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Prescriptive maintenance and routing for a fleet of degrading vehicles

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The digitalization of the economy in the past decades has made data availability grow and become more important. Consequently, in the field of maintenance, in recent years, different jargons appeared, among which prescriptive maintenance has gained remarkable popularity. In this prescriptive maintenance study, we introduce the notion of degradation management to one of the most important combinatorial optimization problems, the vehicle routing problem. A fleet of vehicles has a set of points to visit each day. Each of those points has known deadlines. Additionally, vehicles are subjected to degradation and a stochastic state of health evolves with the distance traveled. Finding the best order of points to visit requires the solution method to account for the long-term degradation, since solving for each day independently in the way that most logistic operations are currently done, can lead to suboptimal solutions and increases the risk of downtime. Here, this optimization problem is stated using a Multi-Integer Linear Programming (MILP) formalism. We also provide numerical experiments, solving it and discussing the implications of considering degradation while optimizing the exploitation of vehicles. In conclusion, our solution proposal can reduce breakdown and maintenance costs.

Keywords: Vehicle routing problem, maintenance, decision-making, numerical optimization, degradation and reliability models.

1. Introduction

With new technologies, the popularization of sensors and the wide adoption of data-centric solutions, the field of maintenance has witnessed relevant conceptual discussions around ways of creating new policies. In this context, the term Prescriptive Maintenance (PsM) appeared Longhitano (2024). However, despite its relative popularity, the most common definitions of PsM Ansari et al. (2019) are not successful in differentiating it from classical maintenance practices, especially predictive ones.

In this paper, PsM conveys the notion of a policy based on decision-making algorithms that minimize a metric related to the overall exploitation cost of the system through actions that affect all system functions Longhitano (2024). In particular, this paper addresses heavy vehicles routing optimization.

Heavy vehicles are crucial for logistics and optimizing their usage is important both from a financial and environmental point of view. As such, the problem of how to assign missions to vehicles is a classic combinatorial problem first introduced by Dantzig and Ramser (1959) and popularly known today as the Vehicle Routing Problem (VRP). Several versions of the VRP exist and, for a systematic review of the most classic variations, we refer to Toth and Vigo (2002). Although the literature on VRPs is vast, there have been only few attempts to include vehicle maintenance in it, among which are Robert et al. (2019); Jbili et al. (2018); Longhitano et al. (2021).

In Robert et al. (2019), authors address vehicle assignment and maintenance operation schedule in the same optimization problem, representing vehicle health as a stochastic quantity. However, authors do not consider the spatial dimension of the problem and they treat fleet management as a scheduling problem instead of a proper VRP.

In Jbili et al. (2018) authors formalize their problem as VRP variation and jointly optimize routing and maintenance planning. However, despite the quality of their work, since they do not address degradation management, optimizing scenarios with neglectable failure probabilities would yield the same solution as classical VRPs. This undermines the long-term nature of combining maintenance and vehicle exploitation decisions, since they always have an impact on vehicle health or degradation.

In Longhitano et al. (2021), the spatial dimension of the VRP and degradation management are considered, but the problem formulation is not capable of ensuring optimality in long-term applications due to its overly simplistic optimization model. In this paper, this contribution is improved with a new optimization formulation that is more robust and realistic. It optimizes maintenance and route decisions in a finite time horizon reducing the overall exploitation cost of managing a fleet of heavy vehicles.

2. Problem Formulation

2.1. *Degradation Model*

The first step to include degradation into a VRP is to define a degradation model capable of connecting routes to vehicle health evolution. For that, we use a first-hitting-time approach in which a health indicator $W(d)$ evolves with traveled distance d. Failure occurs at D_{fail}, when this indicator crosses a threshold w_{crit}.

$$D_{fail} = \inf\{d : W(d) \geq w_{crit} \mid W(0) \leq w_{crit}\} \tag{1}$$

It is common to model vehicle component degradation by a Wiener process Yan et al. (2022); Guérin et al. (2010), therefore the following degradation model is used:

$$W(d) = W_0 + \lambda d + \sigma_b B(d) \tag{2}$$

where W represents the component health indicator, d is the traveled distance, W_0, λ and σ_b are constants and $B(d)$ is the standard Brownian motion. Those parameters can be estimated directly through data, using, for example, maximum likelihood techniques such as in Tang et al. (2014).

