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Impact of common-cause failures on the availability of connected (r,s)-out-of-(m,n):F systems

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There has been a renewed interest in connected bi-dimensional configurations, since they successfully describe real-life systems such as cellular networks, communication and property security solutions, etc. The computation of their availability has long been known as a delicate task, especially for large systems. Most of the published studies have considered independent failures for identical elements.

In this work, we investigate the modification of the availability of connected (r,s)-out-of-(m,n):F systems when common-cause failures or correlations are present. It turns out that the availability may be larger or smaller than its value in the case of purely independent failures; this is also true for the failure frequency. The transition between these two behaviors occurs at a component critical unavailability q_c that depends on the size of the system. Analytical expressions for the asymptotic values of q_c are given in the case of the β-factor model and binomial failure rate models, as well as for correlations described by a degenerate Gaussian copula. A comparison with the case of one-dimensional consecutive configurations is also provided.

Keywords: Connected (r,s)-out-of-(m,n):F lattice system, Network reliability, Availability, Common-cause failures, Correlations, Asymptotic expansions.

1. Introduction

Telecommunication networks may be simply described by a two-dimensional lattice system, initially proposed as a generalization of the linear consecutive k-out-of-n system.Salvia and Lasher (1990); Boehme et al. (1992); Zuo (1993) A (r,s)-out-of-(m,n):F system consists of $m \times n$ elements arranged in n rows and m columns; the system fails if *all* elements in a block $r \times s$ fail. Initial applications of this model ranged from electronic devices to X-ray and disease diagnostics. More recently, wireless sensors networks, video surveillance systems, pattern search systems, and biological systems have been proposed.Aki and Hirano (2004); Beiu and Dăuş (2015); Cheng et al. (2016); Si et al. (2017). Preventive maintenance and the component assignment problem have recently been addressed.Yun and Endharta (2016); Nakamura et al. (2024) Very good surveys are available.Kuo and Zuo (2003); Akiba et al. (2019)

The calculation of the availability of such systems, especially for large ones, is definitely not simple. Zhao et al. (2011) have improved algorithms, while keeping small values of r, s, and m. More complete states of the art can be found in Akiba et al. (2019); Tanguy (2024); the problem has been revisited, showing that the availability for given r, s, and m can be otained through a recursion relation, the order of which increases rapidly with m.Malinowski and Tanguy (2022); Tanguy and Malinowski (2023) For large ($n \gg 1$) systems of identical and independent elements, the probability of operation of a (r,s)-out-of-(m,n):F has the asymptotic form

$$R_n^{(m)} \approx \delta_* \left(\gamma_*\right)^m \left(\chi_*\right)^n \left(\zeta_*\right)^{m\,n} \tag{1}$$

where δ_*, γ_*, χ_*, and ζ_* depend on r, s, and the common unavailability $q = 1 - p$. When presented at the ESREL 2024 conference, this result attracted a question from the audience: "What happens when failures are not independent? Is the power-law still valid?". This question cannot be waved aside. The issue of common-cause failures (CCFs), and more generally of correlations, has long been studied by the reliability community, and is still important, as witnessed by a recent work on the operational resilience of a network.Yuge et al. (2024)

A very nice introduction to dependent failures can be found in Rausand and Høyland (2004). The main point to recall is that the average value of two probabilities of operation p_i and p_j is not equal to the product of the individual probabilities anymore: $\langle\, p_i\, p_j \,\rangle \neq \langle\, p_i \,\rangle \langle\, p_j \,\rangle$. Many approaches and models have been discussed in Fleming (1974); Atwood (1986); Mosleh et al. (1988); Hokstad and Rausand (2008), to mention only a few. This adds another level of complexity to the present problem. Is it possible to say something sensible about the role of CCFs or correlations for connected two-dimensional systems? Our approach is to apply a few CCF and correlation models developed in Tanguy (2009, 2010, 2011) to a connected two-dimensional (r,s)-out-of-(m,n):F system made of identical elements. The notations $\langle\, \mathcal{R} \,\rangle$ and $\langle\, \nu \,\rangle$ will be used to describe the *measured* availability and failure frequency in the presence of correlations, in order to distinguish them from the usual R and ν when elements are independent.

In this work, we show that in the presence of CCFs or correlations, the availability and failure frequency of the system may *increase* or *decrease* with respect to the independent failures case. Equation (1) will therefore be useful as an initial estimate when dependence effects are small.

