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Bayesian Updating for Reliability with Imprecise Probabilities: Julia Implementation and Application to the NASA Langley UQ Challenge 2019

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Model updating has emerged as a technique of great importance in a number of engineering contexts where measurements are employed to infer the parameters of systems. In many applications, the limited availability of experimental data, coupled with significant model uncertainties, presents a considerable challenge for inferring accurate model parameters, particularly in the context of hybrid uncertainties. Nevertheless, accurately quantifying these uncertain parameters is of key importance in order to ensure the reliability of systems within an engineering context. This highlights the need for flexible yet robust methodologies to address systems with imprecise probability models in the context of stochastic model updating. In light of these considerations, we present a Julia implementation of a Bayesian updating technique within a structural reliability framework. The objective is to infer parameters represented by imprecise probabilistic models and obtain imprecise failure probabilities. The methodology is illustrated with reference to the NASA Langley UQ Challenge 2019, which demonstrates the application of Bayesian techniques in updating the uncertainty model of a black-box system and conducting subsequent reliability analysis. A two-step Bayesian updating procedure is employed here. This procedure uses the Transitional Markov Chain Monte Carlo algorithm and applies model reduction techniques to construct an approximate likelihood function from coefficients of time series data using the Euclidean and Bhattacharyya distances.

Keywords: Bayesian Updating, Imprecise Probabilities, Reliability Analysis, Hybrid Uncertainty, Transitional Markov Chain Monte Carlo, Uncertainty Quantification, Julia Programming, NASA Langley UQ Challenge

1. Introduction

In the field of engineering, the accuracy of predicting system behavior is crucial, and this often requires continuously updating models to reflect current conditions. Such model updating is vital across numerous sectors like aerospace, civil engineering, and structural health monitoring. In these applications practitioners often deal with sparse and noisy data (Bi et al., 2023). This task is further complicated by the presence of hybrid uncertainties within the model, arising from both inherent randomness (aleatory uncertainty) and incomplete knowledge (epistemic uncertainty). Traditional probabilistic methods often fall short in representing the full range of these uncertainties, particularly in situations with limited data and high sensitivity to model assumptions.

Imprecise probability models, such as probability boxes (p-boxes) and fuzzy probability models, offer a flexible approach to address these challenges. By bounding probabilities instead of relying on precise estimates, these models can incorporate both types of uncertainty, providing a more robust framework for reliability assessment (Beer et al., 2013; Faes et al., 2021). The Bayesian updating framework, which allows prior knowledge to be updated in light of new data, is particularly effective in this context, as it enables a systematic integration of sparse information with existing model assumptions (Beck and Katafygiotis, 1998). There, Bayes' theorem is employed to derive the posterior distribution of model parameters $\boldsymbol{\theta}$ conditioned on data $\boldsymbol{D}$, which is denoted as $p(\boldsymbol{\theta}|\boldsymbol{D})$. We can obtain said posterior distribution by combining the prior distribution $p(\boldsymbol{\theta})$ and the likelihood function $\mathcal{L}(\boldsymbol{D}|\boldsymbol{\theta})$ using Bayes' theorem:

$$p(\boldsymbol{\theta}|\boldsymbol{D}) \propto \mathcal{L}(\boldsymbol{D}|\boldsymbol{\theta}) \cdot p(\boldsymbol{\theta}). \tag{1}$$

In this paper, we present an implementation of a Bayesian updating framework using the Julia programming language. The methodology employs a two-step Bayesian updating process introduced by Kitahara et al. (2022) to handle imprecise probabilities using p-boxes. There, sampling is performed using the Transitional Markov Chain Monte Carlo (TMCMC) algorithm (Ching and Chen, 2007). Further, a model reduction technique based on Prony's method (Hauer et al., 1990) is employed to construct an approximate likelihood function based on time series data of dynamic systems, using Euclidean and Bhattacharyya distances as metrics (Bi et al., 2019). Afterwards, double-loop Monte Carlo simulation is used to propagate the hybrid uncertainties through the model to estimate the bounds of the failure probabilities (Faes et al., 2021).

