the signal can reflect changes in signal noise.

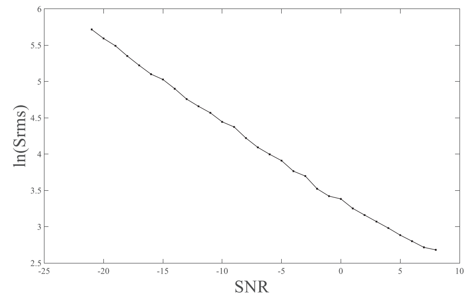

Fig. 1Diagram of the Relationship between Signal to Noise Ratio and $\ln(S_{rms})$

Using 20 sets of signals sampled at 1000 Hz, we aim to determine the effect of different quadratic penalty terms on the decomposition results under varying noise levels. The simulated signal is as shown in Eq.(9).

$$\begin{cases} s_1 = A_1 \sin(2\pi f_1 t) \\ s_2 = A_2 \sin(2\pi f_2 t) \\ s_3 = \eta(q(i)) \\ s(i) = s_1 + s_2 + s_3 \end{cases} \qquad (9)$$

In the equation, $t \in [0,1]$, $f_1 = 15$, $f_2 = 30$, $A_1 = 6$, $A_2 = 12$ where η represents noise with a signal-to-noise ratio (SNR) of $q(i)$, $i = 1, 2 \cdots 20$, $q(i) = -2 \times (0.25 \times i + 1)$. The quadratic penalty term α is set to value $\alpha(j) = j \times 400, j = 1, 2, \cdots 50$, resulting in 50 sets of parameters. Variational Mode Decomposition (VMD) is performed on the generated signals $s(i)$ with the number of decomposition levels set to $k = 3$, and the process is carried out 1000 times in total.

Each VMD decomposition yields three Intrinsic Mode Function (IMF) components: IMF1 represents the noise component, while IMF2 and IMF3 represent the signal components. To determine the effectiveness of the decomposition under different quadratic penalty terms and noise levels, an IMF evaluation metric is constructed. The evaluation metric includes the frequency deviation and amplitude deviation of the separated IMFs, as defined in Eq.(10).

$$\begin{cases} q(i,j) = \sum \left| (f_{IMF_1} - f_1) + (f_{IMF_2} - f_2) \right|_{ij} \\ e(i,j) = \sum \begin{pmatrix} (1 - \left| \dfrac{A_{IMF_1} - A_1}{A_1} \right|) + \\ (1 - \left| \dfrac{A_{IMF_2} - A_2}{A_2} \right|) \end{pmatrix}_{ij} \\ d(i,j) = -10 \times q(i,j) + e(i,j) \end{cases} \quad (10)$$

In the equation, $q(i,j)$ represents the frequency deviation evaluation. If each IMF successfully separates the key frequencies f_1 and f_2, the $q(i,j)$ is zero; otherwise, $q(i,j)$ is greater than zero.

$e(i,j)$ represents the amplitude deviation evaluation. Under the ij-th VMD operation, it is the difference between the amplitude of each IMF's key frequency and the amplitude of the original signal's key frequencies A_1 and A_2. The larger $e(i,j)$ is, the better the signal extraction performance.

$d(i,j)$ is the comprehensive evaluation metric. If the frequencies are not successfully separated, $d(i,j)$ is less than zero; if the frequencies are successfully separated, $d(i,j)$ is greater than zero, and the larger the value, the better the result. The top 20 results with the largest $d(i,j)$ values from each decomposition are selected as the optimal solutions.

Based on the above method, the optimal solutions and the corresponding Root Mean Square (RMS) values under different quadratic penalty terms are obtained, as shown in Fig. 2. It can be observed that as the RMS value increases, the quadratic penalty term corresponding to the optimal solution also increases, indicating a certain relationship between them.