The paper is organized as follows. We present in Section 2 the formulas used in the different CCF and correlation models. We expose in Section 3 a study of the (3,3)-out-of-(5,5):F system in the case of the β-factor model, in order to show the main variations of $\langle\, \mathcal{R} \,\rangle$ and $\langle\, \nu \,\rangle$ with β. The regions corresponding to an increase or a decrease of these two quantities are displayed in a way that makes them easier to visualize. Section 4 is devoted to (r,s)-out-of-(m,n):F systems. The asymptotic expressions for the frontiers of the regions are given in Section 5. Before the Conclusion, Section 6 describes the results for other architectures, including the standard linear consecutive k-out-of-n:F configuration.

2. Assumptions and notations

2.1. Systems of identical components

The assumption of identical components may provide some insight to the behavior of large systems.Kołowrocki (2004) Assuming that the N components of the system are identical, the reliability polynomial may be written $R(p) = \sum_{k=1}^{N} a_k\, p^k$, where each a_k is an integer, with $R(0) = 0$ and $R(1) = 1$. The *true* value of the availability under correlations will then be

$$\langle\, \mathcal{R} \,\rangle = \sum_{k=1}^{N} a_k \,\langle\, p^k \,\rangle \tag{2}$$

Note that $\langle\, \mathcal{R} \,\rangle \neq R(\langle\, p \,\rangle)$, since $\langle\, p^k \,\rangle$ is not equal to $\langle\, p \,\rangle^k$ anymore; its value is model-dependent. $\langle\, p \,\rangle$ is actually the *measured* availability of an element of the system.

2.2. *β-factor model*

This model is a well-known one, in which only two failure rates are considered: λ for the individual failure rate, and Λ for the failure rate of all the elements of the system. The factor β is simply given by $\beta = \frac{\Lambda}{\lambda+\Lambda}$. After some calculations (a unique, single-element repair rate μ was assumed); see Tanguy (2009)), the kth coupled population reads, where Γ is the Euler gamma function

$$\langle\, p^k \,\rangle = \frac{\langle\, p \,\rangle^k}{(1-\beta\,(1-\langle\, p \,\rangle))^k}\, \frac{k!\,\Gamma\Big(\frac{1}{1-\beta\,(1-\langle\, p \,\rangle)}\Big)}{\Gamma\Big(k+\frac{1}{1-\beta\,(1-\langle\, p \,\rangle)}\Big)} \tag{3}$$

When $\beta \to 0$, we recover the independent case.

From the previous equations, it is not difficult to find the derivative of $\langle\, \mathcal{R} \,\rangle$ at $\beta = 0$.Tanguy (2010)

$$\begin{aligned} \left.\frac{\partial \langle\, \mathcal{R} \,\rangle}{\partial \beta}\right|_{\beta=0} &= (1 - \langle\, p \,\rangle) \\ &\times \left\{ p\, R'(p) - \int_0^p \frac{R(r) - R(p)}{r-p}\, dr \right\}_{p=\langle\, p \,\rangle} \end{aligned} \tag{4}$$

The failure frequency is another performance measure of systems. Following the above procedure, one can show — after some work — that the variation of $\langle\, \nu \,\rangle$ at the origin, for the most general reliability polynomial and the β-factor model assumption, reads

$$\left.\frac{\partial\langle \nu \rangle}{\partial \beta}\right|_{\beta=0} = \mu\,(1-\langle p \rangle) \times \Big\{ R(p) - p\,R'(p) + p\,(1-p)\,R''(p) - (1-p)\int_0^p \frac{R'(r)-R'(p)}{r-p}\,dr \Big\}_{p=\langle p \rangle} \quad (5)$$

2.3. *Binomial failure rate model*

In this case, we can define an effective $\widetilde{\beta}$ factor for a total number N of elements (see Tanguy (2010) for a detailed exposition). The kth coupled population is then

$$\langle p^k \rangle = \frac{\langle p \rangle^k}{\prod_{i=1}^k \left(\langle p \rangle + (1-\langle p \rangle)\,\frac{\gamma_i}{i}\right)} \quad (6)$$

$$\gamma_i = \frac{1-(1-\widetilde{\beta})^{\frac{i}{N-1}}}{1-(1-\widetilde{\beta})^{\frac{1}{N-1}}} \quad (7)$$

When $\widetilde{\beta} \to 0$ (independent limit), $\gamma_i \to i$, whereas $\widetilde{\beta} \to 1$ leads to $\gamma_i \to 1$.

