Proceedings of the 35th European Safety and Reliability & the 33rd Society for Risk Analysis Europe Conference
Edited by Eirik Bjorheim Abrahamsen, Terje Aven, Frederic Bouder, Roger Flage, Marja Ylönen
 Published by Research Publishing, Singapore.
doi: 10.3850/978-981-94-3281-3_ESREL-SRA-E2025-P6507-cd

Physics-informed Neural Networks used for Structural Health Monitoring in Civil Infrastructures: State of Art and Current Challenges

Vaibhav Gupta

Data Engineering, Helmut-Schmidt-University/University of the Federal Armed Forces Hamburg, Germany. E-mail: guptav@hsu-hh.de

Francesca Marsili

Chair of Engineering Material and Building Preservation, Helmut-Schmidt-University/University of the Federal Armed Forces Hamburg, Germany. E-mail: francesca.marsili@hsu-hh.de

Sylvia Kessler

Chair of Engineering Material and Building Preservation, Helmut-Schmidt-University/University of the Federal Armed Forces Hamburg, Germany. E-mail: sylvia.kessler@hsu-hh.de

Maria Maleshkova

Data Engineering, Helmut-Schmidt-University/University of the Federal Armed Forces Hamburg, Germany. E-mail: maleshkm@hsu-hh.de

Structural Health Monitoring (SHM) is a fundamental task in the life-cycle assessment and management of civil infrastructures, specifically dams, locks, bridges, and roads. It aids in cost reduction, facilitates the early detection of degradation processes, damages, and structural deficiencies, ensures timely maintenance, and provides early risk warnings. SHM is directly related to the concept of Digital Twin, which is usually defined as a virtual replica of the physical asset. On the one hand, SHM provides the data for the implementation of digital twins, while on the other hand, digital twins can improve the effectiveness of SHM and support data analysis. Together, they represent a powerful combination for managing and maintaining critical infrastructure. A hybrid approach has become increasingly established in recent years, which comprises a combination of physics-based models and data-driven techniques. This approach mitigates the constraints of both models to align the digital twin's behavior more closely with that of the corresponding physical asset. This study explores the hybrid modeling framework known as physics-enhanced machine learning for forecasting potential structural damage. Among the various hybrid modeling approaches, we focus on physics-informed neural networks (PINNs) and their applications in SHM of civil infrastructures. This study provides a comprehensive classification of research employing the PINNs architecture and critically evaluates its associated limitations. Additionally, we explore advanced deep learning architectures that can integrate PINNs within their computational frameworks to enhance SHM performance by addressing its limitations. This work is a foundational reference for understanding state-of-the-art advancements in PINNs for SHM applications.

Keywords: Structural Health Monitoring, Physics Enhanced Machine Learning, Civil Infrastructures, Physics Informed Neural Networks, Graphical Neural Network, Transformers, Neural Operators

1. Introduction

Within the context of the fourth industrial revolution, artificial intelligence, the internet of things, edge computing, and mixed reality are crucial technologies facilitating the deployment of digital twins (DTs). By integrating real-world data with computer modeling and simulation, the DT connects the physical and digital realms, creating an effective tool for virtual sensing of physical assets Drakoulas et al. (2023). Digital twins in civil engineering can be used in different contexts like structural identification, predictive maintenance, etc. One of the key applications includes Structural Health Monitoring (SHM) of civil structures like bridges, dams, roads, etc. According to Farrar and Worden (2012), SHM refers to implementing a damage detection strategy for aerospace, civil, or mechanical engineering infrastructure. DGZfP (2022) describes SHM as the long-term monitoring of an infrastructure system and, in particular, its performance in terms of safety, durability, and serviceability by permanently installing sensors on the structure to detect long-term changes. Damage detection involves five hierarchical steps Rytter (1993): identifying damage, locating it, determining its type, assessing its severity, and predicting the remaining service life. An SHM system uses data analysis algorithms to answer these questions. Unsupervised learning can address the first two steps, while the others require supervised learning, physical models, and probabilistic methods Farrar and Worden (2012). Both data-driven models, and physics-based models present deficiencies: (1) physics-based models require detailed knowledge of the physics behind the problem under assessment, which is often missing; (2) data-driven models require data from the damaged state of the structure, which is usually unavailable. Over time, various approaches have been proposed to integrate both models to mitigate their drawbacks. A hybrid approach has emerged in recent years, aiming to combine the best features of model and data-based SHM, which is referred to as Physics- Enhanced Machine Learning (PEML) (Cicirello (2024)).

