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Parameter Estimation in the SIR Model Using Collocation Method

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In this paper, we present a methodology for estimating the parameters of a system of ordinary differential equations (ODEs) for the SIR model, a critical tool for understanding the dynamics of infectious diseases. The SIR model is essential for predicting outbreak patterns and informing public health interventions, playing a pivotal role in safety analysis. The parameters of the model are estimated from measured data while simultaneously solving the corresponding system of ODEs numerically. Our approach is based on the collocation method, where the solution is expressed as a linear combination of B-spline basis functions and fitted to the data through regression. The square Euclidean measure is used for both regression fitting and minimizing the ODE error. This problem is formulated as a multicriteria optimization task, balancing errors in the model fit and the numerical solution of the ODE system. The entire methodology is implemented in the MATLAB environment. We present numerical results and demonstrate the effectiveness of the approach for parameter estimation in epidemiological models using artificial benchmark datasets.

Keywords: SIR model, epidemiological modeling, parameter estimation, collocation method, regression, ODE.

1. Introduction

In this paper, we address a regression problem over given data, where the considered model is SIR (Susceptible-Infected-Recovered), see Kermack and McKendrick (1927). This model is a fundamental epidemiological model that describes the spread of an infectious disease in a population.

Additionally, this model is used not only in epidemiology but also in cybersecurity and physical security. It helps model the spread of computer viruses, malware, or disinformation in network systems. In security analysis, the SIR model can predict how quickly a threat may spread among vulnerable systems and what strategies can effectively stop or slow its propagation, such as patching vulnerabilities or isolating infected nodes. For more details see, e.g., Zimeras and Diomidous (2018).

The model is based on the division of the population into three groups: S (Susceptible), I (Infected), and R (Recovered). The relationships between the numbers of individuals in each group are formulated as a system of ordinary differential equations (ODEs). Unfortunately, this system does not have an explicit analytical solution for general cases, so this solution cannot be substituted into the cost function of the regression problem, thus making it impossible to solve a simple nonlinear optimization problem with unknown model parameters. Due to this difficulty, the ODEs must be solved numerically, simultaneously solving the regression optimization problem.

In this work, we formulate the problem as a multi-criteria optimization problem. The first criterion represents the regression error with respect to the given data, measured in the least squares sense (see Section 2), which we aim to minimize.

The second criterion corresponds to the error of the numerical solution of the ODEs (see Section 3), and we minimize this error as well. By aggregating these two optimization problems, we obtain the final multicriteria optimization problem (see Section 4), which we solve using subspace iterations (Section 5). The unknowns in this case are the model parameters (i.e., the parameters of the ODEs) and the coefficients in the chosen basis of the corresponding solution with optimal parameters. This section also discusses the numerical solution of individual subproblems. In Section 6, we present results of our implementation in the MATLAB programming environment and discuss the applicability of the proposed solution. The final Section 7 concludes the paper.

2. Least-square regression

Suppose we have measured time-series data, denoted as $\hat{x}_t \in \mathbb{R}^M$, for $t = 0, \ldots, T$, where $M \in \mathbb{N}$ represents the dimensionality of the data (i.e., the number of features), and $T + 1 \in \mathbb{N}$ indicates the total number of time steps. The goal is to construct a model that captures the underlying process responsible for generating the observed data. In other words, we aim to identify a function that characterizes the data generation mechanism. Regression methods are among the most fundamental modeling techniques used for this purpose.

In this context, we assume a parametric model $m_\theta(t)$ with $K \in \mathbb{N}$ parameters $\theta \in \mathbb{R}^K$. This model describes the dependency between time t and the generated state $\hat{x}(t)$, i.e., we assume the approximation $\hat{x}(t) \approx m_\theta(t)$. The regression process involves identifying the parameters of the model, that best fit the observed data in optimal way. The discrepancy between the observed outputs and the theoretical values predicted by the model with parameters θ is quantified using a distance function ρ. The corresponding optimization problem can be formulated as:

$$\theta^* = \arg\min_{\theta} \sum_{t=0}^{T} \rho(\hat{x}_t, m_\theta(t)). \tag{1}$$

To evaluate the magnitude of individual local errors $(\hat{x}_t - m_\theta(t))$, a common choice is the Mean Square Error (MSE)

$$\rho(\hat{x}_t, m_\theta(t)) = (\hat{x}_t - m_\theta(t))^2 \tag{2}$$

and the corresponding regression problem is said to be solved in least-square sense.