Setting $R(p) = \sum_{k=1}^{N} a_k\,p^k$ gives

$$\langle \mathcal{R} \rangle = \sum_{k=1}^{N} a_k \prod_{i=1}^{k} \frac{1}{1+\frac{1-\langle p \rangle}{\langle p \rangle}\frac{\gamma_i}{i}} \quad (8)$$

$$\langle \nu \rangle = \lambda_{\text{eff}} \sum_{k=1}^{N} \gamma_k\,a_k \prod_{i=1}^{k} \frac{1}{1+\frac{1-\langle p \rangle}{\langle p \rangle}\frac{\gamma_i}{i}} \quad (9)$$

The derivative of the availability with respect to $\widetilde{\beta}$ at the origin is

$$\left.\frac{\partial\langle \mathcal{R} \rangle}{\partial \widetilde{\beta}}\right|_{\widetilde{\beta}=0} = \frac{1-\langle p \rangle}{4\,(N-1)} \sum_{k=1}^{N} k\,(k-1)\,a_k\,\langle p \rangle^k = \frac{1-\langle p \rangle}{4\,(N-1)}\,\langle p \rangle^2 \left.\frac{\partial^2 R}{\partial p^2}\right|_{p=\langle p \rangle} \quad (10)$$

The interpretation is straightforward: the slope at the origin has the sign of the second derivative $\mathcal{R}''(\langle p \rangle)$. It vanishes at the point of inflection of $\mathcal{R}$, which is easy to determine visually. Here again, if we keep $\langle p \rangle$ constant and increase the size of the network, the slope at the origin will become positive.

The failure frequency at $\widetilde{\beta} = 0$ is

$$\langle \nu \rangle_{\widetilde{\beta}=0} = \lambda_{\text{eff}}\,\langle p \rangle\,R'(\langle p \rangle) \quad (11)$$

$\left(\lambda_{\text{eff}} = \mu\,\frac{1-\langle p \rangle}{\langle p \rangle}\right)$. From eqs. (7) and (9), one gets

$$\left.\frac{\partial\langle \nu \rangle}{\partial \widetilde{\beta}}\right|_{\widetilde{\beta}=0} = \lambda_{\text{eff}}\,\frac{\langle p \rangle^3}{4\,(N-1)} \times \Big\{ (1-p)\,R'''(p) - 2\,R''(p) \Big\}_{p=\langle p \rangle} \quad (12)$$

The critical availability p_c is a solution of $(1-p)\,R'''(p) - 2\,R''(p) = 0$; it is one of the two extrema of $(1-p)^2\,R''(p)$.

2.4. *Degenerate Gaussian copula*

Our last model of correlated probabilities is the degenerate multivariate normal distribution also known as the Gaussian copula; it has been used in the context of network reliability.Walter et al. (2008); Tanguy (2011) Correlations are described by a single parameter ρ. For identical components, the relevant integrals are

$$\langle p^k \rangle = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}}\,e^{-u^2} \times \left(\frac{1}{2} + \frac{1}{2}\,\text{erf}\left[\frac{\text{erf}^{-1}(2\,p-1)}{\sqrt{1-\rho}} - \frac{\sqrt{\rho}}{\sqrt{1-\rho}}\,u\right]\right)^k du \quad (13)$$

where erf is the standard error function. When $\rho = 0$, we recover the independent case $\langle p^k \rangle = p^k$. The fully correlated situation is reached when $\rho \to 1$ (and $\langle p^k \rangle \to p$). Since small correlations are considered, a Taylor expansion in ρ is what really matters. Using Eq. (13), one obtains $\left(\xi = \left[\text{erf}^{-1}(2\,\langle p \rangle - 1)\right]^2\right)$

$$\langle \mathcal{R} \rangle = R(\langle p \rangle) + \frac{\rho}{4\,\pi}\,e^{-2\xi}\,R''(\langle p \rangle) + O(\rho^2) \quad (14)$$