1.1. *Physics- Enhanced Machine Learning*

PEML involves developing hybrid physics and data-driven models to enhance their predictive and decision-making performance for complex dynamic structures. These hybrid models integrate advanced computational models and data with domain and prior knowledge, which can be mathematical, physical, observational, or empirical. Moreover, first principles and appropriate biases (e.g., physics constraints, model form) are also integrated in the hybrid model. In Physics-Enhanced Machine Learning (PEML), bias refers to guiding principles or constraints incorporated into a machine learning model to prioritize specific solutions over others. These biases help restrict the space of admissible solutions, ensuring that the model converges to physically plausible and generalizable solutions rather than overfitting to limited data. The role of biases in PEML is primarily positive, as they enhance model generalization, accelerate learning, and integrate physical laws into the learning process Cicirello (2024).

Based on the paper, Cicirello (2024), key components of PEML include four Physics and Domain Knowledge Biases: Observational Bias, Learning Bias, Inductive Bias, and Model Form Bias. PEML strategies can be grouped as summarised in Fig. 1. This includes Physics Guided ML, Physics Informed ML, and Physics Encoded ML Faroughi et al. (2023); Cicirello (2024).

Physics Guided ML: This strategy uses a detailed physics-based model as a foundation, guiding the learning process with small-to-medium amounts of informative data. The primary focus is identifying latent parameters, improving predictions, and refining physics-based models with data-driven insights.

Physics Informed ML: This is a data-driven approach where a machine learning model is developed and constrained using physics-based biases. These biases – such as conservation laws, symmetries, and physical constraints – restrict the solution space, ensuring physically consistent and generalizable predictions.

Physics Encoded ML: This can be classified as a hybrid approach that explicitly combines physics-based and data-driven models. It is used when the physics model is incomplete, and data can fill in the gaps by learning missing components or correcting uncertainties in known physics.

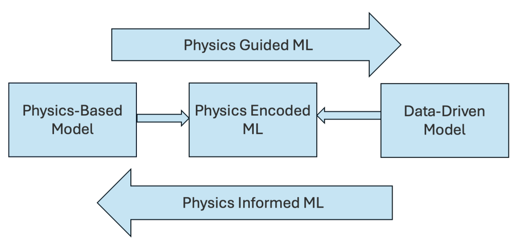

Fig. 1. Components of PEML

This study focuses on Physics Informed Neural Networks (PINNs), a class that comes under the definition of Physics Informed ML (Cicirello (2024); Haywood-Alexander et al. (2024)). In the examined PEML techniques for SHM, PINNs have predominantly been emphasized in research about civil structures and have been substantially incorporated into predictive algorithm frameworks in the last five years. In this study, we review a selection of research works incorporating the PINNs framework in their experiments, though the list is not exhaustive. Based on their application within computational architectures, we categorize these studies to provide a structured understanding of their implementation. This paper highlights the limitations of PINNs and lists alternative deep-learning frameworks that can effectively incorporate PINNs, offering a valuable starting point for researchers seeking to integrate PINNs into their experiments.

The **primary objectives** of this paper are to 1) analyze the architecture of PINNs, 2) categorize studies that integrate the PINNs framework within their computational architectures, 3) identify key challenges associated with PINNs, and 4) explore alternative deep learning architectures that can incorporate PINNs to enhance SHM performance.

The structure of this paper is as follows: Chapter 2 provides an in-depth discussion of the architecture of PINNs. In Chapter 3, we classify the reviewed studies based on their implementation within computational architectures. We then outline the limitations of PINNs and propose alternative deep learning architectures capable of embedding the PINNs framework to address these challenges. In Chapter 4, we summarize our findings, followed by the conclusion presented in Chapter 5.

