Proceedings of the 35th European Safety and Reliability & the 33rd Society for Risk Analysis Europe Conference
Edited by Eirik Bjorheim Abrahamsen, Terje Aven, Frederic Bouder, Roger Flage, Marja Ylönen

doi: 10.3850/978-981-94-3281-3_ESREL-SRA-E2025-P6336-cd

Tracking Reliability and updating the overhaul interval of engineering components: Bayesian approach

Guru Prakash

Department of Civil and Environmental Engineering, University of Waterloo, Waterloo, Canada & Deaprtment of Civil Engineering, IIT Indore, Indore, India. E-mail: gprakash@uwaterloo.ca

M.D. Pandey

Department of Civil and Environmental Engineering, University of Waterloo, Waterloo, Canada. E-mail: mdpandey@uwaterloo.ca

The Bayesian approach is commonly used to track the progression of a degradation process and update the reliability of a component. However, there is a wide variety of components in which the failure cannot be attributed to a single well-defined degradation process, and inspection techniques are unable to quantify the state of degradation. For such components, only maintenance records pertaining to the timing of corrective and preventive maintenance are available. This paper focuses on developing an adaptive method to update the reliability and revise the maintenance interval based on the survival history of a component. The paper presents a Weibull mixture model to account for heterogeneity in the lifetime data. The posterior model parameters are estimated through Bayesian updating. The proposed method is illustrated using the lifetime data obtained for a group of level control valves used in steam generators of a Canadian nuclear power plant.

Keywords: Reliability updating, Weibull mixture, heterogeneous population, Bayesian inference, Markov chain Monte Carlo (MCMC), Overhaul interval

1. Introduction

Reliability-based approaches, such as time-based and condition-based policies, are commonly used to determine the maintenance (or replacement/overhaul) interval for engineering components. The time-based policy, such as the age-based replacement policy, requires only the lifetime distribution of the component, whereas a degradation process model is a key input to the condition-based policy. Both approaches are static in nature, given only the point estimates of the model parameters, i.e., the calculated reliability for any given time interval or an optimum maintenance interval, are fixed quantities for all similar components in the population. Reliability updating or tracking is possible only by conditioning on the evidence of the survival of the component over its life. This tracking is also static, i.e., conditional reliability (or mission reliability) versus component age can be calculated before using the model parameters.

Recognizing this aspect, the Bayesian approaches have been used to build a dynamic reliability updating/tracking model. In this approach, the prior distributions of model parameters are updated based on the component-specific information. In the literature, most studies apply the Bayesian approach to condition-based maintenance policy driven by a stochastic degradation process, such as the gamma process Grall et al. (2002). These models primarily include the parameter uncertainty, which is updated as the new degradation measurements become available. However, such models are only applicable when a well-defined degradation process can be identified and observed through inspections.

In a power plant, there is a wide variety of components in which the failure cannot be attributed to a single well-defined degradation process. Or, inspection techniques are not available to quantify the underlying degradation process. For such components, only time to failure and time to maintenance records are typically available as a basis for the reliability analysis. This paper focuses on developing an adaptive method to revise the maintenance interval for a group of components based on their performance, i.e., survival over a period. An evident approach would be to update the parameter uncertainty associated with the lifetime distribution of the component. However, experience suggests that component failures can be classified into two broad categories: (1) Early failures of components with manufacturing flaws and inadequate overhauls (i.e., weak components), (2) late failure of robust components (i.e., strong components). Therefore, reliability tracking can identify which group the component belongs to. Accordingly, its maintenance interval can be revised.

This paper presents a Bayesian mixture model for reliability tracking of engineering components. A probabilistic mixture model accounts for the existence of underlying sub-populations within a larger population without requiring knowledge of the specific sub-population from which each observed lifetime data originates Titterington et al. (1985); Everitt (2013); Ruhi et al. (2015); Murthy et al. (2004); Carta and Ramirez (2007). The proposed approach is illustrated using the maintenance data obtained from level control valves (LCVs) used at a nuclear power plant in Canada.

