Proceedings of the 35th European Safety and Reliability & the 33rd Society for Risk Analysis Europe Conference
Edited by Eirik Bjorheim Abrahamsen, Terje Aven, Frederic Bouder, Roger Flage, Marja Ylönen
 Published by Research Publishing, Singapore.
doi: 10.3850/978-981-94-3281-3_ESREL-SRA-E2025-P6474-cd

Incorporating Continuous Distributions in Quantum Bayesian Networks for Reliability Assessment

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Operational demands in industries, such as the energy sector, underscore the critical need for reliable equipment capable of withstanding long-term planning and unpredictable factors. Reliability assessment is important for maintaining productivity and optimizing maintenance strategies, especially in scenarios where data limitations challenge traditional assessment methods. In this context, Bayesian inference has emerged as a dynamic tool to update reliability estimates using data from various hierarchical levels. However, conventional simulation techniques may lack computational efficiency when dealing with the reliability estimation of complex systems, creating opportunities to explore alternative approaches such as quantum computation techniques. Quantum Computing leverages principles of quantum mechanics, such as superposition and entanglement, to try to address these computational challenges more effectively. Previous works have applied quantum Bayesian networks using amplitude amplification methods to the context of risk and reliability, focusing on nodes representing discrete probability distributions. This research aims to enhance this approach by incorporating continuous marginal and conditional probabilities into the analysis, which is particularly relevant for systems that rely on these distributions to model events. We explore the encoding of continuous probability distributions within the amplitude amplification framework, aiming to improve the efficiency and precision of probabilistic inference. Additionally, we apply this methodology to Bayesian networks, benchmarking the performance of quantum methods against classical simulation techniques like Monte Carlo to identify scenarios where quantum techniques demonstrate clear advantages.

Keywords: Reliability Assessment, Quantum Computing, Bayesian Networks.

1. Introduction

Reliability Engineering encompasses a broad spectrum of applications, among which Bayesian networks stand out as a fundamental tool for reliability estimation. These networks, extensively explored in the literature, have been applied in diverse contexts such as system diagnostics (Zhou, 2022), failure prediction (Zhang et al., 2024), and risk analysis (Wu et al., 2015). For instance, classical Bayesian networks have been utilized in applications ranging from structure engineering (Hlaing et al., 2022) to human reliability analysis (Podofillini et al., 2023). From a traditional computational perspective, Bayesian networks often rely on Monte Carlo simulations and other probabilistic inference methods to perform reliability assessments. While robust, these techniques are computationally intensive, particularly for large and complex systems.

In recent years, advancements in quantum computing have paved the way for innovative approaches to Bayesian networks, giving rise to quantum Bayesian networks (QBNs) (Borujeni et al., 2021). Studies such as those by (San Martin and Droguett, 2023; San Martin, Parhizkar, et al., 2023) have highlighted the potential of QBNs in reliability contexts.

Quantum computing offers unparalleled advantages in handling complex probabilistic computations due to its inherent parallelism and the exponential scalability of quantum states. This has opened new horizons for applications in reliability engineering, where precise and efficient modeling is crucial. Recent studies highlight, for example, the potential of quantum machine learning (QML): (Lins et al., 2024) applied QML to EEG-based drowsiness detection; (Correa-Jullian et al., 2022) used QML for wind turbine fault detection; and (Maior et al., 2023) explored QML for rotating machinery health management. Beyond QML, quantum computing has also shown promise in combinatorial optimization, as demonstrated by (San Martín et al., 2024), who proposed a quantum-based approach for optimal sensor placement in civil structures. Therefore, these new techniques could also explore the field of inference.

Despite these advancements, a significant gap persists in the application of QBNs: the incorporation of continuous probability distributions. Many practical reliability problems rely on continuous distributions to model uncertainties and dependencies accurately. Existing quantum approaches to Bayesian networks primarily focus on discrete distributions, limiting their applicability in scenarios where continuous variables play a critical role. Bridging this gap is essential for advancing the practical utility of QBNs in reliability engineering.

In this study, we aim to encode continuous probability distributions in quantum Bayesian networks. By combining quantum state preparation with Quantum Amplitude Estimation (QAE), we discretize and encode distributions, especially the Normal distribution, by using rotation and controlled quantum gates. As a proof of concept, we will apply this method to a hypothetical Bayesian network for system reliability estimation and compare its performance against the traditional Monte Carlo method.