Common regression models include linear regression, logistic regression, polynomial regression, ridge regression, lasso regression, elastic net regression, and autoregressive models (see, e.g., Abraham and Ledolter (2006)). These models are widely used across various fields due to their simplicity and effectiveness in modeling relationships within data. The choice of a suitable model in each case depends on the assumptions made about the data under examination and the process that generates this data.

In our research, we focus on processes that can be described by ODEs. Such models are particularly common in physical processes and are widely used in various engineering applications. In this paper, we are interested in a specific application - compartmental models in epidemiology. One of the simplest example could be SI model. It is a basic framework in epidemiology that describes how infectious diseases spread in a population. It divides individuals into two groups: those who are susceptible to infection $s(t)$ and those who are infectious $i(t)$ and can transmit the disease. The system is given by

$$\begin{aligned} \dot{s}(t) &= -\beta s(t) i(t), \\ \dot{i}(t) &= \beta s(t) i(t), \end{aligned} \tag{3}$$

where $\dot{s}(t)$ and $\dot{i}(t)$ represent the time derivatives of $s(t)$ and $i(t)$, respectively.

The model assumes that once an individual becomes infectious, they remain in that state indefinitely, without recovery or immunity. The spread of the disease depends on contact between susceptible and infectious individuals, and the entire population is eventually expected to become infectious. The transmission rate β, representing the likelihood of disease spread per contact, is denoted by the sole parameter of the system, i.e., $K = 1$ and $\theta = \beta$. To eliminate the influence of population size, it is assumed that $s(t)$ and $i(t)$ represent the proportions of the population rather than absolute numbers. This assumption is expressed by the condition

$$\forall t : s(t) + i(t) = 1. \tag{4}$$

In the case of this simple model, it is easy to derive the analytical solution

$$s(t) = \frac{1 - \hat{i}_0}{1 - \hat{i}_0 + \hat{i}_0 e^{\beta t}}, \quad i(t) = \frac{\hat{i}_0 e^{\beta t}}{1 - \hat{i}_0 + \hat{i}_0 e^{\beta t}}, \tag{5}$$

where $\hat{i}_0$ is the initial proportion of infectious individuals at $t = 0$.

To apply this model to regression problem, we substitute the explicit solution (5) to regression fitting problem (1) with MSE (2). We get optimization problem

$$\beta^* = \arg\min_\beta \sum_{t=0}^{T} \left(\left(\hat{s}_t - \frac{1 - \hat{i}_0}{1 - \hat{i}_0 + \hat{i}_0 e^{\beta t}} \right)^2 + \left(\hat{i}_t - \frac{\hat{i}_0 e^{\beta t}}{1 - \hat{i}_0 + \hat{i}_0 e^{\beta t}} \right)^2 \right), \tag{6}$$

where $\hat{x}_t = [\hat{s}_t, \hat{i}_t] \in \mathbb{R}^2$ are given measured data in $t = 0, \dots, T$. The problem (6) can be solved by suitable numerical method. In our case, we are using Matlab and we can solve the problem using *fmincon* optimizer.

Although the previously described approach yields the desired results, its application heavily relies on the existence of an explicit solution, i.e., an explicit model, which allows for the determination of the objective function with the unknown parameters. In the case of more complex models without a known explicit solution, one must solve the ODEs numerically. In such cases, it is not possible to directly establish a relationship between the model and its parameters. Therefore, we cannot directly substitute this solution into the regression problem to obtain the objective function with the unknown model parameters. For a review of methods addressing such problems, see Brewer et al. (2007).