The paper is organized as follows. Section 2 discusses the modeling of a heterogeneous population using the Weibull mixture. Section 3 presents the Bayesian approach for updating the model parameters. Section 4 presents a case study using the example of level control valves in a steam generator. Finally, the conclusion and limitations of the present study are discussed in Section 5.

2. Modeling Heterogeneous Population

2.1. *Background*

General sources of heterogeneity in reliability characteristics are variations in material properties, deviations in the manufacturing process, human error, and changes in operating conditions. Because of this, it is postulated that such statistically different groups exist in the component population. Therefore, the maintenance schedule should depend on the membership of a component to a particular group. This can achieved by the Bayesian approach, which provides an opportunity to update the prior distribution with available lifetime data.

The lifetime distribution in a heterogeneous population can be expressed as:

$$f_M(t|\mathbf{\Theta}) = \sum_{k=1}^{m} w_k f_k(t|\theta_k) \tag{1}$$

where $f_M(t|\mathbf{\Theta})$ is probability density function (p.d.f.) of the mixture model, $f_k(t|\theta_k)$ is the p.d.f. of kth component with parameters, θ_k, and w_k is the weight of kth component, and m is the total number of components.

Each weight w_k represents the proportion of observations expected to come from the associated distribution $f_k(\cdot)$ and sum up to one, i.e., $\sum_{k=1}^{m} w_k = 1$. The various distributions $f_k(\cdot)$ can be from the same family or different families.

2.2. *Weibull Mixture Model*

This paper considers a two-component (i.e., $m = 2$) mixture of Weibull distributions:

$$f_M(t|\mathbf{\Theta}) = \sum_{k=1}^{2} w_k f_k(t|\theta_k) \tag{2}$$

where, $f_k(t|\theta_k)$ denotes the pdf of Weibull distribution given as,

$$f_k(t|\theta_k) = \left(\frac{a_k}{b_k}\right)\left(\frac{t}{b_k}\right)^{a_k-1} e^{-\left(\frac{t}{b_k}\right)^{a_k}} \tag{3}$$

$\theta_1 = (a_1, b_1)$ and $\theta_2 = (a_2 b_2)$ are the shape and scale parameters of the first and second components, respectively.

The number of components in a mixture model can be identified using the Weibull quantile-quantile (QQ) plot. This plot is a straight line for a single Weibull distribution, while for the two components, this is "S-shaped" with one inflection point. Jiang et al. Jiang and Murthy (1995); Jiang and Kececioglu (1992) presented a graphical method to roughly estimate parameters $\hat{\mathbf{\Theta}}$ for a two-component Weibull mixture. Similarly, the Weibull probability paper plot (WPP) and segmented rank regression can be used for an approximate estimate of parameters Deganaksoy et al. (2002). The estimates obtained from these methods are often used as a prior in Bayesian methodology.

3. Bayesian Inference: Mixture model

3.1. *Basic formulation*

This Section presents the Bayesian updating of the parameters, $\boldsymbol{\Theta} = (w_1, a_1, a_2, b_1, b_2)$, of the two-component mixture model (Eq. 2)

The first step is to formulate the likelihood function for a sample of lifetime data, $t_i, i = 1, n$, which includes complete and right-censored lifetimes. Define a two-dimensional binary latent variable $\boldsymbol{z_i}$ associated with lifetime t_i . The random variable $\boldsymbol{z_i}$ has a 1-of-2 representation in which a particular element z_{ik} is equal to 1 and all other elements are equal to 0 and satisfy the following condition for ith lifetime: $z_{ik} \in \{0, 1\}$ and $\sum_k z_{ik} = 1$. Further, let δ_i denote a censorship indicator, which takes value 0 or 1 accordingly:

$$\delta_i = \begin{cases} 1 \text{ If } t_i \text{ is censored.} \\ 0 \text{ If } t_i \text{ is complete.} \end{cases} \tag{4}$$