2. Physics-Informed Neural Networks (PINNs)

PINNs first appear in the merged version of the paper Raissi et al. (2019), introducing a new class of data-driven solvers. They represent a novel approach in scientific machine learning, designed to tackle problems involving partial differential equations (PDEs) using automatic differentiation Cuomo et al. (2022). PINNs are deep learning frameworks that incorporate physics-based knowledge, such as ordinary differential equations (ODEs), PDEs, or constraints, into neural network architectures Al-Adly and Kripakaran (2024). To understand the architecture of the PINNs framework and how physical behavior is incorporated into the framework, here we take an example of a damped pendulum with initial and boundary conditions.

The first component of the framework constitutes the **Neural Network (NN) Representation**. Consider a deep neural network (Eq. 1) that models the function:

$$u_{NN}(x,t;\Theta) : \phi \times [0,T] \mapsto \mathbb{R} \tag{1}$$

where the input $(x,t) \in \phi \times [0,T]$ maps to the output $u_{NN}(x,t;\Theta) \in \mathbb{R}$ through a nonlinear transformation via neural network layers, parametrized by Θ.

This network consists of three layers: **Input Layer**, **Hidden Layer** and **Output Layer**, as presented in the Eq. (2), Eq. (3) and Eq. (4).

$$NN^0 = (x,t) \tag{2}$$

$$NN^l = a(W^l NN^{l-1} + b^l) \tag{3}$$

where $a(\cdot)$ is the activation function, and W^l, b^l are the weights and biases at layer l.

$$NN^L = W^L NN^{L-1} + b^L \tag{4}$$

where L is the total number of layers.

This neural network is then equipped with the physical dynamics of the system. Here, the **Physics Governing Equation** of system (see Eq. 5) is presented, which is a second order ODE.

$$y''(t) + 2\beta y'(t) + \omega_0^2 y(t) = 0 \tag{5}$$

where: $y(t)$ is the angular displacement, β is the damping coefficient, and ω_0 is the natural frequency of the system.

To ensure the neural network solution satisfies physical constraints, we impose the following initial conditions (Eq. 6):

$$y(0) = y_0, \quad y'(0) = y'_0 \tag{6}$$

where y_0 is the initial displacement and y'_0 is the initial velocity.

Therefore, to incorporate the physics governing equation in the neural network, the resultant loss function consists of three main components: the data loss L_d, the boundary condition loss L_b, and the residual loss L_r for the governing equation.

The total loss function (Eq. 7) is the linear combination of the three loss functions.

$$L(X;\Theta) = \lambda_1 L_r(X_r;\Theta) + \lambda_2 L_b(X_b;\Theta) + \lambda_3 L_d(X_d;\Theta) \quad (7)$$

here λ_1, λ_2 and λ_3 are the assigned weights to control the importance of each loss functions. The mathematical formulation of the three loss function is presented in Eq. (8), Eq. (9) and Eq. (10) below.

$$L_b(X_b;\Theta) = \frac{1}{|X_b|} \sum_{(x_b,t_b=0)} |u_{NN}(x_b,t_b;\Theta) - y_0|^2 + \frac{1}{|X_b|} \sum_{(x_b,t_b=0)} \left| \frac{\partial}{\partial t} u_{NN}(x_b,t_b;\Theta) - y'_0 \right|^2 \quad (8)$$

$$L_r(X_r;\Theta) = \frac{1}{|X_r|} \sum_{j=1}^{|X_r|} \left| \frac{\partial^2}{\partial t^2} u_{NN}(x_j,t_j;\Theta) + 2\beta \frac{\partial}{\partial t} u_{NN}(x_j,t_j;\Theta) + \omega_0^2 u_{NN}(x_j,t_j;\Theta) \right|^2. \quad (9)$$

$$L_d(X_d;\Theta) = \frac{1}{|X_d|} \sum_{(x_d,t_d)} |u_{NN}(x_d,t_d;\Theta) - u_d|^2 \quad (10)$$

X_b, X_r and X_d are sets of points where boundary conditions are enforced, the governing differential equation is evaluated to ensure physics compliance, and observed or synthetic data is used to enhance real-world accuracy, respectively.