2.2. *Optimization model*

The proposed degradation model, although simple, correlates degradation to traveled distance and therefore can be used to connect classical maintenance actions and routing. Routing and maintenance have an interconnected nature. Sending vehicles that are more degraded to do less severe missions can postpone maintenance operations. The PsM problem becomes defining mission plans and choosing maintenance dates in a way that this relationship is exploited to reduce long-term costs. The assumptions are:

- Long-term optimization happens over a finite time horizon H, composed of different *working sessions* t.
- At each working session t, a set of missions must be performed. Each mission can be seen as a delivery with a known address and deadline. All relevant addresses are represented by a graph $G_t = (N_t, E_t)$ with $N_t = C_t \cup 0$. C_t represent mission addresses while 0 is the headquarters, where vehicles start and to where they must return by the end of t. The edges E_t represent the shortest paths between those addresses.
- The set of mission addresses does not change during a working session.
- Each vehicle in the fleet, represented by a set Z, has constant known degradation parameters λ_z and σ_z.
- Maintenance is treated in an opportunistic setup. It can only take place after specific working sessions which define a set V of maintenance opportunities with $V \subset H$.
- Each maintenance operation takes the component back to an as good as new state.

Figure 1 represents this decision-making problem which comes down to minimizing the exploitation cost over H. However, every routing process must be made ensuring that operational constraints such as mission deadlines, are re-

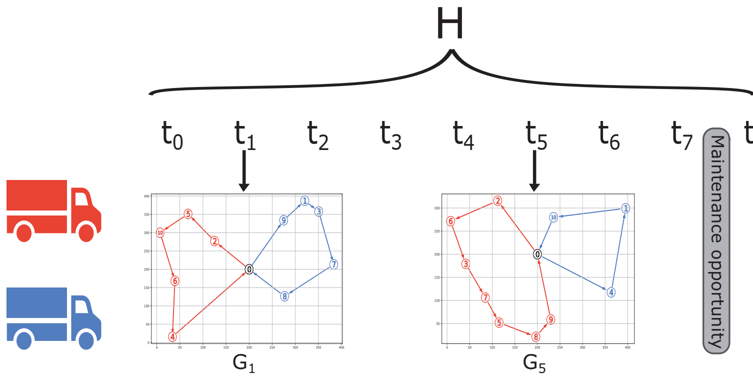

Fig. 1. Visual representation of the problem statement. At each working session, a graph G_t is used to define routes, as explicitly shown for t_1 and t_5. In this case, $V = [7]$, meaning maintenance operations can occur between t_7 and t_8.

spected. Furthermore, it is also important to keep failure probabilities under a maximum acceptable threshold to avoid downtime.

To transform the aforementioned assumptions into an optimization model, the traditional MILP approach is used. We start by defining the following decision variables:

$$x_{tzij} = \begin{cases} 1, & \text{if vehicle } z \text{ goes from node } i \text{ to node } j \text{ with } i, j \in N_t,\ z \in Z \text{ and } t \in H \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$m_{vz} = \begin{cases} 1, & \text{if vehicle } z \text{ performs maintenance at } v \text{ with } v \in V \text{ and } z \in Z \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

The cost function of the optimization problem becomes:

$$\min C_{psm} = \sum_{t \in H} \sum_{z \in Z} \sum_{j \in N_t} \sum_{i \in N_t} c_{km} x_{tzij} d_{tij} + \sum_{v \in V} \sum_{z \in Z} c_{maint} m_{vz} \tag{5}$$

where c_{km} is a constant related to the fuel cost and average fuel consumption while d_{tij} is the distance between each node i and $j \in N_t$. c_{maint} is the cost of preventive maintenance operations. In order to obtain solutions that represent valid routes, x_{tzij} must respect classical VRP constraints in each working session:

$$\sum_{z \in Z} \sum_{j \in N_t} x_{tzij} = 1 \quad \forall i \in N_t - \{0\}, t \in H \tag{6}$$