Now, given $\boldsymbol{z_i} = \{z_{i1}, z_{i2}, \cdots, z_{in}\}$, $\boldsymbol{t} = \{t_1, t_2, \cdots, t_n\}$, and $\boldsymbol{\delta} = \{\delta_1, \delta_2, \cdots, \delta_n\}$, the likelihood function $p(\boldsymbol{t}, \boldsymbol{\delta}, \boldsymbol{z}|\boldsymbol{\Theta})$ of two-component mixture model can be written as:

$$p(\boldsymbol{t}, \boldsymbol{\delta}, \boldsymbol{z}|\boldsymbol{\Theta}) = \left[\prod_{i=1,\delta_i=0}^{n} \prod_{k=1}^{2} \left(w_k f_k(t_i|\theta_k)\right)^{z_{ik}} \right] \times \left[\prod_{i=1,\delta_i=1}^{n} \prod_{k=1}^{2} \left(w_k R_k(t_i|\theta_k)\right)^{z_{ik}} \right] \tag{5}$$

The first term in Eq. 5 is the contribution of complete lifetimes, while the second term is due to the right censored lifetimes.

According to Bayes' theorem, the posterior distribution of parameters $p(\boldsymbol{\Theta}|\boldsymbol{t}, \boldsymbol{\delta}, \boldsymbol{z})$ given the data $(\boldsymbol{t}, \boldsymbol{\delta}, \boldsymbol{z})$ can be given as:

$$p(\boldsymbol{\Theta}|\boldsymbol{t}, \boldsymbol{\delta}, \boldsymbol{z}) = \frac{\pi(\boldsymbol{\Theta}) p(\boldsymbol{t}, \boldsymbol{\delta}, \boldsymbol{z}|\boldsymbol{\Theta})}{\int \pi(\boldsymbol{\Theta}) p(\boldsymbol{t}, \boldsymbol{\delta}, \boldsymbol{z}|\boldsymbol{\Theta}) d\boldsymbol{\Theta}} \tag{6}$$

$$\propto \pi(\boldsymbol{\Theta}) p(\boldsymbol{t}, \boldsymbol{\delta}, \boldsymbol{z}|\boldsymbol{\Theta}) \tag{7}$$

$$\propto \pi(\boldsymbol{\Theta}) \left[\prod_{i=1,\delta_i=0}^{n} \prod_{k=1}^{2} \left(w_k f_k(t_i|\theta_k)\right)^{z_{ik}} \right] \times \left[\prod_{i=1,\delta_i=1}^{n} \prod_{k=1}^{2} \left(w_k R_k(t_i|\theta_k)\right)^{z_{ik}} \right] \tag{8}$$

The model parameters can be estimated using sampling methods. The methods like Metropolis-Hastings, Affine-invariant Ensemble sampler, and Transitional Markov Chain Monte Carlo (MCMC) require careful tuning of proposal distributions to achieve efficient sampling, which can be can be time-consuming. On the other hand, Gibbs sampling relies on conditional distributions, which can directly be sampled without the need for proposal distributions. This makes Gibbs sampling more efficient and easier to implement. Hence, in this paper, the MCMC algorithm has been used to sample from the posterior distribution using a Gibbs sampler. The Gibbs sampling is an iterative procedure that generates samples from the full conditional distribution rather than the joint distribution of parameters Gelfand (2000). The central concept behind Gibbs sampling is to update the value of a parameter (say α_1) by keeping other variables fixed. This creates a Markov chain, which, after several iterations, converges to the joint distribution. The algorithmic details of Gibbs sampling for WMM parameters is given in Algorithm 1 on the next page.

3.2. *Prior distributions*

Generally, the prior $\pi(\boldsymbol{\Theta})$ is chosen based on the available domain knowledge, which can be informative, semi-informative, or non-informative. Assuming the independence of parameters the joint prior distribution $\pi(\boldsymbol{\Theta}) = \pi(a_1, a_2, b_1, b_2, w_1)$ can be re-written as $\pi(\boldsymbol{\Theta}) = \pi(a_1)\pi(b_1)\pi(a_2)\pi(b_2)\pi(w_1)$. The integration involved in Eq. 6 is often challenging to perform, and hence, joint posterior distribution $p(\boldsymbol{\Theta}|\boldsymbol{t}, \boldsymbol{\delta}, \boldsymbol{z})$ is obtained through MCMC sam-