3. A simple example: (3,3)-out-of-(5,5):F system and β-factor model

Before turning to the behavior of a general connected (r,s)-out-of-(m,n):F system in the next Sections, we first consider the (3,3)-out-of-(5,5):F configuration, as already studied in the literature Yun and Endharta (2016). In that case, the reliability for identical components is given by (the expression of the reliability polynomial is shorter when using q instead of p)

$$R = 1 - 9\,q^9 + 12\,q^{12} + 8\,q^{14} - 16\,q^{15} + 12\,q^{16} - 14\,q^{17} + 8\,q^{18} - 10\,q^{19} + 12\,q^{20} - 4\,q^{21} \quad (15)$$

From (15), one can compute the values of $\langle \mathcal{R} \rangle$ and $\langle \nu \rangle$ as functions of $\langle p \rangle$ and β, as well as their derivatives with respect to β. The latter are displayed in Fig. 1. In the case of $\langle \mathcal{R} \rangle$, the sign of the slope is positive for $\langle p \rangle$ less than a critical value $p_c \approx 0.208549$, and negative above it. The observed availability $\langle \mathcal{R} \rangle$ will increase or decrease — with respect to its expected value for independent failures — depending on $\langle p \rangle$. In the case of the failure frequency, two critical points exist for $\langle p \rangle$, at 0.054153 and 0.362715, leading to another increase/decrease behavior. Similar variations are observed for $\beta = 1$: the critical value for the availability is 0.076027, while they are 0.016598 and 0.221638 for the failure frequency (see Fig. 1).

In order to make things more explicit, one can focus on the $\beta = 0$ limit which corresponds to a vanishingly small influence of CCFs. In Fig. 2, the colored regions for $\langle p \rangle$ indicate what kind of behavior is to be expected for the true availability and failure frequency with respect to their values in the absence of CCFs. The color code is

$\langle \mathcal{R} \rangle \nearrow \langle \nu \rangle \nearrow$	$\langle \mathcal{R} \rangle \nearrow \langle \nu \rangle \searrow$
$\langle \mathcal{R} \rangle \searrow \langle \nu \rangle \searrow$	$\langle \mathcal{R} \rangle \searrow \langle \nu \rangle \nearrow$

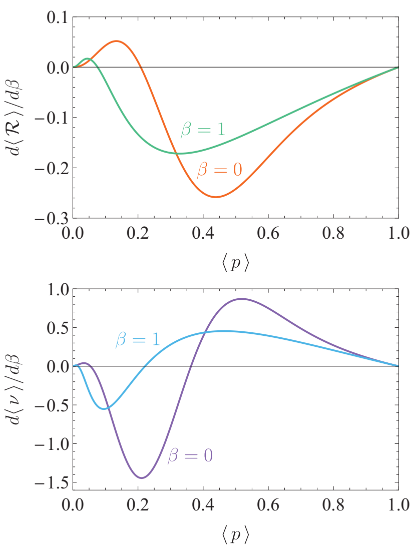

Fig. 1. Slopes of $\langle \mathcal{R} \rangle$ (top) and $\langle \nu \rangle$ (bottom) at $\beta = 0$ and $\beta = 1$ for the (3,3)-out-of-(5,5):F system.

In the following, the dependence of these two derivatives at the origin $\beta = 0$ will be considered for various $(r,2)$-out-of-(m,n):F systems, for the β-factor model and the two other models described in Section 2. This will allow to assess the influence of the system size on the global behavior.

4. $(r,2)$-out-of-(m,n):F systems

4.1. *β-factor model in the limit $\beta \to 0$*

Vanishingly small contributions of CCFs are again considered. The critical value $p_c(n)$ has been calculated for different values of r, but in this Section, the focus is made on the case $r = 3$. When m goes from 3 to 12, the reliability polynomials of $(r,2)$-out-of-(m,n):F systems can be calculated exactly throught recursion relations. Tanguy and Malinowski (2023); Tanguy (2024) One can observe in Fig. 3 that p_c — the value of $\langle p \rangle$ for which the introduction of CCFs does not change the observed availability — increases with m and n. One may have a better understanding of the expected behavior of large systems by also considering the failure frequency, and investigate how the various regions of decrease/increase of $\langle \mathcal{R} \rangle$ and $\langle \nu \rangle$ evolve for a given m, when n increases. Starting with $m = 3$ (see top of Fig. 4) one notes that the four regions vary with n; the case $m = 12$ provides a displaced set of curves and regions, as shown at the bottom of Fig. 4.