The example is SIR model. It is an extension of the SI model (3) that incorporates an additional state, recovered $r(t)$, to account for individuals who have either recovered from the infection or gained immunity. An infectious individual eventually recovers, and is no longer able to spread the disease. This transition is governed by a recovery rate γ. The ODE system is given by

$$\begin{aligned} \dot{s}(t) &= -\beta s(t) i(t), \\ \dot{i}(t) &= \beta s(t) i(t) - \gamma\, i(t), \\ \dot{r}(t) &= \gamma\, i(t) \end{aligned} \tag{7}$$

and assumption (4) has a form

$$\forall t : s(t) + i(t) + r(t) = 1. \tag{8}$$

In practice, the SIR model is typically solved numerically using methods such as Runge-Kutta or similar approaches, as discussed in Rafei et al. (2007). In such cases, the parameters $\theta = [\beta, \gamma] \in \mathbb{R}^2$ must be known and fixed beforehand. While an analytical solution to the SIR model exists, it is limited to specific cases; see, for example, Kröger and Schlickeiser (2020).

However, even when the model parameters are not known, the numerical solution of ODEs can be incorporated into the so-called *shooting* method. The method begins by guessing initial values for the unknown parameters and solving the corresponding system (7) numerically. The resulting solution is then compared with the observed data by evaluating the objective function of the regression problem (1), and the parameters are adjusted to minimize the discrepancy between the model and the data. This process is repeated iteratively, adjusting the parameters each time to improve the model's fit to the data. This approach can be interpreted as 0-order optimization, where only the values of the objective function are used in the minimization process.

In our paper, we present an alternative method, which uses collocation method.

3. Collocation method

The collocation method involves approximating the solution of the ODE by a function from a chosen set of basis functions and ensuring that the approximated solution satisfies the ODE at specific points called collocation points. Each component of $x(t) \in \mathbb{R}^M$ is approximated by

$$x_m(t) \approx \sum_{i=-1}^{T+1} \phi_i(t) C_{i,m}, \tag{9}$$

where $C \in \mathbb{R}^{T+3,M}$ are coefficients of approximated functions in used basis and ϕ_i are basis functions. In our analysis, we employ cubic B-spline functions, see Fig. 1. Such an approach is common in the numerical solution of differential equations, widely employed in methods like the Finite Element Method, see Reddy (2006), or in the application of cubic splines for solving mechanical problems, see Bobková and Pospíšil (2021).

Since the derivative is linear mapping, the derivative of the solution is approximated by

$$\dot{x}_m(t) \approx \sum_{i=-1}^{T+1} \dot{\phi}_i(t) C_{i,m}. \tag{10}$$

Let us denote the collocation points as $n = 0, \dots, N$, where $N + 1 \in \mathbb{N}$ is a number of collocation points. For simplicity, we arrange the basis functions at the collocation points into a matrix $Q \in \mathbb{R}^{N+1,T+3}$ and the derivatives of basis functions into $\dot{Q} \in \mathbb{R}^{N+1,T+3}$. We require that the ODE equations are satisfied at these collocation points.

In the case of the SIR model, we have data dimension $M = 3$, i.e., $x(t) = [s(t), i(t), r(t)] \in \mathbb{R}^3$. We denote the unknown coefficients of the approximated functions $s(t)$, $i(t)$, and $r(t)$ by c^{S}, c^{I}, and $c^{\mathrm{R}} \in \mathbb{R}^{T+3}$, respectively and introduce a matrix of all coefficient by $C = [c^{\mathrm{S}}, c^{\mathrm{I}}, c^{\mathrm{R}}] \in \mathbb{R}^{T+3,M}$. The corresponding discretization of the system (7) is given (for all collocation points $n = 0, \dots, N$) as:

$$\begin{aligned}
\langle \dot{Q}_{n,:}, c^{\mathrm{S}} \rangle &= -\beta \langle Q_{n,:}, c^{\mathrm{S}} \rangle . \langle Q_{n,:}, c^{\mathrm{I}} \rangle, \\
\langle \dot{Q}_{n,:}, c^{\mathrm{I}} \rangle &= \beta \langle Q_{n,:}, c^{\mathrm{S}} \rangle . \langle Q_{n,:}, c^{\mathrm{I}} \rangle - \gamma \langle Q_{n,:}, c^{\mathrm{I}} \rangle, \\
\langle \dot{Q}_{n,:}, c^{\mathrm{R}} \rangle &= \gamma \langle Q_{n,:}, c^{\mathrm{I}} \rangle,
\end{aligned} \tag{11}$$

where $\langle\, .\, ,\, .\, \rangle$ denotes the standard dot product and $Q_{n,:}$ denotes the n-th row of matrix Q.