2. NASA Langley UQ Challenge 2019

To demonstrate the effectiveness of the proposed methodology, we apply it to a case study from the NASA Langley Uncertainty Quantification Challenge 2019 (Crespo and Kenny, 2021). The challenge involves a complex system with hybrid uncertainties, where the objective is to calibrate the model parameters and estimate the reliability of the system under uncertain conditions. The system consists of an integrated system $\boldsymbol{z}$ and a subsystem y, which are given as:

$$\begin{aligned} \boldsymbol{z} &\equiv z(\boldsymbol{a}, \boldsymbol{e}, t) : \mathbb{R}^5 \times \mathbb{R}^4 \times [0, T] \to \mathbb{R}^2, \\ y &\equiv y(\boldsymbol{a}, \boldsymbol{e}, t) : \mathbb{R}^5 \times \mathbb{R}^4 \times [0, T] \to \mathbb{R}^1. \end{aligned} \tag{2}$$

Here, $\boldsymbol{a} \sim p_{\boldsymbol{a}}(\boldsymbol{a})$ represents five aleatory parameters, $\boldsymbol{e} \in \boldsymbol{E}$ represents four epistemic parameters, and $T = 5$ denotes the final time step of the analysis.

The objective of the first part, the Model Calibration, is to infer the uncertainty model of these parameters $\langle p_{\boldsymbol{a}}(\boldsymbol{a}), \boldsymbol{E} \rangle$. Therefore, a set of time-series data of the subsystem, denoted as $\boldsymbol{D}_1$, is provided. The goal is to update the uncertainty model of the parameters $\boldsymbol{a}$ and $\boldsymbol{e}$ using the provided data.

The second part, the Reliability Analysis of the Baseline Design, involves conducting a reliability analysis of the integrated system using the updated parameters. Therefore, three performance criteria g_1, g_2 and g_3, and a worst-cast performance w are provided to evaluate the reliability of the system. The goal is to estimate the failure probabilities of the system under the given performance criteria.

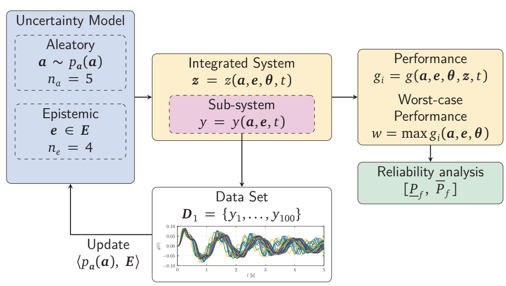

Fig. 1. Flowchart of the performed tasks of the NASA 2019 UQ Challenge.

A flowchart summarizing the tasks, the provided systems and the data-set is shown in Fig 1. Other parts of the challenge, such as the reliability based design optimization, are not considered in this paper.

2.1. *Bayesian Updating and Parameter Calibration*

The Bayesian updating methodology applied in this study utilizes a two-step approach, as proposed by Kitahara et al. (2022), specifically designed for imprecise probability models under hybrid uncertainties. The primary objective of this method is to refine parameter estimates iteratively by incorporating observed data $\boldsymbol{D}_1$ from the subsystem and adjusting the posterior distribution of the model parameters $\boldsymbol{a}$ and $\boldsymbol{e}$.

A first important step of the model calibration is to introduce a model order reduction technique to reduce the complexity of the model output from the time-series data to a limited number of coefficients. This is achieved by applying Prony's method to decompose the time-series data into primary modes. This approach was also used in the contribution of Agrell et al. (2022) to the NASA UQ Challenge 2019. The method decomposes a time-series $x(t)$ of N_t equally spaced samples into a linear combination of p damped cosines:

$$x(t) = \sum_{k=1}^{p} A_k \cos(\omega_k t + \theta_k) e^{\alpha_k t}, \qquad (3)$$

with varying amplitudes A_k, damping factors α_k, frequencies ω_k, and phase angles θ_k. Hence, a time signal can be efficiently represented by a reduced number of parameters, when the signal is dominated by a few primary modes. Here, it was found that the signals of the data-set $\boldsymbol{D}_1$ can be well approximated by three primary modes. Therefore, a time series is represented by a total of twelve parameters, three for each mode. The Prony decomposition is used to reduce the dimensionality of the data by shifting from the time domain with 5001 discrete time steps to the frequency domain with twelve Prony coefficients. For comparison, the original time series data and the Prony decomposition of the data are shown in Fig 2.