$$\sum_{i \in N} x_{tzij} - \sum_{i \in N_t} x_{tzji} = 0 \quad \forall i \in N_t, z \in Z, t \in H \tag{7}$$

$$\sum_{t \in H} \sum_{z \in Z} \sum_{j \in C_t} x_{tz0j} \leq |Z| \quad \forall i \in N_t - \{0\}, t \in H \tag{8}$$

$$x_{tijz} \in \{0, 1\} \tag{9}$$

$$m_{tz} \in \{0, 1\} \tag{10}$$

Constraint 6 ensures that every mission address is visited once. Constraint 7 ensures flux continuity and constraint 8 limits the number of vehicles used to the fleet size. Constraints 9 and 10 determine decision variables support. However, those constraints are not sufficient since they do not guarantee solutions without sub-tours and do not address operational constraints. Both problems can be fixed by adding decision variables y_{tzij} in the formulation. They are continuous variables representing the arrival time of vehicle z at node j coming from node i in a working session t. The following constraints are then required:

$$\sum_{j\in N_t} y_{tzij} - \sum_{j\in N_t} y_{tzji} \geq \Delta y_{tij} x_{tzji} \quad \forall i \in N_t, \forall z \in Z \forall t \in H \tag{11}$$

$$y_{tzij} \geq t_{0j} x_{tz0j} \quad \forall j \in N_t \forall z \in Z \tag{12}$$

$$y_{tzij} \leq l_j \quad \forall i \in N_t, j \in N_t, z \in Z, t \in H \tag{13}$$

$$y_{tzij} \quad \in [0, \infty[\tag{14}$$

with Δy_{tij} representing the time necessary to go from node i to j at t $\forall i, j \in N_t$ and $t \in H$. Constraint 11 makes y_{tzij} coherent with displacements duration when $x_{tzji} = 1$ and force them to 0 otherwise. It also acts as a sub-tour elimination constraint. Constraint 12 establishes the first arrival time of each vehicle while 13 ensures that mission deadlines - represented by l_j - are respected. Constraint 14 defines the correct support for variables y_{tzij}.

Including constraints on the failure probability directly through the correspondent cumulative distribution function is inconvenient. It would introduce non-linearities that would needlessly complicate the model from a computational point of view. It is possible to limit failure probabilities indirectly through the quantiles of D_{fail}. For example, imposing an acceptable failure probability q, is equivalent to limiting traveled distances to D^q_{max} where:

$$D^q_{max} = d : P(D_{fail} \leq d) = q \tag{15}$$

Therefore, acceptable failure probabilities can be guaranteed by ensuring that, between every maintenance operation, vehicle z does not travel more than D_{max_z}. This is achieved by introducing decision variables $D_{vz} \forall v \in V, z \in Z$ and the following constraint, referred to as *safety constraint*:

$$D_{vz} + \sum_{v}^{t} \sum_{i\in N_t} \sum_{j\in N_t} x_{tzji} d_{tij} \leq D_{max_z} \quad \forall v \in V, t > v \tag{16}$$

In order for Constraint 16 to work as intended, it is necessary that $D_{vz} = 0$ if a replacement happens at v or that $D_{vz} = D_{v-1z} + \sum_{v-1}^{t-1} \sum_{j\in N_t} \sum_{i\in N_t} x_{tzji} d_{tij}$, otherwise. This conditional logic is achieved through the following constraints:

$$D_{vz} \geq D_{v-1z} + \sum_{t-1}^{v-1} \sum_{j\in N_t} \sum_{i\in N_t} x_{tzji} d_{tij} - m_{tz} * K \quad \forall v, v-1 \in V z \in Z \tag{17}$$

$$D_{0z} = 0 \quad \forall z \in Z \tag{18}$$

$$D_{vz} \quad \in [0, \infty] \tag{19}$$

Constraint 19 establishes the support for D_{vz}. Constraint 17 is used to establish the conditional logic previously discussed. If constant K is large enough and a maintenance operation is performed at t, the right hand side of the inequality becomes negative, forcing D_{vz} to zero. Otherwise, D_{vz} becomes the accumulated distance until the last maintenance opportunity. It is also necessary to add $t = 0$ to the set V so that constraint 17 can be computed at the first maintenance opportunity. Additionally, it is required that $D_{0z} = 0 \quad \forall z \in Z$ which is guaranteed by Constraint 18. Constraint 19 defines the range of D_{tz}.