If we solve this system in the least-squares sense, we obtain a problem that always has a solution, unlike the original system (11), which may lack a solution because it is only an approximation (9) and does not necessarily satisfy the system exactly at the collocation nodes.

This least-square solution is formulated as a minimization problem

$$C^* = \arg \min_{C \in \Omega_C} \psi(C) \tag{12}$$

with objective function

$$\begin{aligned}
\psi(C) = \sum_{n=0}^{N} \Big(& \langle \dot{Q}_{n,:}, c^{\mathrm{S}} \rangle + \beta \langle Q_{n,:}, c^{\mathrm{S}} \rangle . \langle Q_{n,:}, c^{\mathrm{I}} \rangle \Big)^2 \\
& + \Big(\langle \dot{Q}_{n,:}, c^{\mathrm{I}} \rangle - \beta \langle Q_{n,:}, c^{\mathrm{S}} \rangle . \langle Q_{n,:}, c^{\mathrm{I}} \rangle \\
& + \gamma \langle Q_{n,:}, c^{\mathrm{I}} \rangle \Big)^2 + \Big(\langle \dot{Q}_{n,:}, c^{\mathrm{R}} \rangle - \gamma \langle Q_{n,:}, c^{\mathrm{I}} \rangle \Big)^2
\end{aligned} \tag{13}$$

and the feasible set (which represents condition (8)) is given by

$$\Omega_C = \{ c^{\mathrm{S}}, c^{\mathrm{I}}, c^{\mathrm{R}} \in \mathbb{R}^{T+3} \,|\, Qc^{\mathrm{S}} + Qc^{\mathrm{I}} + Qc^{\mathrm{R}} = \mathbb{1} \}, \tag{14}$$

where $\mathbb{1} \in \mathbb{R}^{N+1}$ is a vector of ones. The initial condition at $t = 0$ can be easily enforced by additional equality conditions

$$Q_{0,:} c^{\mathrm{S}} = \hat{s}_0, \quad Q_{0,:} c^{\mathrm{I}} = \hat{i}_0, \quad Q_{0,:} c^{\mathrm{R}} = \hat{r}_0. \tag{15}$$

With known parameters β, γ, one can obtain an approximated solution of the discretized system (7) solving the optimization problem (12).

4. Multicriteria aggregation

Let us return to the original regression problem (1). The model $m_\theta(t)$ is now given by the numerical solution of ODEs given by approximation (9). In this case, we compare the approximation of the solution in the measured $t = 0, \dots, T$ with the given data and minimize the MSE error.

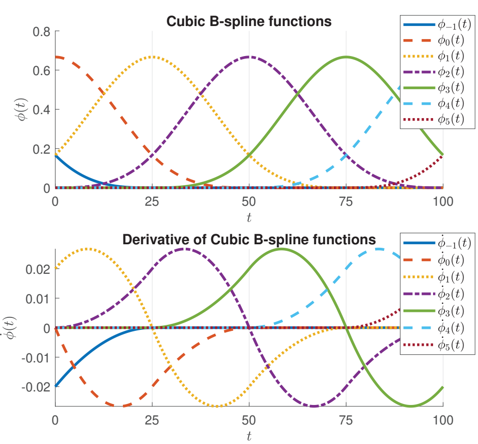

Fig. 1. Cubic B-spline functions and derivatives.

We define a matrix $\hat{Q} \in \mathbb{R}^{T+3,T+3}$, constructed from the basis functions evaluated at the data points. The cost function of the regression problem (1) then takes the form

$$\varphi(C) = \sum_{t=0}^{T} \left(\hat{s}_t - \langle \hat{Q}_{t,:}, c^{\mathrm{S}} \rangle\right)^2 + \left(\hat{i}_t - \langle \hat{Q}_{t,:}, c^{\mathrm{I}} \rangle\right)^2 + \left(\hat{r}_t - \langle \hat{Q}_{t,:}, c^{\mathrm{R}} \rangle\right)^2. \tag{16}$$

The goal is to minimize both function (13) and (16) simultaneously. For this purpose, we will use multicriteria optimization, see, e.g., Ehrgott (2013).