In this approach, the distribution of the aleatory model parameters $p_{\boldsymbol{a}}(\boldsymbol{a})$ are represented as staircase random variables (Crespo et al., 2018). Staircase random variables are characterized by a piecewise constant probability density function

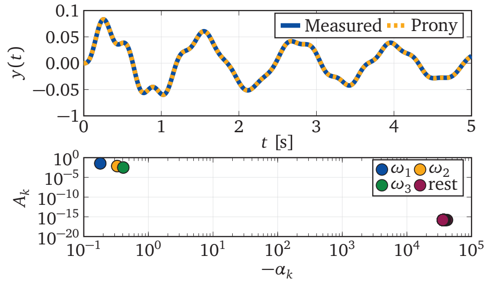

Fig. 2. First measured time series $y(t)$ from the dataset $\boldsymbol{D}_1$ along with its Prony fit and a scatter plot of the amplitudes A_k over the damping coefficients α_k.

(PDF), determined on predefined subintervals that partition the support set of the variable. Each aleatory parameter is defined over the support set $\mathcal{D}_a = [0,\ 2]$. Then, the piecewise constant PDF is obtained by the solution of an optimization problem which maximizes the entropy of the PDF, subject to the constraints that the PDF is non-negative, integrates to one, and is consistent with the available information on first four central moments of the random variable.

The moments of the staircase variables are inferred using a Bayesian updating procedure. Initially, a prior is placed on the parameters such that the moments align with the interval constraints of the variables $\boldsymbol{a}$. Subsequently, a p-box based on the staircase random variables is constructed for each of the five aleatory variables. These p-boxes are derived from the ranges of the moments of the staircase random variables which are determined from the posterior distribution. This methodology has been introduced by Kitahara et al. (2022) and applied by Lye et al. (2022) to the NASA UQ Challenge 2019.

Thus, Bayesian updating is performed to determine the $5 \times 4 = 20$ moments of the staircase random variables that model the aleatory parameters $\boldsymbol{a}$, as well as the four epistemic parameters $\boldsymbol{e}$. To achieve this objective, a comparison is made between the model's predictions and the observed data utilizing the twelve Prony coefficients, as previously delineated. An approximate likelihood is constructed using a stochastic distance metric d between the distribution of the Prony coefficients of the model and the observed data. This likelihood is given as:

$$\mathcal{L}(\boldsymbol{D}|\boldsymbol{\theta}) \propto \exp\left(-\frac{d}{\varepsilon}\right)^2. \tag{4}$$

Here, ε is a scaling factor that is used to adjust the sensitivity of the likelihood function to the distance metric d.

In the first part of the two-step updating approach, the Euclidean distance d_E is used to quantify the difference between the mean Prony coefficients from the simulated data and the observed data. The Euclidean distance is given as:

$$d_E = \sqrt{(\bar{\boldsymbol{x}}_{\text{obs}} - \bar{\boldsymbol{x}}_{\text{sim}})\,(\bar{\boldsymbol{x}}_{\text{obs}} - \bar{\boldsymbol{x}}_{\text{sim}})^T} \tag{5}$$

where $\bar{\boldsymbol{x}}_{\text{obs}}$ represents the mean of the observed features, and $\bar{\boldsymbol{x}}_{\text{sim}}$ denotes the mean of the simulated features. In this case, the features denote the twelve Prony coefficients of the data-set and model predictions, respectively. The Euclidean distance serves to identify an intermediate posterior distribution in the first step, shifting the mean samples towards accordance with the observed data.