The goal of the PINNs architecture is to learn optimal parameters Θ such that the approximated solution u_{NN} minimizes the total error, while satisfying physical constraints. The next sections explore the incorporation of PINNs in the context of SHM in civil structures.

3. Application of PINNs in SHM

The evolution of PINNs and frameworks that incorporate PINNs as a component in SHM has demonstrated significant advancements, particularly in accuracy, robustness, scalability, and real-time applicability. In this section, we classify existing studies based on their implementation of PINNs within computational architectures. Additionally, we analyze their limitations and list alternative deep learning architectures that can incorporate PINNs in their framework to overcome these challenges.

3.1. *Categorization for PINNs-Based SHM Architectures*

Based on the implementation of PINNs in computational architecture, we have divided the studies into three broad categories: PINNs as physics-controlled solvers, PINNs embedded within Deep Learning Architecture, and PINNs in Hybrid Learning and System Identification.

3.1.1. *PINNs as Physics-Constrained Solvers*

These studies are grouped in this category as they employ PINNs primarily as numerical solvers for forward or inverse problems, where the focus is on enforcing physical constraints in solving PDEs related to SHM. The inverse problem is defined as the problem of automated damage detection from measured sensor data Yuan et al. (2020).

As per the reviewed studies, Yuan et al. (2020) is one of the first to introduce and incorporate Physics-Informed Learning in the architecture for forward and inverse problems. They incorporate physics-governing equations with Dirichlet and high-order boundary conditions in the loss function of an Artificial Neural Network (ANN) for the inverse problem. The goal is to reconstruct the entire displacement field of the beam from sparse measurements. In the subsequent year, Lai et al. (2021) presents Physics-Informed Neural ODEs (PiNODE), in which Neural ODEs are recast as a two-level representation with a physics-informed term (prior physical knowledge of the system) and a discrepancy term (from a Feedforward Neural Network) in the residual minimization process. They also propose using Sparse Identification of Nonlinear Dynamical Systems (SINDy). This ML technique identifies governing equations of a dynamical system using a sparse set of basis functions, ensuring that only the most relevant terms are retained. The term sparse implies that most potential functions in the model are discarded, retaining only the minimal essential terms to characterize the system's behavior effectively. Here, PINNs apply to solving governing equations of vibration modes in structural dynamics. Al-Adly and Kripakaran (2024) examines how PINNs solve Kirchhoff–Love plate bending problems under variable temporal loading. This study applies PINNs to model the bending behavior of Kirchhoff–Love plates in structural analysis. Here, PINNs act as numerical solvers, constrained by the governing PDEs of plate deformation.

3.1.2. *PINNs embedded within Deep Learning Architecture*

PINNs architecture's adaptability was enhanced in 2022 when the PINNs were coupled with deep learning architecture. A deep learning architecture is a structured framework of multiple neural network layers designed to automatically extract, learn, and process hierarchical features from data for classification, prediction, and decision-making tasks. Lai et al. (2022) proposes a framework called Neural Modal Ordinary Differential Equations (Neural Modal ODEs), which couples a dynamical variational autoencoder with a Physics-Informed Neural ODEs (PiNODE) scheme. Similarly, Ni et al. (2022) proposes a convolutional neural network to reconstruct structural response under rare events. The innovation encompass two key aspects: a multi-end autoencoder with skip connections and a physics-based loss function derived from the dynamic equilibrium equation to guide the training process toward physically consistent solutions. They use skip connections, as these allow information to bypass certain layers in a neural network, preventing vanishing gradients and improving feature retention.