This formulation leads to a method for combining maintenance and routes, reducing long-term costs. Since this MILP is linear, classical branch and bound methods can be used to solve it. Through a set of numerical experiments, different aspects of this formulation are shown, discussing its characteristics and potential benefits.

3. Numerical experiments

3.1. *Simulation set-up and benchmark model*

Due to the randomness of the degradation process, to empirically validate our proposal, we perform numerical simulations. In those simulations, two fleets of vehicles will carry out the exact same missions for the same time horizon H. Graphs G_t are obtained by randomly sampling 10 mission addresses from an uniform distribution on a 400x400 square km and d_{tij} are computed as the Euclidian distance between each node in N_t. In each working session the headquarters (indicated by 0) position is the same, with coordinates [200,200]. Deadlines were set guaranteeing that, at each working session, there is at least a feasible route.

The first fleet will take decisions on maintenance and routes by solving the optimization model presented in Section 2.2. The second fleet is a benchmark model representing real-life practices for deciding maintenance and routes. Routes are assigned to vehicles, at each working session t, according to the following more classical VRP that does not take into account the degradation:

$$\min \quad \sum_{z\in Z}\sum_{j\in N_t}\sum_{i\in N_t} c_{km} d_{tij} x_{tzji} \tag{20}$$

Subjected to:

$$\sum_{z\in Z}\sum_{j\in C_t} x_{tzij} = 1 \qquad \forall i \in N_t - \{0\} \tag{21}$$

$$\sum_{i\in N_t} x_{tzij} - \sum_{i\in N_t} x_{tzji} = 0 \qquad \forall j \in N_t, z\in Z \tag{22}$$

$$\sum_{z\in Z}\sum_{j\in C_t} x_{z0j} \leq |Z| \quad \forall i \in N_t - \{0\} \tag{23}$$

$$x_{tijz} \in \{0,1\} \tag{24}$$

$$\sum_{j\in N} y_{tzij} - \sum_{j\in N_t} y_{tzji} \geq \Delta y_{tij} x_{tzji} \qquad \forall i \in N_t, \forall z\in Z \forall t \in H \tag{25}$$

$$y_{t0jz} \geq t_{0j} x_{tz0j} \quad \forall j \in N_t \forall z \in Z \tag{26}$$

$$y_{tijz} \leq l_j \qquad \forall i \in N_t, j\in N_t, z\in Z, t\in H \tag{27}$$

$$y_{tijz} \in [0,\infty[\tag{28}$$

and for maintenance, a simple strategy emulating preventive maintenance policies is applied. For each $v \in V$, components are replaced if a vehicle has traveled more than a predefined threshold distance $D_{benchmark}$. This threshold is defined in such a way that it mimics the order of magnitude of real preventive maintenance intervals.

Once routes and maintenance dates are established they are simulated. For each simulation, if, at any point W_z exceeds w_{crit}, a failure is considered to happen. In the next working session, degradation returns to a good as new state, as a consequence of a corrective operation. The cost is assessed for the simulated histories as:

$$C_{simu} = \sum_{t\in H}\sum_{z\in Z}\sum_{j\in N_t}\sum_{i\in N_t} c_{km} x_{tzji} d_{tij} + c_{fail} n_{fail} + c_{maint} n_{maint} \tag{29}$$

where c_{fail} is a constant representing the monetary value of a failure, n_{fail} is the number of failures occurred and n_{maint} the number of maintenance operations performed. For all simulations, cost constants are $c_{km} = 0.4$, $c_{maint} = 100$ and $c_{fail} = 2000$.

Maximum acceptable failure probabilities q_{max} are defined by:

$$q_{max} = \frac{c_{maint}}{c_{fail}} = 5\% \tag{30}$$

as a thumb rule to achieve reasonable failure costs.

The degradation model parameters are chosen so that the component must be replaced several times throughout the life of the vehicle, forcing this PsM solution to address the routing and maintenance decisions together.