Algorithm 1 MCMC sampling for WMM parameters using Gibbs sampler

1: **Initialize**:
2: Set initial values for model parameters $\alpha_1, \alpha_2, \beta_1, \beta_2, w_1$.
3: **Iterate:**
4: **for** $i = 1$ to N **do** ▷ N is the number of iterations.
5: $z_j \sim p(z_j|t_j, w_1, \alpha_1, \alpha_2, \beta_1, \beta_2)) \propto w_1^{z_j}(1-w_1)^{1-z_j} f(t_j|\alpha_{z_j}, \beta_{z_j})$ ▷ Sample component assignment
6: $w_1 \sim p(w_1|z_j, \alpha_1, \alpha_2, \beta_1, \beta_2) \propto w_1^{\sum z_j}(1-w_1)^{n-\sum z_j}$ ▷ Sample mixture weight w_1
7: $\alpha_1 \sim p(\alpha_1|\boldsymbol{t}, \delta, z, \alpha_2, \beta_1, \beta_2)$ ▷ Sample shape parameter α_1
8: $\beta_1 \sim p(\beta_1|\boldsymbol{t}, \delta, z, \alpha_1, \alpha_2, \beta_2)$ ▷ Sample scale parameter β_1
9: $\alpha_2 \sim p(\alpha_2|\boldsymbol{t}, \delta, z, \alpha_1, \beta_1, \beta_2)$ ▷ Sample shape parameter α_2
10: $\beta_2 \sim p(\beta_2|\boldsymbol{t}, \delta, z, \alpha_1, \alpha_2, \beta_1)$ ▷ Sample scale parameter β_2
11: **end for**
12: **Post-processing:**
13: Discard the initial burn-in samples and retain the remaining for inference.

pling. The gamma distribution has been chosen as a prior for shape and scale parameters due to its positive support, and the beta distribution is selected for mixture weight:

$$\begin{aligned} &w_1 \sim B(\alpha_{w_1}, \beta_{w_1}) \\ &a_m|m \sim Ga(\alpha_{a_m}, \beta_{a_m}); \quad \forall\ m = 1, 2 \\ &b_m|m \sim Ga(\alpha_{b_m}, \beta_{b_m}); \quad \forall\ m = 1, 2 \end{aligned} \tag{9}$$

where, $Ga(\cdot)$ denotes the gamma distribution with hyper-parameters (α_x, β_x) and $B(\cdot)$ denotes the beta distribution with hyper-parameters (α_y, β_y). The hyper-parameters can be estimated by equating the mean (μ_{a_m}, μ_{b_m}) and variance $(\sigma^2_{a_m}, \sigma^2_{b_m})$ of parameters obtained from MLE or rank regression method, as given in Eq. 10.

$$\begin{aligned} \mu_{a_m} &= \frac{\alpha_{a_m}}{\beta_{a_m}}; \sigma^2_{a_m} = \frac{\alpha_{a_m}}{\beta^2_{a_m}}; \ \forall\ m = 1, 2 \\ \mu_{b_m} &= \frac{\alpha_{a_m}}{\beta_{a_m}}; \sigma^2_{a_m} = \frac{\alpha_{a_m}}{\beta^2_{a_m}}; \ \forall\ m = 1, 2 \end{aligned} \tag{10}$$

3.3. *Reliability Updating*

Once the posteriors of WMM parameters are estimated, they can be used to update the reliability function of the component. Further, the posterior reliability function can be used to track the reliability of a specific group of components, given their survival history.

The reliability function $R_M(t|\boldsymbol{\Theta})$ of the mixture model can be written as:

$$R_{Mix}(t\,|\,\boldsymbol{\Theta}) = \sum_{k=1}^{2} w_k R_k(t\,|\,\theta_k) \tag{11}$$

The mission reliability $R_M(t+x|t)$, i.e., the probability that the valve will survive more than $(t+x)$ years given that it has survived till time t, is given by:

$$R_M((t+x)\,|\,t) = \frac{\sum_{k=1}^{2} w_k R_k(t+x|\theta_k)}{\sum_{k=1}^{2} w_k R_k(t|\theta_k)} \tag{12}$$

where x is the mission duration or overhaul interval of the component.