In the second step, the updating process transitions to a likelihood function based on the Bhattacharyya distance d_B, which is able to capture a higher level of similarity between the distributions. This distance metric evaluates the overlap between the PDFs of the features of the observed data and the model predictions. In the continuous case, the Bhattacharyya distance is defined as:

$$d_B = -\log\left[\int_{\mathcal{D}_{\boldsymbol{x}}} \sqrt{p_{\text{obs}}(\boldsymbol{x})p_{\text{sim}}(\boldsymbol{x})}\,\mathrm{d}\boldsymbol{x}\right]. \tag{6}$$

However, here we consider the discrete case, where the Bhattacharyya distance is given in terms of higher-dimensional probability mass functions (PMFs). Therefore, a binning algorithm introduced by Bi et al. (2019) is employed to determine the Bhattacharyya distance between the PMFs of the observed data and the model predictions. The goal of the second step of the updating is to refine the posterior distribution of the moments of the parameters $\boldsymbol{a}$ and the parameters $\boldsymbol{e}$, by adjusting the distribution to better match the observed data due to use of higher order statistical information in the Bhattacharyya distance.

Sampling is performed using the TMCMC algorithm (Ching and Chen, 2007). TMCMC is a variant of the Metropolis-Hastings algorithm, which is designed to sample from multimodal distributions by sampling from a sequence of intermediate distributions. These transitional densities are defined as:

$$p^j(\boldsymbol{\theta}) \propto \mathcal{L}(\boldsymbol{D}|\boldsymbol{\theta})^{\beta_j} \cdot p(\boldsymbol{\theta}) \tag{7}$$

where $j \in \{1, 2, \ldots, m\}$ represents the transition steps, with the corresponding tempering parameter β_j progressing from $\beta_0 = 0$ to $\beta_m = 1$ through intermediate values $\beta_0 = 0 < \beta_1 < \beta_2 < \cdots \beta_m = 1$. This gradual transition allows the prior density $p(\boldsymbol{\theta})$ to smoothly evolve into the posterior density $\mathcal{L}(\boldsymbol{D}|\boldsymbol{\theta}) \cdot p(\boldsymbol{\theta})$.

Figure 3 illustrates the flowchart of the Bayesian updating procedure for the NASA 2019 UQ Challenge. The two-step updating process is performed iteratively to refine the model parameters and obtain more accurate reliability estimates.

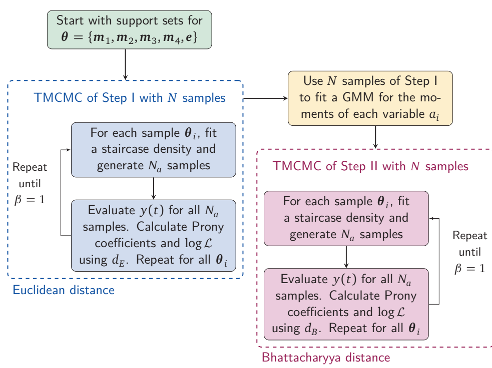

Fig. 3. Flowchart of the Bayesian updating procedure for the NASA 2019 UQ Challenge.

2.2. *Reliability Analysis with Imprecise Probabilities*

With the updated model parameters from the Bayesian calibration, a reliability analysis is performed while considering the hybrid uncertainties given by the p-boxes of the aleatory parameters $p_{\boldsymbol{a}}(\boldsymbol{a})$ and the bounds of the epistemic parameters