We introduce aggregation, which combines these objectives into a single scalar function using a weighted sum, with normalization applied for comparability. The resulting aggregated problem is formulated as

$$[\theta^*, C^*] = \arg \min_{C \in \Omega_C} f_\alpha(C, \theta) \tag{17}$$

with objective function f_α given by

$$f_\alpha(C, \theta) = \frac{\alpha}{3(N+1)} \psi(C, \theta) + \frac{1-\alpha}{3(T+1)} \varphi(C) \tag{18}$$

and the feasible set given by (14). The introduced weighting accounts for the number of squared terms in the functions, and the parameter $\alpha \in (0, 1)$ controls the trade-off between the two objective functions, defining their relative importance. By varying α, different compromises can be explored, ranging from fully prioritizing one function to balancing both.

5. Numerical solution

To solve the proposed optimization problem (17), we employ the subspace algorithm. The problem is solved by alternately fixing one variable and minimizing the objective function with respect to the other. Starting with an initial guess, the method iteratively updates each variable until convergence, simplifying the optimization process by reducing it to a series of problems with one variable, see Alg. 1. It can be easily shown that the algorithm generates a sequence with non-increasing objective function values, see, e.g., Gerber et al. (2020).

Alg. 1: Subspace algorithm

Choose initial approximation $\theta^{\langle 0 \rangle} \in \mathbb{R}^2$
Set algorithm tolerance $\mathrm{tol} \geq 0$
Set initial $f_\alpha^{\langle 0 \rangle} = \infty$
Set iteration counter $\mathrm{it} = 0$
repeat

$$C^{\langle \mathrm{it}+1 \rangle} = \arg \min_{C \in \Omega_C} f_\alpha(C, \theta^{\langle it \rangle})$$

$$\theta^{\langle \mathrm{it}+1 \rangle} = \arg \min_{\theta \in \mathbb{R}^2} f_\alpha(C^{\langle \mathrm{it}+1 \rangle}, \theta)$$

$$f_\alpha^{\langle \mathrm{it}+1 \rangle} = f_\alpha(C^{\langle \mathrm{it}+1 \rangle}, \theta^{\langle \mathrm{it}+1 \rangle})$$

$$\mathrm{it} = \mathrm{it} + 1$$

until $|f_\alpha^{\langle \mathrm{it} \rangle} - f_\alpha^{\langle \mathrm{it}-1 \rangle}| < \mathrm{tol}$;

In the case of the θ-problem, when the variable C is fixed, the system of ordinary differential equations (7) becomes linear in the parameters, making the corresponding objective function (18) quadratic in variable θ, see theory of least-square solution of the system of linear equations, e.g., Nocedal and Wright (2003) or Boyd and Vandenberghe (2004). By applying the necessary optimality conditions, we derive a system of linear equations $A\theta = b$ with the symmetric positive definite system matrix

$$A = \begin{bmatrix} 2 \sum_{n=0}^{N} s_n^2 i_n^2 & -\sum_{n=0}^{N} s_n i_n^2 \\ -\sum_{n=0}^{N} s_n i_n^2 & 2 \sum_{n=0}^{N} i_n^2 \end{bmatrix} \tag{19}$$

and right hand-side vector from the image of A given by

$$b = \begin{bmatrix} \sum_{n=0}^{N} (\dot{i}_n - \dot{s}_n) s_n i_n \\ \sum_{n=0}^{N} (\dot{i}_n + \dot{r}_n) i_n \end{bmatrix} \tag{20}$$

and consequently, the solution of θ problem is reduced to the solution of this system of linear equations. This system has always solution.

In the case of the C-problem, when the variable θ is fixed, the situation is more complex because the ODE system is nonlinear in the functions $s(t), i(t), r(t)$, and consequently, the system (11) exhibits additional nonlinearities in C.

Since we are using MATLAB, we solve the

problem with the *fmincon* optimizer. To enhance convergence, we supply the algorithm with a vectorized implementation of the derived gradient.

6. Results

We implemented the presented methodology in a MATLAB environment and applied it to synthetic data. Specifically, we began by selecting a set of true parameters θ_{true} for the system and used them to numerically solve the ODEs using the Runge-Kutta method, generating a clean dataset X_{true} that accurately represents the underlying dynamics. To simulate real-world measurements, we added random noise to the numerical solution, mimicking measurement inaccuracies typically encountered in practice. The noisy dataset was then used as input for our parameter estimation framework. The objective was to reconstruct the original parameters by minimizing the discrepancies between the noisy measurements and the modeled data. This process not only tests the robustness of our approach in handling noisy inputs but also provides a realistic evaluation of its effectiveness in practical scenarios.