For a given time t and mission duration x, the Bayesian inference of mission reliability is performed by drawing samples of $\boldsymbol{\Theta}$ from the posterior distribution of model parameters $p(\boldsymbol{\Theta}|\boldsymbol{t}, \boldsymbol{\delta}, \boldsymbol{z})$ using MCMC, and evaluating the expression $R_M((t+x)\,|\,t)$ for each sample. Additionally, the simulated samples are used to estimate the empirical density, mean, and credible interval of parameters.

4. Case study: LCVs

4.1. *Data*

The level control valves (LCVs) are used to control the flow of water in a steam generator to maintain the required water level for steam production. In a Canadian nuclear reactor, 4 steam generators (SGs) are used, and each SG is equipped with a set of 3 LCVs. There is one small valve (6” size) and two large (10” size) ones. The three valves are arranged in parallel (called a valve station) and operated by a control program. During regular operation at high unit load, i.e., reactor power greater than 40% capacity, only one large LCV is normally kept in service at any one time, the other large valve in standby mode. In this case, the small valve is kept in a close position. The small valve operates during the low unit load with reactor power less than 20% full power.

In this study, maintenance data for LCVs were obtained from a 4-unit station consisting of 16 steam generators and 32 from the data obtained for large LCVs (10” size) Pandey et al. (2024).

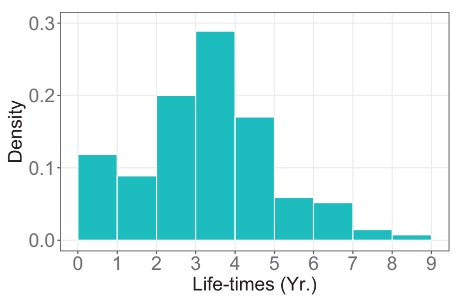

Fig. 1. Histogram of LCV lifetimes

For each valve, 10 to 20 years of maintenance data are available from the maintenance work order (WO) database, which provides information about occurrences of various types of failures/malfunctions, repair actions, and the history of overhauls. Based on WO information, the life histories of 32 LCVs were extracted; a histogram of these lifetimes is presented in Fig. 1.

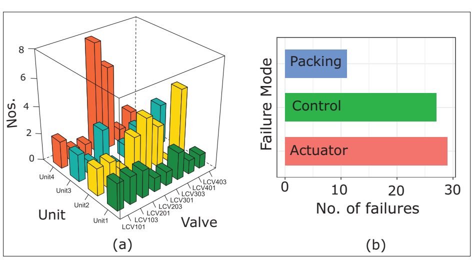

Fig. 2. Number of valve malfunctions (a) across various units (b) in different modes

Figure 2 presents the total number of valve malfunctions for a given valve type across various units and across various failure modes. There is a high degree of variability in the number of malfunctions across the valve population. Similarly, most of the failures are due to the malfunctioning of the actuator, followed by the control failure. This paper analyzes the failure of LCVs due to actuator malfunctions.

4.2. *Model Fitting*

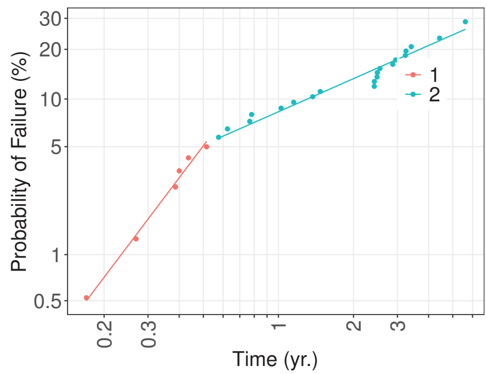

Fig. 3. Weibull probability paper plot for LCV data

The complete lifetime data are plotted on the Weibull probability paper as shown in Figure 3. Figure 3 reveals the presence of two distinct segments with a noticeable slope change occurring around 0.6 years. This justifies the use of a two-component mixture model.

The lifetime data consists of 27 complete and 108 right-censored lifetimes. The heavily censored nature of the LCV data set poses significant challenges for parameter estimation using MLE. In such scenarios, the EM algorithm often struggles to converge or converge to a local maximum rather than the global optimum Elmahdy and Aboutahoun (2013); Moon (1996). Hence, segmented rank regression has been used to estimate prior parameters. Table 1 presents the estimated parameters of two-component WMM using this method.