$\boldsymbol{E}$. The reliability analysis utilizes a double-loop Monte Carlo simulation (MCS) approach to propagate both aleatory and epistemic uncertainties. In this setup, the outer loop samples N_e realizations of epistemic uncertainties $\boldsymbol{E}$, while the inner loop samples N_a realizations of aleatory uncertainties for each epistemic sample (see Faes et al., 2021). The nested MCS structure estimates bounds for each of the system's failure probabilities, $P_{f,1}$, $P_{f,2}$, $P_{f,3}$, and a worst-case failure probability denoted as P_f, corresponding to the respective performance criteria g_1, g_2, g_3 and w. The bounds of the (individual) failure probabilities are represented as:

$$P_{f,i} = \left[\min_{\boldsymbol{e} \in \boldsymbol{E}} \mathbb{P}\left[g_i \geq 0\right], \ \max_{\boldsymbol{e} \in \boldsymbol{E}} \mathbb{P}\left[g_i \geq 0\right] \right]. \quad (8)$$

The procedure is repeated across a range of confidence levels $\alpha = \{0.20, 0.50, 0.75, 0.90, 1.00\}$, producing probability bounds that reflect the degree of epistemic uncertainty remaining after Bayesian updating. At $\alpha = 1.0$, the failure probability bounds converge to a precise cumulative distribution function (CDF), while lower α-levels provide increasing levels of imprecision, visualized as fuzzy probability sets (Beer et al., 2013).

3. Julia Implementation

All algorithms used to address the NASA UQ Challenge 2019 tasks were implemented in the Julia programming language.

The original model for the NASA UQ Challenge was provided in MATLAB, but a compiled C version, named `uqsim`, was later released by the challenge authors. The Julia implementation developed in this study integrates directly with `uqsim`, facilitating seamless evaluation of the model. The Julia code communicates with the executable via input and output files, enabling the specification of aleatory, epistemic, and design variables, as well as the extraction of responses $y(t)$, $z_1(t)$, $z_2(t)$, and performance measures g_1, g_2, and g_3. This interaction is visualized in Fig 4.

The methods of the two-stage model updating approach were implemented in Julia, including the Prony decomposition of the output, the construction of the approximate likelihood functions,

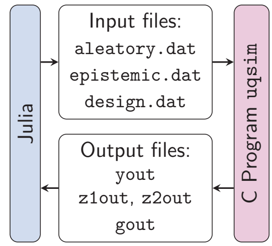

Fig. 4. Visualization of the connection between the Julia framework and the executable `uqsim`.

and the fitting of the staircase random variables to the observed data. This implementation was based on a Bayesian updating framework from the `UncertaintyQuantification.jl` package (Behrensdorf et al., 2024) along with its TMCMC implementation. There, parallel computing capabilities within Julia are used to accelerate the evaluation of approximate likelihood functions, which is crucial for the double-loop approach in model updating arising from the hybrid uncertainties. Each evaluation of the likelihood functions corresponds to a sample in the 24-dimensional input space (5×4 moments and four epistemic parameters). Then, staircase densities are constructed and 500 samples are drawn from the staircase densities to evaluate the model. Finally, the likelihood is obtained by comparing the Prony coefficients of the model and the observed data using the respective stochastic distance metric. In this setup, we define the likelihood function as a `ParallelModel` provided by the `UQ.jl` package. Then, the evaluations of said 'model' are performed in parallel using Julia's `pmap` function. The computations were performed on a local high performace computing cluster using 50 cores.

Crucial to the two-step approach is that the posterior of the first step is used as the prior for the second step. Therefore, a Gaussian Mixture Model (GMM) was fitted to the samples of the posterior of the first step as shown in Fig 3. The GMM was used to build the prior for the second step. This GMM was fitted using the `GaussianMixtures.jl` package (van Leeuwen et al., 2024) which provides a representation of the PDF of the GMM and allows to draw samples from it.

4. Results

Finally, the implemented methodology was applied to the NASA Langley UQ Challenge 2019.

4.1. *Model Calibration*

First, the model calibration was performed using said Bayesian updating framework. The two-step updating process was applied to refine the model parameters represented by imprecise probability models. The results of the sampling process of the Bayesian updating using TMCMC are shown in Table 1 below. This table compares the setup and results of the two steps of the Bayesian updating process, including the distance metric used, the number of samples, the number of accepted samples, the acceptance ratio, and the number of TMCMC steps, to showcase the characteristics of the sampling process.