The remainder of this paper is organized as follows: Section 2 provides an overview of quantum computing principles. Section 3 outlines our proposed methodology for incorporating continuous distributions in quantum Bayesian networks. Section 4 presents the experimental setup and results. Finally, Section 5 concludes the paper and highlights directions for future research.

2. Quantum Computing

The definitions and concepts presented in this section are grounded in established principles of quantum computing and quantum mechanics. For a more comprehensive understanding, readers are encouraged to refer to the foundational literature in this field (Nielsen et al., 2010; Rieffel et al., 2011; Scherer, 2019).

Quantum computing is an emerging computational paradigm that processes information using the principles of quantum mechanics. Unlike classical computing, which relies on bits that take values of either 0 or 1, quantum computing utilizes quantum bits (qubits) that can exist in superposition states.

A qubit can be described as a two-level quantum system, represented as a linear combination of basis states:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{1}$$

Where α, and β are complex probability amplitudes satisfying the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. This superposition allows quantum computers to explore multiple computational paths simultaneously.

Building on this concept, quantum gates manipulate qubits through unitary transformations. For instance, the Hadamard (H) gate transforms a qubit into a superposition where it has an equal probability (50%) of collapsing to either $|0\rangle$ or $|1\rangle$ upon measurement:

$$H = \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{2}$$

The Pauli matrices are fundamental quantum gates that perform single-qubit operations. The *X* gate, also known as the quantum NOT gate, flips the state of a qubit ($|0\rangle$ to $|1\rangle$ and vice-versa). The *Y* gate introduces a phase shift while flipping the qubit state, and the *Z* gate applies a phase flip, leaving unchanged but transforming $|1\rangle$ into $-|1\rangle$:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{3}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \tag{4}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{5}$$

Rotation Gates introduce controlled transformation to qubits, adjusting their phase and, in some cases, their amplitude (R_x and R_y), allowing for finer control over quantum states. The rotation gates around different axes are given by:

$$R_x(\theta) = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -i\sin\left(\frac{\theta}{2}\right) \\ -i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix} \tag{6}$$

$$R_y(\theta) = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix} \tag{7}$$

$$R_z(\theta) = \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix} \tag{8}$$

Another important concept is quantum entanglement. It is a phenomenon where qubits become correlated such that the state of one qubit instantaneously influences the state of another, regardless of distance. For example,

$$CNOT = \begin{bmatrix} 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,0\,1 \\ 0\,0\,1\,0 \end{bmatrix} \tag{9}$$

Moreover, quantum measurement collapses the quantum state into one of the basis states, with probabilities given by the squared magnitudes of the coefficients. This irreversible process fundamentally distinguishes quantum computing from classical probability-based computation.

Additionally, in quantum computing, interference occurs when quantum states combine in ways that enhance (constructive interference) or suppress (destructive interference) certain outcomes. This phenomenon allows quantum algorithms to direct computation toward the correct solutions while reducing the likelihood of incorrect ones, improving efficiency.

Finally, quantum algorithms leverage the principles of superposition, entanglement, and interference to achieve exponential or quadratic speedups over classical approaches in specific problem domains. Some of the most well-known algorithms include: Shor's algorithm for integer factorization, which runs in polynomial time using quantum Fourier transform; and Grover's algorithm, which achieves quadratic speedup in unstructured search problems by iterating an amplitude amplification process $O(\sqrt{N})$ compared to the classical $O(N)$ complexity. This makes Grover's algorithm particularly useful for searching unsorted databases and solving combinatorial optimization problems.

3. Quantum Amplitude Estimation

QAE is a fundamental quantum algorithm that extends Grover's search to estimate the amplitude of a specific quantum state (Tanaka et al., 2022). The problem addressed by QAE involves a unitary operator *U* acting on *n+1* qubits. This operator is constructed such that:

$$U|0\rangle = \sqrt{a}|\psi_1\rangle|1\rangle + \sqrt{(1-a)}|\psi_0\rangle|0\rangle \tag{10}$$

Where $|\psi_1\rangle$ and $|\psi_0\rangle$ are normalized quantum states, and $a \in [0,1]$ represents the amplitude to be estimated. The objective of QAE is to accurately determine a, which corresponds to the probability of measuring the ancillary qubit in the state $|1\rangle$. An ancillary qubit is an additional qubit introduced to facilitate a quantum computation, often used for error correction, intermediate storage, or measurement purposes (Zoufal et al., 2019).