From Table 1, it can be seen that the 1^{st} component has a shape parameter greater than one,

Table 1. Parameters estimate using rank regression

	Shape	95% C.I.	Scale	95% C.I.	R^2 value
Component-1	2.17	(1.97, 2.41)	1.96	(1.60, 2.39)	0.98
Component-2	0.72	(0.67, 0.78)	29.50	(23.40, 37.20)	0.96

indicating an increasing hazard rate and early failure. On the other hand, the 2^{nd} component shows a decreasing hazard rate and captures improving conditions due to maintenance or a smaller number of failures with time. In other words, the two components of WMM are able to model the heterogeneity present in the lifetimes of LCVs.

For the Bayesian inference, gamma distributions have been selected as priors for the shape and scale parameters, while a beta distribution for the weight parameter. The hyper-parameters are obtained using the method of moments, i.e., the mean and variance of the estimates reported in Table 1 were used to calculate hyper-parameters. Thus, the prior distributions are specified as follows: $a_1 \sim Ga(24, 11)$, $b_1 \sim Ga(45, 22)$, $a_2 \sim Ga(250, 350)$, $b_2 \sim Ga(33, 1.2)$, and $w_1 \sim B(4, 6)$. These distributions are also plotted in Fig. 4.

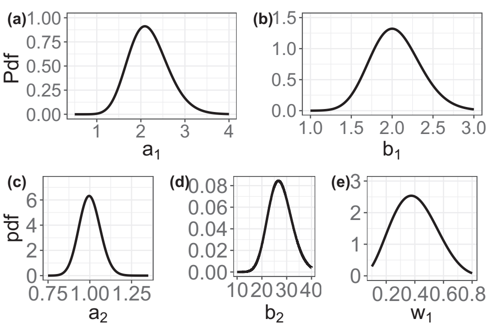

Fig. 4. Prior for parameters $(a_1, b_1, a_2, b_2, w_1)$

Using these priors, 10,000 samples were drawn from the joint posterior distribution, and the first 5000 samples were discarded as burn-in, and the subsequent 5000 samples were retained for posterior analysis.

The trace plots exhibit rapid oscillations around a central value for all the parameters, showing no long-term trends or drifts (see autocorrelation plot in Fig 6). This indicates good mixing and confirms that the chain has converged to its target distribution. Furthermore, the running mean and

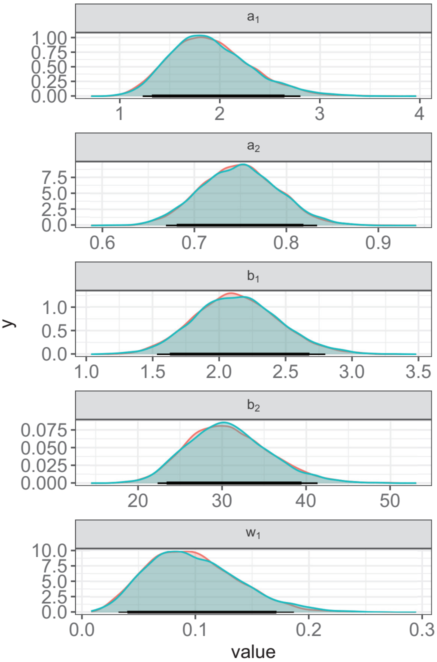

Fig. 5. Estimated density of WMM Parameters

auto-correlation plots for the two simulated chains reveal that the correlation becomes negligible after three lags, indicating that the samples are drawn from a stationary distribution. Moreover, it was found that the model converged after 500 samples. In other words, sufficient draws have been obtained to summarize the posterior distribution accurately, and additional samples will not significantly influence the parameters estimate.