3.1.3. *PINNs for Hybrid Learning & System Identification*

PINNs are increasingly included in hybrid learning frameworks, integrating physical constraints with state-space models, Bayesian inference, and probabilistic modeling. Haywood-Alexander and Chatzi (2023) highlights this hybrid method by merging PINNs with state-space models and adaptive loss balancing approaches, allowing uncertainty-aware learning in the SHM process. Similarly, Liu et al. (2022) explores the integration of PINNs into Physics-Guided Deep Markov Models, allowing for probabilistic inference in structural system identification. This method enables PINNs to account for uncertainty in structural behavior, making it particularly useful for scenarios where sensor data is sparse or noisy. Additionally, Haywood-Alexander et al. (2024) presents Bayesian PINNs (B-PINNs), a methodology that combines Bayesian inference with PINNs-based modeling to assess uncertainty in physics-constrained learning. This probabilistic augmentation makes PINNs more robust in SHM applications, ensuring that models do not overfit to deterministic physics priors but adjust according to confidence intervals in structure behavior predictions.

3.2. *Current challenges in PINNS*

In the reviewed studies, PINNS have demonstrated significant potential as a hybrid approach for modeling the nonlinear dynamics of complex and high-dimensional engineered systems.

However, PINNs face several challenges, mainly when applied to multi-scale dynamic PDE systems. One major limitation is their sensitivity to hyperparameter selection, requiring carefully fine-tuned regularization strategies to balance physics-based constraints with data-driven learning. Another challenge lies in optimizing high-dimensional latent spaces while maintaining long-term dependencies imposed by PDE constraints. Additionally, PINNs often struggle with generalization, as they tend to learn the physical constraints of a single PDE and require re-optimization for different instances. Finally, their reliance on dense, high-quality sensor data for enforcing physical consistency poses difficulties in real-world deployment, especially in environments where missing data and sensor noise are prevalent Li et al. (2024); Lai et al. (2022); Ni et al. (2022).

To overcome these challenges for SHM tasks, current research has explored alternative deep learning architectures such as Graph Neural Networks (GNNs), Transformer-based PINNs and Neural Operators, which can embed PINNs framework in their architecture and mitigate the limitation of learning from sparse, noisy, and dynamic SHM data while preserving physical consistency.

3.3. *Alternative Deep Learning Frameworks for PINNs Integration*

3.3.1. *Graphical Neural Network (GNN)*

Graphs have been incorporated in the deep learning framework for more than a decade, where models are used to process data structured as graphs, making them suitable for applications involving interconnected systems or learning the dynamics of physical systems. The learning capability of GNN through passing messages between nodes allows them to work on non-Euclidean domains Wettewa et al. (2024). This capability of GNN gives a promising use case in SHM where civil structures like bridges, dams, and buildings could be presented as a graph-like network with nodes representing sensors and edges capturing mechanical interactions such as stress, strain, and force distribution Wettewa et al. (2024).

SHM framework could be significantly enhanced by the integration of GNN with PINNs. Recently, Di Lorenzo et al. (2024) proposes a novel architecture that combines GNN with elements of Finite Element Method (FEM). This gives a prospect of their incorporation to be used for SHM problems. It can enhance spatial representation learning and handle sparse sensor data. The current standard PINNs framework depends upon fully connected networks, which struggle to capture localized stress propagation and global structural dependencies. The graph based physics constraints can enable better generalisation across different structural geometries and various loading conditions. Additionally, the graph attention mechanism can prioritize crucial regions, making PINNs-based SHM models effective at early damage identification.

3.3.2. *Transformer-Based PINNs*

Transformers have emerged as the frontrunner in deep learning architectures for capturing long-term dependencies in sequential data Vaswani et al. (2023). The current traditional recurrent networks (LSTM) face problems of vanishing gradients and computational inefficiency in long-term forecasting, while transformers learn global contextual relationships across entire time-series data Devlin et al. (2019). This makes their use case ideal for damage detection scenarios in SHM. Transformers have been used in natural language processing (NLP) and time-series forecasting, but their use case in SHM-related problems has been unexplored.