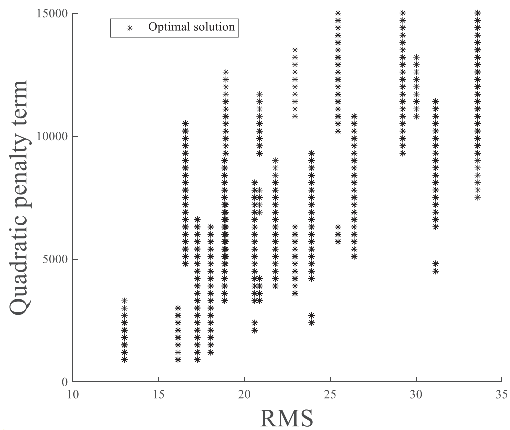

Fig. 2 The relationship between optimal solution and signal variance under different penalty factor

Based on the above calculations, Eq.(11) is proposed as the pre-optimization function for the quadratic penalty term.

$$\alpha = \left\lfloor 2 \times e^{(x/d)} - 1 \right\rfloor \times 1000 \quad (11)$$

In the formula, x represents the Root Mean Square (RMS) value, and d is the shape parameter. If the overall noise in the signal is large, d takes a smaller value; if the overall noise is small, d takes a larger value, within the range of [19, 21], typically set at 20.

3. Constructing the VMD Optimization Method Based on the Pre-Optimization Algorithm

For practical signals with large data volumes, using default parameters for VMD calculations often fails to separate key frequencies, and iterative optimization methods may have low computational efficiency. The VMD optimization method based on the pre-optimization algorithm improves computational efficiency by performing a certain type of spectral segmentation on the amplitudes of key frequencies and calculating the number of decomposition levels and quadratic penalty terms based on the Root Mean Square (RMS) value, thus avoiding iterative computations. The specific calculation steps are as follows (with the process illustrated in Fig. 3).

(i) Apply the Fourier transform to obtain the frequency-domain signal $\hat{s}$.
(ii) Sort the amplitudes of the frequency-domain

signal and compute the minimum amplitude threshold q.

(iii) Count the number of frequencies with amplitudes greater than the minimum amplitude threshold q to determine the number of decomposition levels .

(iv) Compute the RMS value of the input signal and use the quadratic penalty optimization function to determine the required quadratic penalty term α for VMD.

(v) Carry out the VMD using the obtained k and α.

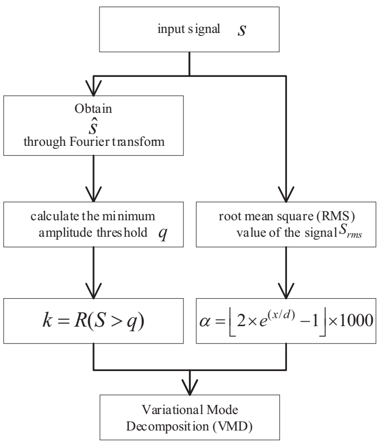

Fig. 3 VMD optimization process based on pre algorithm

4. Simulation Calculation and Analysis

4.1. *Construction of Simulation Signal*

The effectiveness of the optimization algorithm is validated through the use of a simulation signal, taking the cylinder head vibration simulation signal of a certain type of diesel engine as an example. The diesel engine generates a periodic vibration signal due to the sequential firing of its cylinders, which drives the main shaft rotation and also exhibits a vibration signal with a fixed frequency. Additionally, the reciprocating motion of the piston or internal faults produces impact signals. The constructed signal includes deterministic frequency sinusoidal signals s_2(30HZ)、s_3(40HZ) and s_4(80HZ) and periodic impact signals s_1(180HZ). Gaussian white noise(25dBw) is added to the simulation signal, with a sampling frequency of 1024 Hz.

4.2. *The calculation of a fast optimized VMD algorithm based on a pre-processing algorithm*

To determine modal decomposition number k based on the optimization algorithm. First, perform a fast Fourier transform on the simulated signal to obtain its frequency spectrum information, then sort it and calculate the minimum amplitude threshold. The deformation parameter is set to 0.3, and the calculation results are shown in Eq.(13).

$$q = 0.026 + 0.3 \times (15.92 - 0.026) = 4.7942 \quad (12)$$

Based on $k = R(S_m > q)$, we obtain $k = 6$.