The estimated mean and various quantiles of parameters a_1, a_2, b_1, b_2, w_1 are given in Table 2. A close observation of the numerical value of two Weibull distributions, i.e., $W_1 \sim (1.91, 2.13)$ and $W_2 \sim (0.75, 30.80)$, reveals that they correspond to the weak and strong weak components, respectively. The shape parameter of the weak component is greater than 1, indicating an increasing hazard rate and early failure. However, the strong component, the major contributor ($w_2 = 1 - w_1 = 0.9$), shows a decreasing hazard rate and a larger

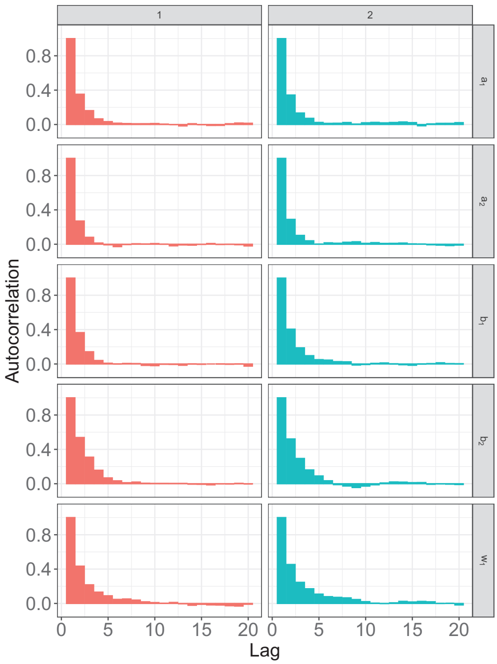

Fig. 6. Auto-correlation plot for two MCMC chains

scale factor.

Table 2. Posterior estimates of WMM parameters

	Mean (sd)	2.5%	25%	50%	75%	97.5%
a_1	1.91 (0.40)	1.25	1.64	1.87	2.16	2.80
a_2	0.75 (0.04)	0.67	0.72	0.75	0.78	0.83
b_1	2.13 (0.32)	1.53	1.90	2.11	2.33	2.78
b_2	30.80 (4.77)	22.35	27.43	30.46	33.81	40.92
w_1	0.10 (0.04)	0.03	0.07	0.09	0.12	0.18

4.3. *Reliability analysis*

Figure 7 shows the probability density function of WMM and its two components. The posterior mixture model has been used to compute the valve reliability curve (see Eq. 11). Figure 8 plots the median reliability and 80% credible interval. This can be used to determine the overhaul interval of a newly installed valve. For example, for a 4-year overhaul interval, the median reliability is 0.73 with an 80% credible interval of (0.69 to 0.77).

The overhaul interval of an in-service valve can

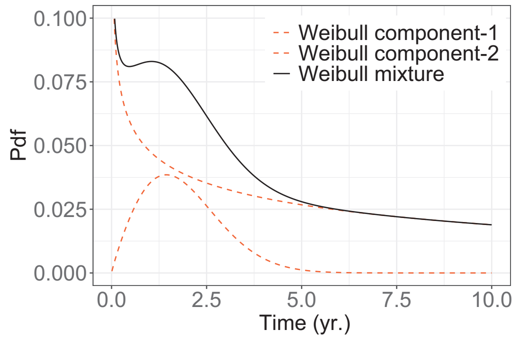

Fig. 7. PDFs of the Weibull components and the mixture

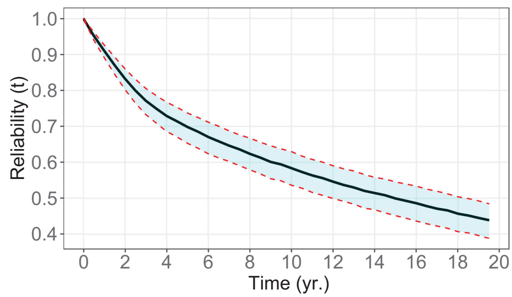

Fig. 8. Median valve reliability and 80% credible interval

be updated based on the conditional reliability curve, also known as mission reliability. For a valve of age t, its reliability over an overhaul interval x can be computed using Eq. 12.