Table 1. Setup of the two-step Bayesian updating.

	Step I	**Step II**
Distance $\mathcal{L}$	Euclidean	Bhattacharryya
Samples	1000	1000
Accepted	288	145
Ratio	0.288	0.145
TMCMC steps	10	9

It is notable that the acceptance rate for both steps of the TMCMC algorithm is relatively low. This can be attributed to the high dimensionality of the parameter space, which presents a significant challenge in effectively exploring the parameter space. Additionally, the acceptance rate for the second step is observed to be lower than that of the first step. This can be attributed to the higher complexity of the likelihood function in the second step.

Despite the high rejection rate, the distribution of the log-likelihood indicates that the posterior samples represent an improvement over the prior samples, shown by improved ranges of log-likelihood values. This suggests that new, valuable samples were found during the process. Nevertheless, the distributions resulting from the posterior samples must be evaluated with a degree of skepticism.

The results of the Bayesian updating are visualized in Fig 5, which shows the p-boxes of the aleatory parameters $\boldsymbol{a}$ after the calibration process. The p-boxes were determined from the posterior distributions of the moments of the staircase density functions of the aleatory parameters $\boldsymbol{a}$. Then, using the histogram of those moments, the ranges of the moments were determined for each α-level to obtain the p-boxes, as also explained in more detail by Lye et al. (2022). Further, it is seen in Fig 5 that the p-boxes are refined in the two-step updating process, with the second step narrowing the bounds of the aleatory parameters substantially.

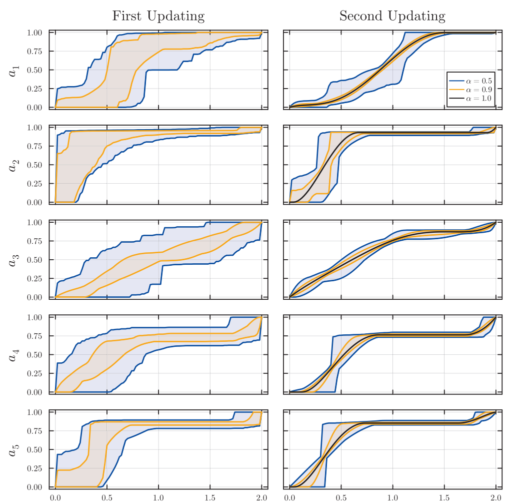

Fig. 5. P-boxes of the aleatory parameters $\boldsymbol{a}$ after the Bayesian updating.

To complement the p-boxes, the results of the epistemic parameters $\boldsymbol{e}$ are shown in Table 2. The bounds of the epistemic parameters shown in that table were determined from the corresponding samples of the posterior distribution of the second updating step. Similarly, different α-levels were considered to obtain the bounds of the epistemic parameters for varying levels of precision.

When comparing the bounds of the updated parameters to their initial bounds given as $\boldsymbol{E}_0 = [0,2]^4$, it becomes clear that the bounds are much narrower after the second updating step. Even for the largest level of imprecision considered, *i.e.*,

Table 2. Bounds of the four epistemic parameters $\boldsymbol{e}$ for different levels of precision α.

α	e_1	e_2	e_3	e_4
0.2	[0.63, 0.77]	[1.02, 1.20]	[1.35, 1.67]	[1.34, 1.43]
0.50	[0.65, 0.74]	[1.04, 1.16]	[1.44, 1.64]	[1.37, 1.41]
0.75	[0.66, 0.70]	[1.06, 1.13]	[1.52, 1.62]	[1.38, 1.40]
0.90	[0.67, 0.69]	[1.09, 1.12]	[1.54, 1.60]	[1.39, 1.40]
1.00	0.68	1.11	1.57	1.39

$\alpha = 0.20$, the resulting bounds are substantially narrower than their initial values.