As n increases, p_c continues increasing. The value of m plays a similar role, too, as expected in

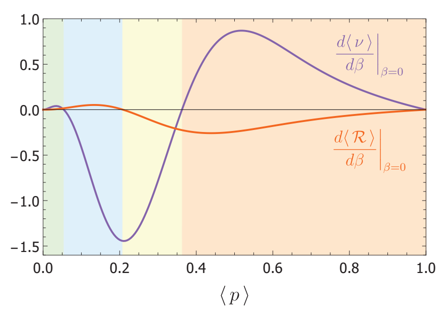

Fig. 2. Slopes of $\langle \mathcal{R} \rangle$ and $\langle \nu \rangle$ at $\beta = 0$ for the (3,3)-out-of-(5,5):F system. $\langle \mathcal{R} \rangle \nearrow \langle \nu \rangle \nearrow$ $\langle \mathcal{R} \rangle \nearrow \langle \nu \rangle \searrow$ $\langle \mathcal{R} \rangle \searrow \langle \nu \rangle \searrow$ $\langle \mathcal{R} \rangle \searrow \langle \nu \rangle \nearrow$

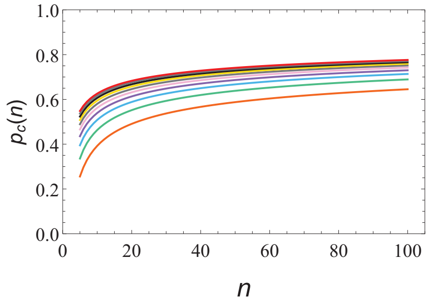

Fig. 3. Variation of p_c with n. The lowest curve corresponds to $m = 3$, the highest one to $m = 12$.

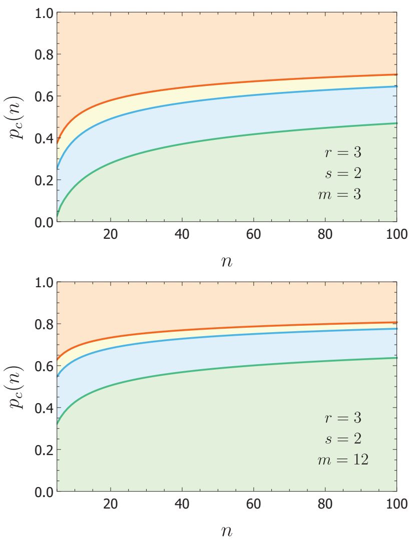

Fig. 4. Regions at $\beta = 0$ for ($r = 3$,$s = 2$,$m = 3$) (top) and ($r = 3$,$s = 2$,$m = 12$) (bottom).

such a connected two-dimensional system. Note that the increase with n of the three curves is rather slow. Their asymptotic variations will be explicited in a later Section.

4.2. *Binomial failure rate model ($\tilde{\beta} \to 0$)*

The derivatives of the availability and the failure frequency have been computed when $\tilde{\beta} \to 0$ using Eqs. (10) and (12), again in the limit of a vanishing CCF contribution. The variation of the domains is displayed in Fig. 5 for $m = 3$ and $m = 12$.

The *general* behavior is the same as for the β-factor model, with quantitatively slightly different numerical values. The increase with n of the separation curves is still very slow.

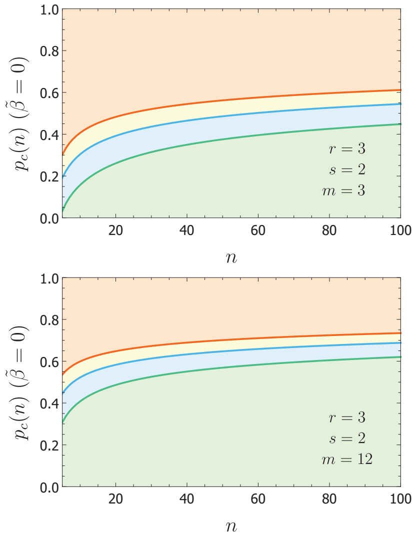

Fig. 5. Same as Fig. 4 at $\tilde{\beta} = 0$.

4.3. *Degenerate Gaussian copula ($\rho \to 0$)*

The derivatives of the availability and failure frequency in the limit of vanishingly small correlations can be computed at $\rho = 0$, so that the critical values p_c or $q_c = 1 - p_c$ may be computed very easily from the zeros of $R''(p)$ and $(p\,R'(p))'$, respectively. The result is displayed in Fig. 6 for the configuration $r = s = 2$, and $m = 7$.