3.2. *Results*

3.2.1. *Experiment 1: Homogeneous fleet*

In the first numerical experiment, we consider a homogeneous fleet with parameters described in Table 1. We also have, $V = \emptyset$ and H = $[0, 1, ..., 19]$, *i.e*: no maintenance opportunities and a horizon of 20 working sessions.

Table 1. Fleet parameters for experiment 1

Vehicle	λ	σ_B	W_0
0	0.002	0.1	0
1	0.002	0.1	0

Results are shown in Table 2. In this experiment, both models use exactly the same routes. This happens because in such a short horizon H, safety constraints could be easily respected and both models minimized the total traveled distance. As failure probabilities were negligible, the empirical costs obtained were also the same. As expected, like the benchmark model, the PsM model minimizes fuel consumption costs when failure probabilities are not significant. A second experiment is performed to highlight the differences between both models in a heterogeneous fleet.

Table 2. Simulation result of experiment 1

	Consumption cost	$\bar{C}_{simu}$
PsM	10583.5	10583.5
Benchmark	10583.5	10583.5

3.2.2. *Experiment 2: Heterogeneous fleet*

To address a case in which both models behave differently, a second experiment is performed with H = $[0, 1, ..., 35]$ and a fleet with parameters shown in Table 3. This fleet is heterogeneous, since Vehicle 1 has a grater σ. This corresponds to a greater degradation variance, leading to a lower $D_{max}^{q=5\%}$. With more restrictive safety constraints and a longer horizon, routing strategies were different in some working sessions. Figure 2 shows the routes used by both models in those cases.

The routes used by the PsM model are suboptimal from the point of view of fuel consumption, since they do not minimize the total traveled distance. However, these routes allow feasible solutions in terms of safety constraints, making it possible for Vehicle 1 to travel less than D_{max_1} and limiting the probabilities of failure under 5%. Consequently, although the consumption cost of the solution found by the PsM is higher,the cost observed in the simulation is considerably lower since failures occurred more often with the benchmark model. The results are given in Table 4.

3.2.3. *Experiment 3: Maintenance decision*

The two models also behave differently in terms of maintenance management. In the following numerical experiment, the fleet used is presented in Table 5. It has smaller λ values when compared to the previous fleets used. As a consequence, it allows for longer horizons in which decisions on maintenance become relevant. To illustrate this, four instances of the problem with the same set G_t and a horizon $H = [0, 1...70]$ are solved. In the first instance, $V = \emptyset$ *i.e.* maintenance is not allowed. In the second, third and fourth instances, maintenance opportunities occur at $V = [20]$, $V = [50]$, $V = [68]$, respectively. The results are shown in Table 6.

Table 3. Fleet parameters for experiment 2.

Vehicle	λ	σ_B	W_0
0	0.002	0.1	0
1	0.002	0.5	0

The results in Instance 1 show that there is a feasible solution in which respecting safety constraints is possible. As in previous examples, the PsM model is capable of limiting failure probabilities by using sub-optimal routes, reducing failure costs. In Instance 2, a maintenance opportunity is available at the beginning of the horizon. Since the vehicles had not yet traveled more than $D_{benchmark}$, no maintenance is performed by the benchmark model, leading to significant failure costs. On the other hand, the PsM model performs a maintenance operation on Vehicle 1, which allows it to take shorter routes when compared to instance 1, reducing consumption costs. In Instance 3, the benchmark model chooses maintenance operations for both vehicles, since it makes decisions purely based on traveled distance. The PsM model performs maintenance only for Vehicle 1. As a maintenance opportunity was available in the middle of the horizon, it was possible to take only optimal routes on this instance and both models achieved the same consumption cost. Finally, in Instance 4, maintenance opportunities were at the end of H. The benchmark model replaced both components, however, the PsM model did not perform a maintenance operation. Since the maintenance opportunity only occurred at the end of H, savings in fuel consumption due to the possibility of choosing better routes afterwards did not compensate for the extra maintenance operation.