The algorithm builds upon the concept of amplitude amplification, which generalizes Grover's operator Q. The amplification operator is defined as :

$$Q = -U_0 S U^{\dagger} S \tag{11}$$

Where $S = I - 2|\psi\rangle\langle\psi|$ reflects about the initial state, and $U_0 = I - 2|0\rangle\langle 0|$ reflects about the ancillary qubit state. Repeated applications of Q increase the amplitude of the desired state (Zoufal et al., 2019; Tanaka et al., 2022).

A crucial aspect of QAE is the quantum phase estimation, which extracts the amplitude encoded as a phase. The eigenvalues of the amplification operator Q are given by $\lambda = e^{\pm 2i\theta}$, where θ satisfies $a = sin^2(\theta)$.

Using phase estimation, the angle θ is determined, and the amplitude a is computed as $a = sin^2(\theta)$. The algorithm involves three primary steps. First, the initial state is prepared using the unitary operator U $(U|0\rangle|0\rangle = \sqrt{a}|\psi 1\rangle|1\rangle + \sqrt{(1-a)}|\psi 0\rangle|0\rangle$.

Next, the quantum phase estimation is applied to the amplification operator Q to estimate phase θ. Finally, the amplitude a is extracted using the relation $a = sin^2(\theta)$. The accuracy of QAE depends on the number of queries to Q and improves with the number of iterations m: $\Delta a \sim O(1/m)$ *(Zoufal et al., 2019)*.

This leads to a quantum speedup, as QAE requires $O(1/\varepsilon)$ queries to achieve an error ε (for $\varepsilon < 1$), compared to $O(1/\varepsilon^2)$ in classical Monte Carlo methods. However, its practical implementation often necessitates fault-tolerant quantum computers due to its reliance on quantum phase estimation. To address this limitation, variants such as Iterative Amplitude Estimation (IAE) have been proposed, which reduce resource requirements by bypassing phase estimation.

4. Proposed framework for Quantum Bayesian Networks with Continuous Distributions and Quantum Amplitude Estimation

In this paper, we apply a quantum-enhanced Bayesian network framework to incorporate continuous probability distributions for reliability assessment. The methodology involves three main steps: (1) discretization and quantum encoding of continuous distributions, (2) construction of a quantum circuit to represent the Bayesian network, and (3) probabilistic inference using amplitude estimation. Additionally, we compare the quantum approach with classical Monte Carlo simulation to evaluate its performance.

To demonstrate the proposed methodology, we consider a specific symple Bayesian network with four nodes, designed to model a reliability scenario in the context of industrial equipment maintenance (Fig. *1*).

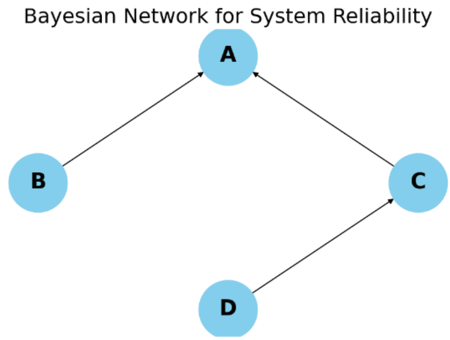

Fig. 1. Example of a Bayesian network illustrating reliability modeling.

The network consists of the following nodes:

- D (equipment wear level): represents the wear level of a critical component in the equipment, modeled as a continuous variable following a normal distribution $N(\mu = 0.5, \sigma = 0.1)$;
- C (environmental conditions): represents the environmental conditions affecting the equipment, such as temperature or humidity. It depends on the wear level D, with the conditional probability $P(C = 1 \mid D) = 0.3 + 0.6D$. This reflects that harsher conditions are more likely as the wear level increases;
- B (maintenance quality): represents the quality of maintenance performed on the equipment. It is an independent binary variable with $P(B = 1) = 0.6$, indicating a 60% chance of high-quality maintenance;

- A (equipment failure): represents the probability of equipment failure, which depends on both the maintenance quality B and the environmental conditions C. The conditional probabilities are defined as:
    - $P(A=1 \mid B=0, C=0)=0.1$: Low failure probability under good maintenance and mild conditions;
    - $P(A=1 \mid B=0, C=1)=0.3$: Moderate failure probability under good maintenance but harsh conditions;
    - $P(A=1 \mid B=1, C=0)=0.6$: High failure probability under poor maintenance but mild conditions;
    - $P(A=1 \mid B=1, C=1)=0.9$: Very high failure probability under poor maintenance and harsh conditions.