In the benchmark, we consider a hypothetical disease with parameters $\beta_{\text{true}} = 0.6$ and $\gamma_{\text{true}} = 0.05$, with the initial condition $[\hat{s}_0, \hat{i}_0, \hat{r}_0] = [0.99, 0.01, 0]$ over the interval $t \in [0, 100]$. Measurements are taken at an equidistant time grid $t = 0, 1, \ldots, 100$, and the problem is solved using the Runge-Kutta method. This solution is treated as the exact solution X_{true}.

To simulate the data from real-world measurements, we add additive noise with a normal distribution $\epsilon \sim \mathcal{N}(0, \sigma)$ and project the resulting values onto the interval $[0, 1]$, i.e.,

$$\begin{aligned}\hat{s}_t &= P_{[0,1]}(s_{\text{true},t} + \epsilon_{s,t}), \quad \epsilon_{s,t} \sim \mathcal{N}(0, \sigma)\\ \hat{i}_t &= P_{[0,1]}(i_{\text{true},t} + \epsilon_{i,t}), \quad \epsilon_{i,t} \sim \mathcal{N}(0, \sigma)\\ \hat{r}_t &= P_{[0,1]}(r_{\text{true},t} + \epsilon_{r,t}), \quad \epsilon_{r,t} \sim \mathcal{N}(0, \sigma)\end{aligned} \tag{21}$$

where the projection can be computed using

$$P_{[0,1]}(x) = \max\{0, \min\{1, x\}\}. \tag{22}$$

We use 20 times more collocation points than measurements; specifically, we include the same collocation points as the measurement points,

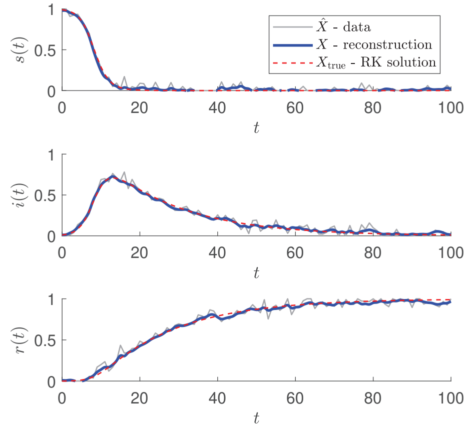

Fig. 2. Solution of the benchmark with noise parameter $\sigma = 0.05$ and aggregation parameter $\alpha = 0.5$.

along with an additional 18 collocation points between each pair of measurement points. The final solutions for the noise parameter $\sigma = 0.05$ and the aggregation parameter $\alpha = 0.5$ are presented in Fig. 2. In this case, the solution for parameters is $\beta^* = 0.6168$ and $\gamma^* = 0.0518$. The difference from original parameters is $\|\theta_{\text{true}} - \theta^*\|_2 = 0.0169$.

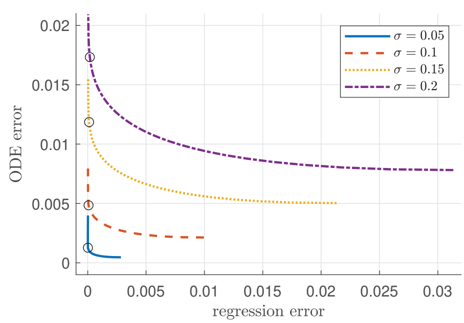

Fig. 3. The L-curve for various values of the noise parameter. The *regression error* is given by the value $\psi(C^*, \theta^*)/(3(N+1))$, and the *ODE error* is given by the value $\varphi(C^*, \theta^*)/(3(T+1))$.

We solve the problem for various values of the noise parameter σ and several values of the aggregation parameter $\alpha \in [0, 1]$. The resulting

L-curves (see Hansen and O'Leary (1993)) are presented in Fig. 3. Each point on a curve corresponds to a specific solution for a given noise parameter and a particular value of the aggregation parameter. When the aggregation parameter is small, the minimization process focuses primarily on reducing the ODE error, with the influence of the provided measured data being almost negligible. Conversely, for larger values of the aggregation parameter, the model relies more on the data, effectively suppressing the numerical error of the ODE solution.