Figure 9 presents the mission reliability curves for $x = 4$ and 6-year overhaul (OH) intervals. Consider the case of $x = 4$ year, a valve of age 2-year will have a median reliability of 0.8 for the next 4-year interval. As the valve age increases, the mission reliability gradually increases. A valve of age 6-year will have 0.85 reliability. If the target reliability of a valve is 0.8 over its OH interval, then this requirement is satisfied for a valve with an age greater than 2 years and an OH interval of 4 years. In the case of a 6-year OH interval, a valve of age 4-years will achieve the median reliability of 0.8. In this manner, the updated OH interval can be estimated for in-service valves.

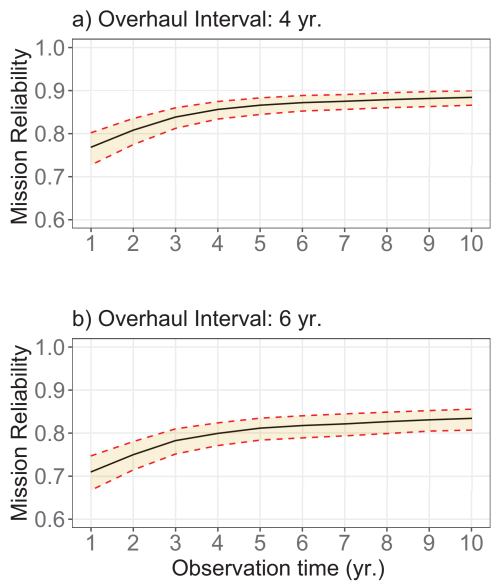

Fig. 9. Conditional reliability curves for reliability tracking

5. Conclusion

In this paper, a Weibull mixture model has been presented to model the lifetime data of heterogeneous component populations. The Bayesian method is proposed to update the reliability and overhaul interval of in-service valves in a more adaptive way. The reliability of a specific component can be tracked as a function of its age along with appropriate credible intervals. As an illustration, The proposed approach is applied to update the overhaul interval of level control valves in a nuclear power plant. In future, the sensitivity of prior parameters on the reliability prediction will be examined.

Acknowledgement

The authors gratefully acknowledge financial support for this study provided by the Natural Science and Engineering Council of Canada (NSERC) and the University Network of Excellence in Nuclear Engineering (UNENE).

References

Carta, J. and P. Ramirez (2007). Analysis of two-component mixture weibull statistics for estimation of wind speed distributions. *Renewable energy 32*(3), 518–531.

Deganaksoy, N., G. Hahn, and W. Meeker (2002). Reliability analysis by failure mode-a useful tool for product reliability evaluation and improvement. *QUALITY PROGRESS 35*(6), 47–52.

Elmahdy, E. E. and A. W. Aboutahoun (2013). A new approach for parameter estimation of finite weibull mixture distributions for reliability modeling. *Applied Mathematical Modelling 37*(4), 1800–1810.

Everitt, B. (2013). *Finite mixture distributions*. Springer Science & Business Media.

Gelfand, A. E. (2000). Gibbs sampling. *Journal of the American Statistical Association 95*(452), 1300 – 1304. Cited by: 196.

Grall, A., C. Bérenguer, and L. Dieulle (2002). A condition-based maintenance policy for stochastically deteriorating systems. *Reliability Engineering System Safety 76*(2), 167–180.

Jiang, R. and D. Murthy (1995). Modeling failure-data by mixture of 2 weibull distributions: a graphical approach. *IEEE Transactions on Reliability 44*(3), 477–488.

Jiang, S. and D. Kececioglu (1992). Graphical representation of two mixed-weibull distributions. *IEEE Transactions on Reliability 41*(2), 241–247.

Moon, Todd, K. (1996). The expectation-maximization algorithm. *IEEE Signal processing magazine 13*(6), 47–60.

Murthy, D. P., M. Xie, and R. Jiang (2004). *Weibull models*. John Wiley & Sons.

Pandey, M., M. Goodchild, A. Sherzad, and R. Wang (2024, 05). Reliability analysis of steam generator level control valves. In *Asset Integrity Management of Critical Infrastructure*. ASME.

Ruhi, S., S. Sarker, and M. Karim (2015). Mixture models for analyzing product reliability data: a case study. *SpringerPlus 4*, 1–14.

Titterington, D. M., A. F. Smith, and U. E. Makov (1985). *Statistical analysis of finite mixture distributions*. John Wiley and Sons.