One of the main limitations of PINNS in time series modeling is their incapability to capture long-term structural behavior and dynamic responses, mainly in the scenarios of temporal variable loading conditions. Zhao et al. (2024) introduces a novel Transformer-based framework called 'PINNsFormer,' which enables PINNs to capture temporal dependencies via generated pseudo sequences, thereby improving generalization and approximation accuracy in solving PDEs. By incorporating structures like PINNsFormer, SHM models can enforce physics constraints while leveraging self-attention mechanisms to extract long-range dependencies from sensor data. This could result in a more stable and generalizable model capable of accurately predicting structural responses under varying operational and environmental conditions.

3.3.3. *Neural Operators for PINNs*

Neural operators have been one of the current advancements in deep learning frameworks, designed to learn the mapping between the functions, making them efficient for solving PDEs Li et al. (2024). Unlike traditional neural networks, which approximate functions at discrete points, Neural Operators (e.g., Fourier Neural Operators

(FNOs) and DeepONets) generalize across different input distributions, significantly improving computational efficiency in physics-based modeling. These models have been successfully applied to fluid dynamics, material science, and engineering problems, but their integration with PINNs for SHM remains unexplored.

One of the primary drawbacks of standard PINNs is their computational cost, as they require solving physics-informed loss functions pointwise across all training samples, making them impractical for large-scale SHM applications. Neural Operators can replace traditional PINNs architectures, allowing for faster physics-constrained learning by leveraging global function mappings instead of local pointwise approximations Lu et al. (2022); Li et al. (2024). By integrating FNOs with PINNs, the model can efficiently infer global stress-strain relationships, leading to real-time structural simulations faster than traditional PINNs solvers. This makes Neural Operator-PINNs hybrids a promising solution for large-scale SHM tasks, such as monitoring entire bridge networks.

4. Discussion

The evolution of PINNs has demonstrated significant advancements in recent years, transitioning from their initial application in solving PDEs to becoming integral components of deep learning architectures. To elucidate the fundamental principles of PINNs, this study employs the example of a damped pendulum, offering a mathematical overview of their underlying architecture. Furthermore, this study provides a state-of-the-art analysis of research incorporating PINNs within various computational frameworks. It systematically classifies these studies into three broad categories: (1) PINNs as Physics-Constrained Solvers, (2) PINNs embedded within Deep Learning Architecture, and (3) PINNs for Hybrid Learning and System Identification. Through this classification, the study presents a comprehensive evaluation of the methodologies, applications, and contributions of PINNs across various SHM tasks.

Despite significant advancements over the past five years, they still inherit several limitations. These include sensitivity to hyperparameter selection, challenges in optimizing high-dimensional latent spaces while maintaining long-term dependencies imposed by PDE constraints, and a lack of generalization. To address these issues, recent research has introduced frameworks with strong potential for SHM applications. Approaches such as Graphical Neural Networks, Transformer PINNs, and Neural Operators have demonstrated promising results and could be widely adopted in future SHM developments. This study provides an overview of advancements in the PINNs framework and explores how integrating advanced deep learning techniques can enhance its effectiveness for SHM tasks

5. Conclusion

In the modern era, Digital Twin-based Structural Health Monitoring (DT-SHM) has emerged as a critical component in developing and maintaining concrete civil structures. The inherent limitations of purely physics-driven and data-driven approaches have necessitated the adoption of hybrid Digital Twin frameworks that effectively integrate both methodologies. A hybrid approach termed as PEML has emerged over the years, in which one of the classification-PINNs have gained significant prominence in SHM applications over the past five years.

This study presents a comprehensive review of state-of-the-art research in the PINNs domain, highlighting existing limitations and challenges. Furthermore, we outline the integration of PINNs within deep learning architectures to enhance SHM tasks, providing insights into their potential for improving predictive accuracy and structural assessment.

References

Al-Adly, A. and P. Kripakaran (2024, 03). Physics-informed neural networks for structural health monitoring: a case study for kirchhoff–love plates. *Data-Centric Engineering 5*.

Cicirello, A. (2024, December). Physics-enhanced machine learning: a position paper for dynamical systems investigations. *Journal of Physics: Conference Series 2909*(1), 012034.