Another aspect of the analysis is the computational cost of the two-step updating approach with the TMCMC algorithm. An improvement could be achieved by optimizing the sampling algorithm, *e.g.* by using Kriging-assisted TMCMC (Angelikopoulos et al., 2015) or a variational inference-based method such as transport maps (Grashorn et al., 2024) to reduce the computational cost of the updating process.

4.2. *Reliability Analysis*

With the calibrated p-boxes for $\boldsymbol{a}$ and the updated ranges of $\boldsymbol{e}$, a reliability analysis using a double-loop MCS as introduced in sec. 2.2 was performed to estimate the failure probabilities of the black-box system under the given performance criteria. In this procedure, the outer loop involves sampling $N_{\boldsymbol{e}} = 500$ realizations of the epistemic parameters $\boldsymbol{e} \in \boldsymbol{E}$ and the epistemic moments of the p-boxes, followed by the inner loop, where $N_{\boldsymbol{a}} = 1000$ aleatory samples are generated from the resulting staircase densities.

Then, for each of the epistemic parameters, the failure probabilities are estimated using the $N_{\boldsymbol{a}}$ evaluations of the limit-state functions. Repetition of that procedure to all 500 epistemic samples yields the lower and upper bounds on the failure probabilities. Finally, the double-loop analysis is repeated for different levels of precision α to asses the sensitivity of the failure probabilities to the epistemic uncertainty.

A summary of the reliability analysis is given by the bounds of the failure probabilities shown in Table 3. There, the bounds of the individual performance criteria $P_{f,1}$, $P_{f,2}$, and $P_{f,3}$, as well as the worst-case failure probability P_f, are presented for different levels of precision α.

Table 3. Bounds of the failure probabilities $P_{f,i}$ for the individual performance criteria and worst–case estimate for different levels of precision α.

α	$P_{f,1}$	$P_{f,2}$	$P_{f,3}$	P_f
0.20	[0.13, 0.24]	[0.31, 0.60]	[0.13, 0.27]	[0.35, 0.63]
0.50	[0.13, 0.22]	[0.33, 0.56]	[0.13, 0.23]	[0.37, 0.59]
0.75	[0.14, 0.22]	[0.38, 0.55]	[0.14, 0.24]	[0.43, 0.58]
0.90	[0.13, 0.21]	[0.41, 0.51]	[0.14, 0.22]	[0.45, 0.56]
1.00	0.17	0.48	0.17	0.51

It becomes evident that the failure probabilities are relatively large, especially when comparing this solution with other contributions to the NASA UQ Challenge (see Lye et al., 2022; Agrell et al., 2022). Due to the small variations of the epistemic input, the variations of the failure probabilities of the output are rather small. Only for the second and worst case failure probability, a significant reduction of the ranges of the values of the failure probability can be observed when reducing the imprecision of the inputs.

When judging the results we note that the dataset used for model calibration was relatively small, consisting of only 100 time series for $y(t)$. Moreover, this dataset only included data from the subsystem $y(t)$, which was used for updating, while the reliability analysis was performed on the full system response $\boldsymbol{z}(t)$. In the NASA UQ Challenge, participating teams were allowed to perform updating on $\boldsymbol{z}(t)$ and could also restrict the space of the epistemic parameters by consulting with the authors of the challenge. These options were not available here, which may have contributed to the challenges encountered in the analysis.

5. Conclusions

This study presents a Bayesian updating framework implemented in Julia for reliability analysis under hybrid uncertainties. The framework employs two-step updating with an approximate likelihood function based on the Euclidean and Bhattacharyya distance metrics. In order to construct the likelihood a Prony decomposition of the time signal is introduced. The Julia implementation is based on the Bayesian updating module of the `UncertaintyQuantification.jl` package. Further, the implementations were evaluated through using an application to the NASA Langley UQ Challenge 2019.

Despite these positive aspects, the study also encountered limitations. The resulting failure probabilities were larger than those of other solutions presented in the challenge. This may result from the small size of the dataset and the high dimensionality of the parameters. Additionally, the complexity of the system and the lack of data on the integrated system may have contributed to the challenges faced in the analysis.

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