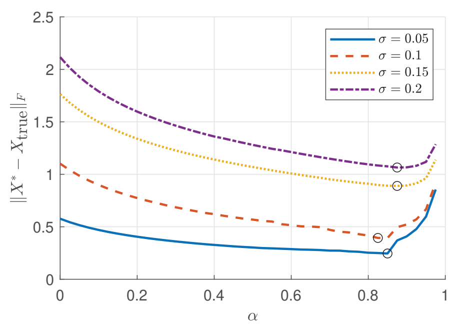

Fig. 4. Distance between the original Runge-Kutta solution and the optimization problem solution (18) for various α. The highlighted solution minimizes the distance and matches the one in Fig. 3 and Tab. 1.

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The highlighted value (black circle) in Fig. 3 represents the optimal value, chosen to minimize the distance between the Runge-Kutta solution (without noise) X_{exact} and the reconstructed solution $X = Qc$, as shown in Fig. 4. The optimal parameters corresponding to these regularization values are presented in Table 1.

In practical scenarios, the absolute error cannot be utilized because the true solution is unknown. However, the previous example demonstrates that the L-curve and the Pareto-optimal point offer a viable approach for estimating the optimal value. An alternative approach, particularly when sufficient data is available, is the cross-validation method; see Stone (1974). Based on the provided results, we recommend using the optimal value $\alpha = 0.85$. For this value, we solve the problem

Table 1. The values of optimal parameters corresponding to highlighted solution presented in Fig. 3 and Fig. 2.. The original values used to generate X_{true} are $\beta_{\text{true}} = 0.6$ and $\gamma_{\text{true}} = 0.05$.

σ	α	β^*	γ^*
0.05	0.85	0.6026	0.0484
0.1	0.825	0.6039	0.0512
0.15	0.875	0.4511	0.0473
0.2	0.875	0.3523	0.0516

with various noise parameters σ to evaluate the denoising capability of the proposed method, as shown in Fig. 5. In this analysis, data with 10 random additive noise realizations is used, and we report the average error, minimal error, and maximal error values.

7. Conclusion

In this paper, we introduced a methodology for parameter estimation in the SIR model using a collocation-based approach. By representing the solution as a linear combination of B-spline basis functions and employing regression techniques, we simultaneously minimized the ODE error and the regression fitting error, addressing the problem as a multicriteria optimization problem. The implementation in MATLAB demonstrated the feasibility and effectiveness of this method, providing accurate parameter estimates for epidemiological models. Our results highlight the potential of this approach for improving the understanding of disease dynamics and supporting public health interventions. Future work could extend this methodology to more complex models and real-world datasets.

The MATLAB code reproducing the results presented in this work will be shared upon reasonable request.

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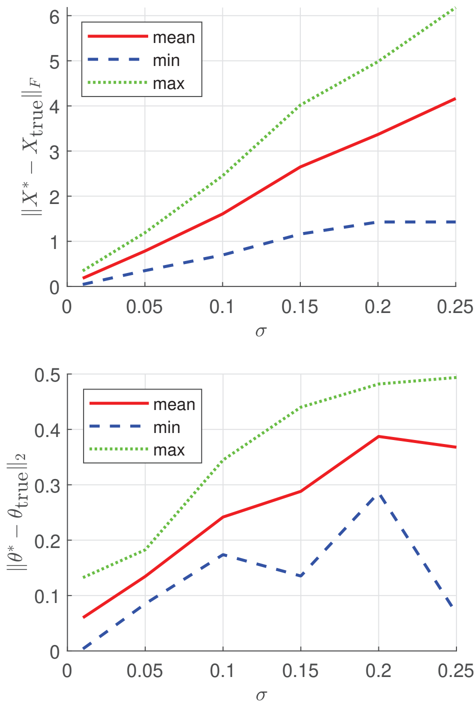

Fig. 5. The distance between original Runge-Kutta solution with true parameters and the solution obtained by solving optimization problem (18) (top) and the distance between true parameters and recoved parameters (bottom) for aggregation parameter $\alpha = 0.85$ for several noise parameters. We processed data with 10 realization of additive noise.

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