4.1. Discretization and quantum encoding of continuous distributions

Continuous probability distributions can be discretized into a finite set of values to enable their representation in a quantum state. It is important to highlight that the methods and steps described in this work are specifically designed and analyzed under the assumption of a normal distribution, ensuring that the continuous variables are represented and processed accordingly within the quantum framework.

For a continuous random variable *D*, for example, with a normal distribution $N(\mu, \sigma^2)$, the probability density function (PDF) is given by:

$$f_D(x)=\frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \tag{12}$$

where μ is the mean and σ is the standard deviation. The distribution is discretized into *n* points over the range [0,1]. The discretized probabilities p_i' are calculated as:

$$p_i'=\frac{f_D(x_i)}{\sum_{j=0}^{n-1} f_D(x_j)} \tag{13}$$

These probabilities are encoded into the amplitudes of a quantum state $\mid \psi_D\rangle$ using rotation gates (R_y):

$$\mid \psi_D\rangle=\sum_{i=0}^{n-1} \sqrt{p_i'}|i\rangle \tag{14}$$

Here, $\sqrt{p_i'}$ represents the amplitude associated with the basis state $|i\rangle$. When the quantum state is measured, the probability of obtaining $|i\rangle$ is given by $\mid \sqrt{p_i'} \mid^2 = p_i'$, ensuring the correct encoding of the discretized distribution. To construct this state in a quantum circuit, each basis state $i\rangle$ is prepared using an $R_Y(\theta_i)$ gate with an angle computed as $\theta_i = 2arcsin(\sqrt{p_i'})$.

4.2. Construction of a quantum circuit to represent the Bayesian network

The quantum circuit (Fig. 2) is designed to represent a Bayesian network with discrete and continuous nodes as follows (Borujeni et al., 2021):

- As initialization, Hadamard gates (*H*) are applied to the qubits q_0, q_1, and q_2 to create a superposition state for *D*;
- Node D is encoded into a quantum state $\mid \psi_D\rangle$ using rotation gates (*RY*) and *n* = 8 points, requiring $log_2(8)=3$ qubits. These qubits are labeled, in Fig. 2, as q_0, q_1, and q_2;
- The conditional probability $P(C=1 \mid D)$ is modeled as a linear function of *D* ($P(C=1 \mid D)=0.3+0.6D$). This is implemented using controlled rotation gates $CRY(\theta_i)$, where:

$$\theta_i = 2\text{arcsin}\left(\sqrt{P(C=1 \mid D=x_i)}\right) \tag{15}$$

The qubit q_3 (Fig. 2), represents *C*, and its state depends on the qubits q_0, q_1, and q_2 (which encode *D*)

- The discrete node *B* is independent and encoded using a single-qubit (q_4) rotation gate $RY(\theta_B)$, where:

$$\theta_B = 2\text{arcsin}\left(\sqrt{P(B=1)}\right) \tag{16}$$

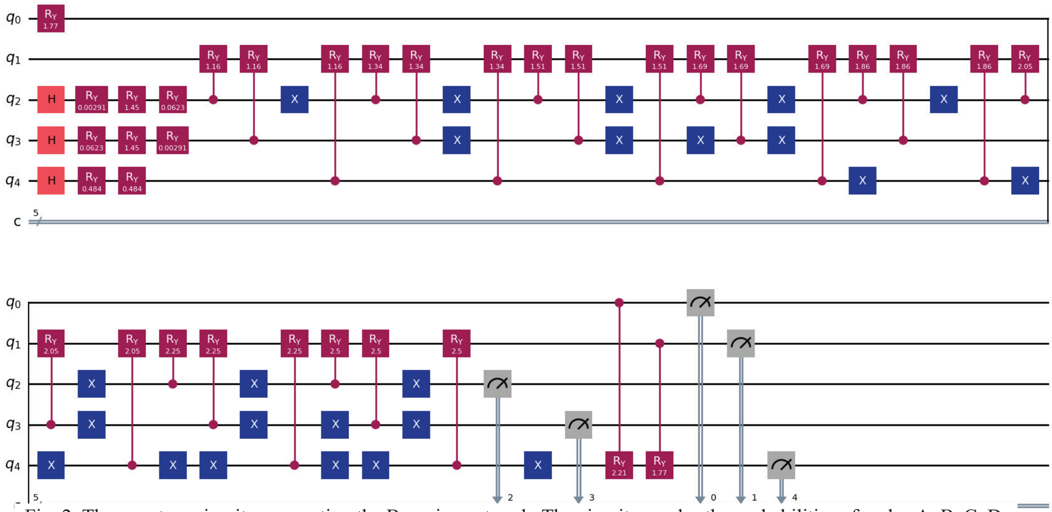

Fig. 2. The quantum circuit representing the Bayesian network. The circuit encodes the probabilities of nodes A, B, C, D.

- The conditional probabilities of $P(A = 1|B, C)$ are encoded into the circuit using multi-controlled rotation gate $MCRY(\theta)$. In Fig. 2, the qubit q_5 represents A, and its state depends on the qubits q_3 (for C) and q_4 (for B).

4.3. Probabilistic inference using amplitude estimation

The methodology was implemented using Qiskit, an open-source quantum computing framework developed by IBM. Qiskit provides tools for designing quantum circuits, simulating quantum algorithms, and running experiments on real quantum hardware. The Qiskit Aer simulator was employed to simulate the quantum circuit. This simulator allows for noise-free execution of quantum algorithms.

The probabilistic inference is performed using the IAE algorithm. The goal is to estimate the probability $P(A = 1)$ of the target node A. The IAE algorithm iteratively refines the estimate to achieve the desired precision (ε) and confidence level $1-\alpha$. Here we used 0.0001 and 0.95, respectively.

The IAE algorithm iteratively refines the estimate of $P(A = 1)$ by performing quantum phase estimation on the target qubit q_5. The results were compared with a classical Monte Carlo simulation (100,000 samples). The estimated probability $P(A = 1|B, C)$ and associated metrics are summarized below:

Table 1. Comparison of Quantum and Monte Carlo Methods.

Method	Estimated Probability	95% Confidence Interval	Standard Error
QAE	0.51876	[0.51875, 0.51878]	1.57e-05
Monte Carlo	0.58800	[0.55749, 0.61851]	0.01556

The results demonstrate notable differences between the quantum and Monte Carlo approaches in estimating the $P(A = 1)$. The quantum method yielded an estimated probability of 0.51876, with [0.51875, 0.51878] as 95% confidence interval and a low sample mean standard error of 1.57e-05. In contrast, the Monte Carlo method produced a higher estimated probability of 0.588, but with a considerably wider 95% confidence interval, [0.55749, 0.61851], and a significantly larger standard error of the sample mean (0.01556). While the quantum approach provides a much narrower confidence interval and lower variability, the discrepancy in estimated probabilities suggests the need for further analysis.

5. Conclusions

The results highlight the potential of quantum methods for probabilistic inference in Bayesian networks, offering lower variance compared to the traditional Monte Carlo approach. While the quantum method demonstrated high precision, the discrepancy in probability estimates suggests the need for further validation to assess potential biases introduced by quantum encoding and estimation processes. Additionally, despite its advantages in variance reduction, practical implementation on near-term quantum hardware remains a challenge, especially in terms of computational cost and scalability. Expanding this approach to more complex Bayesian networks will be essential to fully understand its applicability.

For reliability engineering, these findings point to quantum computing as a promising tool for risk assessment and decision-making under uncertainty. However, further exploration is needed to refine the method and ensure its robustness. Future research should test alternative probability distributions beyond the Normal distribution, such as Exponential, Lognormal , and Weibull, to evaluate whether quantum techniques consistently improve inference accuracy. Additionally, investigating key parameters – such as the number of qubits, IAE configurations, and the number of shots used in simulations — could help optimize performance and enhance reliability assessments.

Inference remains a promising area for the application of quantum computing, with opportunities to refine algorithms and explore hybrid quantum-classical approaches. Continued research in this direction could lead to more efficient methods for reliability analysis, particularly in computationally demanding scenarios.

Acknowledgement

The authors thank CNPq, FACEPE, and PRH 38.1 managed by ANP and FAPESP for the financial support through research grants. This study was financed in part by CAPES – Finance Code 001.

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