Determine the secondary penalty term using the optimization algorithm. Calculate the root mean square value of the signal, as shown in Eq.(14).

$$S_{rms} = \sqrt{\frac{1}{N}\sum_{n=1}^{N} |s_n|^2} = 26.41 \quad (13)$$

Using the formula for penalty factor, calculate the optimized penalty factor with the shape parameter d set to 17. The calculation results are shown in Eq.(15).

$$\alpha = \left\lfloor 2 \times e^{(26.41/20)} - 1 \right\rfloor \times 1000 = 6000 \quad (14)$$

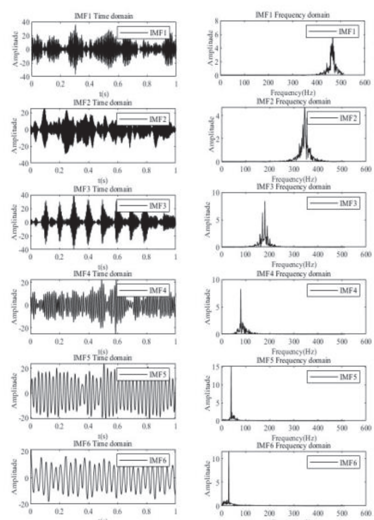

Fig. 4 Decomposition results of optimized VMD algorithm based on pre algorithm

Perform VMD decomposition on the simulated signal S using the optimized values of k and α.

The results are shown in Fig. 4. It can be observed that IMF4 to IMF5 represent the extraction of key signals, where frequencies f_2 , f_3 and f_4 , are extracted as independent components with minimal mixed signals. In terms of the corresponding amplitude extraction for $A_{IMF4} = 8.25$, $A_{IMF5} = 15.18$, $A_{IMF6} = 11.50$, the deviation between the extracted amplitudes and the original signal's frequency amplitudes is controlled within 6%. IMF3 represents an impact signal, and it partially extracts the key frequencies and corresponding amplitudes of the impact signal. Three key frequencies around 180Hz are successfully extracted, but the amplitude deviation for the corresponding key frequencies is 24%. This is due to the large amount of noise added to the original signal and the relatively poor adaptability of the VMD algorithm to impact signals.

4.3. *Comparison of Methods*

To validate the effectiveness of the pre-optimization algorithm, we made a comparison. The compared algorithms include the VMD algorithm with default parameters, the Complete Ensemble Empirical Mode Decomposition with Adaptive Noise (CEEMDAN) algorithm, and the VMD algorithm optimized by Particle Swarm Optimization (PSO), which will be hereafter referred to as PSO-VMD. The focus of the comparison was on the differences in computational efficiency and decomposition effectiveness among the optimization algorithms.

When comparing the accuracy of the signal decomposition of the three algorithms, the primary focuses are whether the decomposed signal components can separate the key frequencies of the input signal and what the amplitude deviation of the separated frequencies is. The comparison results of amplitude deviation (in percentage) are shown in Table 1. It can be seen that the VMD optimization based on the pre-processing algorithm is better at separating signals whose frequencies are closely spaced and has higher accuracy in separating sine signals, outperforming other optimization methods. For impact signals(180Hz), since impact signals exhibit multiple peaks across the entire frequency band in the frequency domain, their characteristics conflict with the VMD algorithm's frequency domain-based signal extraction method, resulting in the amplitudes of the extracted impact signals being higher than the original signal.

Table.1 Comparison of three methods

Frequency (Hz)	Optimized VMD	Default VMD	CEEMDAN
30	4.17%	Not separated	Not separated
40	5.13%	Not separated	Not separated
80	3.00%	3.25%	Not separated
180	15.09%	9.47%	Not separated
Required time(s)	138.93	138.91	Not separated

5. Conclusion

From the simulation results, the optimized VMD algorithm based on the pre-processing algorithm can address the mode mixing issues that occur under the default parameters, reducing the noise components in each IMF without compromising the decomposition accuracy of sine signals. And the optimized VMD algorithm can also avoid the mode mixing and false signal issues while correctly separating impact signals.

With 1,024,000 sampling points, the optimized VMD algorithm based on the pre-processing algorithm and the VMD algorithm with determined parameters are respectively used for decomposition. By comparing the calculation time in Table.1, it is found that the former takes 138.93 seconds, and the latter takes 138.91 seconds. The time difference between the two is relatively small. The time spent on the pre-processing calculation for parameter determination is only 0.02 seconds, accounting for 0.01% of the total calculation time. It can be seen that the pre-processing algorithm occupies a relatively small amount of computation and can optimize the VMD algorithm at a relatively low computational cost.

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