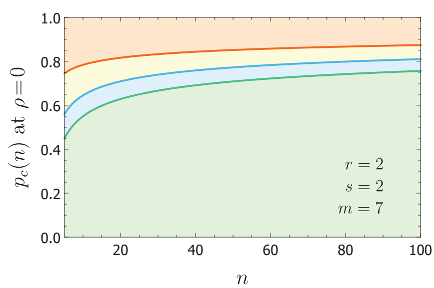

Fig. 6. Regions at $\rho = 0$ for $r = 2$, $s = 2$ et $m = 7$.

5. Asymptotic limits

The purpose of the present Section is to provide the leading term of the large n behavior of the various critical values delimiting the different regimes for the increase or decrease of the availability and the failure frequency with respect to an independent failures situation, in the limit of small common-cause failures. Basically, one must find the zeros of the derivatives of $\langle \mathcal{R} \rangle$ and $\langle \nu \rangle$ in the limit $n \to \infty$.

The availability of the connected (r,2)-out-of-(m,n):F system has been shown, for arbitrary r and $s = 2$, to be of the form of Eq. (1). Further calculations for other values of $s > 2$ indicate that the leading terms of the Taylor expansions of ζ_*, χ_*, γ_*, and δ_* are

$$\ln \zeta_* = -q^{rs} + \cdots \tag{16}$$

$$\ln \chi_* = (r-1)\, q^{rs} + \cdots \tag{17}$$

$$\ln \gamma_* = (s-1)\, q^{rs} + \cdots \tag{18}$$

$$\ln \delta_* = -(r-1)\,(s-1)\, q^{rs} + \cdots \tag{19}$$

This leads to

$$R_n \approx e^{-(n-s+1)\,(m-r+1)\,q^{rs}} \tag{20}$$

The critical values of $\langle p \rangle$, p_c, or their complements to 1, q_c, can be found from the cancellation of the derivatives of $\langle \mathcal{R} \rangle$ and $\langle \nu \rangle$ at the respective origins (see Section 2).

5.1. *Binomial failure rate model*

The zeros of the derivatives of the availability and the failure frequency are now given by the solutions of $R''(p) = 0$ and $(1-p)\,R'''(p) - 2\,R''(p) = 0$, respectively. Using Eq. (20), one finds that the critical value for the availability is

$$q_c \to \left(\frac{r\,s-1}{r\,s\,(n-s+1)\,(m-r+1)} \right)^{\frac{1}{r\,s}} \tag{21}$$

In the case of the failure frequency, the two critical values are given by

$$(q_c)_\pm \to \left(\frac{3\,r\,s-1 \pm \sqrt{5\,r^2\,s^2-2\,r\,s+1}}{2\,r\,s\,(n-s+1)\,(m-r+1)} \right)^{\frac{1}{r\,s}} \tag{22}$$

5.2. *Degenerate Gaussian copula model*

For this model, the relevant equations for the critical parameters are $R''(p) = 0$ (again) and $(r\,R'(p))'' = 0$. This means that the critical q_c for the availability is the same as in Eq. (21). However, for the failure frequency, slightly different $(q_c)_\pm$ are obtained:

$$(q_c)_\pm \to \left(\frac{3\,(r\,s-1) \pm \sqrt{(r\,s-1)\,(5\,r\,s-1)}}{2\,r\,s\,(n-s+1)\,(m-r+1)} \right)^{\frac{1}{r\,s}} \tag{23}$$

5.3. *β-factor model*

The calculations of the critical values are a bit more involved because of the integrals in Eqs. (4) and (5). Following the derivation in Tanguy (2010), one finds

$$q_c \to \left(\frac{\ln(n-s+1)\,(m-r+1)+\gamma}{r^2\,s^2\,(n-s+1)\,(m-r+1)} \right)^{\frac{1}{r\,s-1}} \tag{24}$$

where $\gamma \approx 0.577...$ is Euler's constant. The critical values related to the failure frequency have different power-law behaviors and no $\ln$ term:

$$(q_c)_- = \left(\frac{1}{r^2\,s^2\,(n-s+1)\,(m-r+1)} \right)^{\frac{1}{r\,s-1}} \tag{25}$$

$$(q_c)_+ = \left(\frac{1}{(n-s+1)\,(m-r+1)} \right)^{\frac{1}{r\,s}} \tag{26}$$

5.4. *Discussion*

The above expressions show that all the critical values q_c tend to zero as $n \to \infty$. They also agree with the observed behavior exhibited in the preceding Section. For large enough systems, one will therefore reach a point located in the green zone; this implies that the true availability and failure frequency would both increase when even a small fraction of CCFs is introduced. Note that even though the global evolution with size is similar, the n-dependences are slightly different. Remember that the above equations only provide the leading term of expressions that converge *very slowly* to zero, and that the following terms of the expansions decrease only slightly faster than the prevailing one.