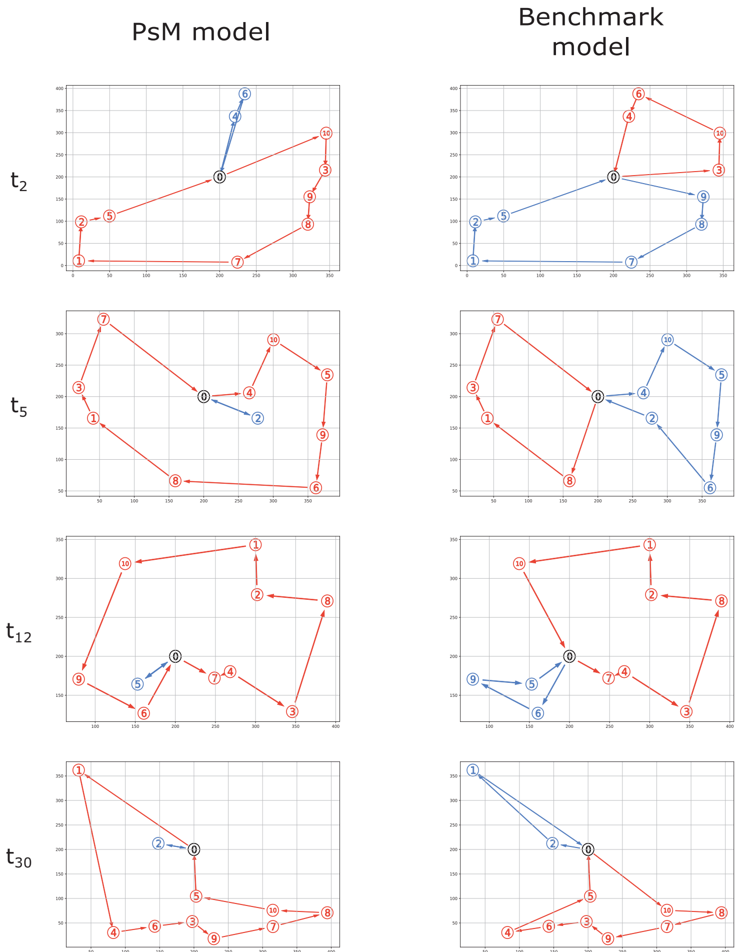

Fig. 2. Working sessions in which routes used by each model were different. Notice that the PsM model chooses routes that allow Vehicle 1 (represented in blue) to travel shorter distances and respect its safety constraints.

Table 4. Simulation result of experiment 2. Columns D^{vh0}_{total} and D^{vh1}_{total} represent the total traveled distance for Vehicle 1 and 0 respectively.

	D^{vh0}_{total}	D^{vh1}_{total}	Consumption cost	$\bar{C}_{simu}$
PsM model	33619.5	7693.4	16525.4	16670.6
Benchmark model	22297.3	18980.4	16511.1	16788.7

Table 5. Fleet parameters for experiment 3

Vehicle	λ	σ_B	W_0
0	0.0005	0.1	0
1	0.0005	0.5	0

4. Conclusion

In this paper, a PsM application was developed. It combines the usage and maintenance decision for heavy vehicles by extending the VRPs with the notion of degradation management. This allows vehicles to be allocated effectively, not only from

Table 6. Simulation result of experiment 3

Instance	PsM model			Benchmark model		
	Consump. cost	**Maint. cost**	**Failure cost**	**Consump. cost**	**Maint. cost**	**Failure cost**
1	35884	0	191	35498.8	0	641
2	35679	100	207	35498.8	0	652
3	35498.8	100	217	35498.8	200	241
4	35884	0	211	35498.8	200	619

a fuel consumption point of view but also reducing downtime and breakdown costs in heterogeneous fleets. Through a series of experiments, it was shown that our proposition yields better results when compared to traditional approaches, which were represented by a benchmark model.

In future publications, we would like to address more complex degradation phenomena where, for example, multi components reliability models could be used, representing the different systems of a vehicle. It would also be interesting to relax some of the hypothesis made and consider more complex variations of VRPs as a basis. For example, in future works we could allow for new missions to arrive during working sessions, leading to a dynamic VRP.

Furthermore, for a PsM solution to be complete, it is important to "close the decision loop" by collecting degradation data periodically and reducing the variance of the performance of the solutions. Finally, a very interesting research path is to develop algorithms that are efficient for this new optimization problem.

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