Cuomo, S., V. S. Di Cola, F. Giampaolo, G. Rozza, M. Raissi, and F. Piccialli (2022, September). Scientific machine learning through physics–informed neural networks: Where we are and what's next. *J. Sci. Comput. 92*(3).

Devlin, J., M.-W. Chang, K. Lee, and K. Toutanova (2019). Bert: Pre-training of deep bidirectional transformers for language understanding.

DGZfP (2022). *Code of Practice B 09 - Permanent monitoring of engineering structures*. Technical Committee for Non-Destructive Testing in Civil Engineering Berlin. in German.

Di Lorenzo, D., V. Champaney, C. Ghnatios, E. Cueto, and F. Chinesta (2024, 11). Physics-informed and graph neural networks for enhanced inverse analysis. *Engineering Computations*.

Drakoulas, G. I., T. V. Gortsas, and D. Polyzos (2023). Physics-based reduced order modeling for uncertainty quantification of guided wave propagation using bayesian optimization.

Faroughi, S. A., N. Pawar, C. Fernandes, M. Raissi, S. Das, N. K. Kalantari, and S. K. Mahjour (2023). Physics-guided, physics-informed, and physics-encoded neural networks in scientific computing.

Farrar, C. R. and K. Worden (2012). *Structural health monitoring: a machine learning perspective*. John Wiley & Sons.

Haywood-Alexander, M., G. Arcieri, A. Kamariotis, and E. Chatzi (2024, 10). Response estimation and system identification of dynamical systems via physics-informed neural networks.

Haywood-Alexander, M. and E. Chatzi (2023, 09). Physics-informed neural networks for one-step-ahead prediction of dynamical systems.

Lai, Z., W. Liu, X. Jian, K. Bacsa, L. Sun, and E. Chatzi (2022, 11). Neural modal ordinary differential equations: Integrating physics-based modeling with neural ordinary differential equations for modeling high-dimensional monitored structures. *Data-Centric Engineering 3*.

Lai, Z., C. Mylonas, S. Nagarajaiah, and E. Chatzi (2021, September). Structural identification with physics-informed neural ordinary differential equations. *Journal of Sound Vibration 508*, 116196.

Li, Z., H. Zheng, N. Kovachki, D. Jin, H. Chen, B. Liu, K. Azizzadenesheli, and A. Anandkumar (2024, May). Physics-informed neural operator for learning partial differential equations. *1*(3).

Liu, W., Z. Lai, K. Bacsa, and E. Chatzi (2022, October). Physics-guided deep markov models for learning nonlinear dynamical systems with uncertainty. *Mechanical Systems and Signal Processing 178*, 109276.

Lu, L., X. Meng, S. Cai, Z. Mao, S. Goswami, Z. Zhang, and G. E. Karniadakis (2022). A comprehensive and fair comparison of two neural operators (with practical extensions) based on fair data. *Computer Methods in Applied Mechanics and Engineering 393*, 114778.

Ni, P., L. Sun, J. Yang, and Y. Li (2022). Multi-end physics-informed deep learning for seismic response estimation. *Sensors 22*(10).

Raissi, M., P. Perdikaris, and G. Karniadakis (2019). Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics 378*, 686–707.

Rytter, A. (1993). *Vibrational Based Inspection of Civil Engineering Structures* (Edition ed.). Aalborg: Dept. of Building Technology and Structural Engineering, Aalborg University. PhD Thesis.

Vaswani, A., N. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, A. N. Gomez, L. Kaiser, and I. Polosukhin (2023). Attention is all you need.

Wettewa, S., L. Hou, and G. Zhang (2024). Graph neural networks for building and civil infrastructure operation and maintenance enhancement. *Advanced Engineering Informatics 62*, 102868.

Yuan, F.-G., S. Zargar, Q. Chen, and S. Wang (2020, 04). Machine learning for structural health monitoring: challenges and opportunities. pp. 2.

Zhao, Z., X. Ding, and B. A. Prakash (2024). Pinnsformer: A transformer-based framework for physics-informed neural networks.