6. Comparison with other architectures

6.1. *Circular connected two-dimensional structures*

The first obvious architecture is the circular or cylindrical connected (r,s)-out-of-(m,n):F configuration. It has been shown in Tanguy (2024) that in that configuration, the asymptotic availability reads

$$R_n^{(C)} \approx (\gamma_*)^m\,(\zeta_*)^{m\,n} \tag{27}$$

leading to $R_n^{(C)} \approx e^{-m\,(n-s+1)\,q^{rs}}$. One can therefore expect a behavior similar to that of the simple connected configuration of the preceding Sections. In all the asymptotic limits, one merely has to replace $(m-r+1)$ by m. The change is marginal, but means that all the critical values q_c should be smaller in the circular configuration. This is indeed observed in Fig. 7.

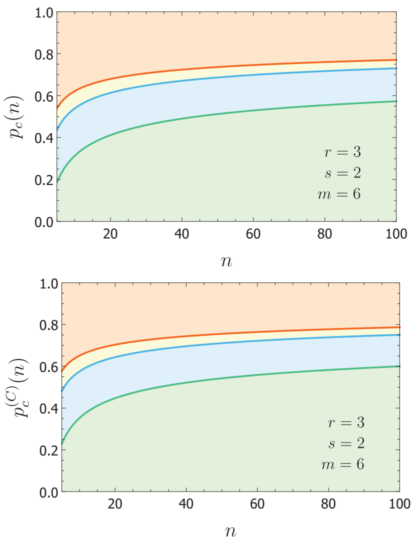

Fig. 7. Regions at $\beta = 0$ for LinCon and CirCon/(3,2)-out-of-(6,n):F.

6.2. *Con/k-out-of-n:F*

The method used in the previous Sections can be used to look at the influence of CCFs or correlations for the well-known linear connected k-out-of-n:F system. In that configuration,

$$R_n \approx e^{-(n-k+1)\,q^k}\,. \tag{28}$$

leading, in the case of the β-factor model, to

$$q_c \to \left(\frac{\ln(n-k+1)+\gamma}{k^2\,(n-k+1)}\right)^{\frac{1}{k-1}} \tag{29}$$

One has to replace s by k and remove all the terms $(m-r+1)$. The particular case $k = 6$ is displayed in Fig. 8.

7. Conclusion and outlook

We have considered the inclusion of various models of common-cause failure to assess how they

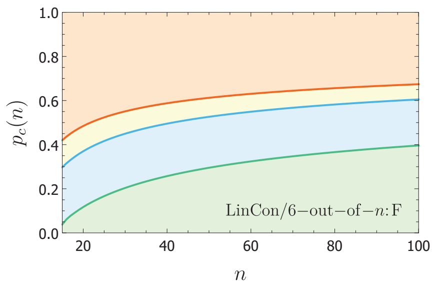

Fig. 8. Regions at $\beta = 0$ for LinCon/6-out-of-n:F.

can, even when their contribution is very small, modify the availability and the failure frequency of a two-dimensional (r,s)-out-of-(m,n):F system. We have shown that depending on the size of the system and the average *perceived* availability $\langle\, p\,\rangle$, both these performance indices may increase or decrease with respect to a situation where these CCFs and correlations are absent. There are therefore four different regions, the extension of which depends on r, s, m, n and $\langle\, p\,\rangle$. The asymptotic limits of their separations have been provided and shown to increase *very* slowly with the system size. For extremely large sytems ($n \gg 1$), the introduction of CCFs should translate into an *increase* of both availability and failure frequency. This is also true for linear consecutive k-out-of-n:F systems.

The method described in this paper can be readily transposed to other configurations. Even though the results will quantitatively depend on the specific architecture of the system and its size, the general behavior should remain, for the CCF models under consideration, qualitatively the same.

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