

Demonstrating Reliability for Actual Field Load Efficiently

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Empirical life tests are used to demonstrate product reliability, typically defined by a fixed service life, reliability values, and a set load (e.g. for 95 % customer). However, actual field usage varies, requiring methods to estimate reliability based on real usage conditions and load variation. Physical tests are essential for reliability demonstration but are challenging to plan. The Probability of Test Success serves as a key evaluation metric to address those challenges. The established hypothesis test framework aids in developing easy-to-implement algorithms and procedures for identifying optimal tests in individual cases. This paper combines efficient reliability test planning with the estimation of actual field usage reliability, applicable to Success-Run tests, End-of-Life tests, and accelerated tests. The result is a method for identifying the best test to demonstrate and estimate a product's real-world reliability. This is achieved by establishing necessary equations and definitions and developing a Monte Carlo-based algorithm to evaluate these equations. Results show significant benefits in terms of test expenditure and highlight drawbacks of common test load profiles, which this approach can remedy.

Keywords: Reliability Demonstration, Reliability Testing, Probability of Test Success, Accelerated Testing, Field Load, Success Run.

1. Introduction

Product development must ensure the product's integrity and function throughout its design life, verified by physical reliability demonstration tests before market entry [1, 2]. The reliability team faces the challenge of selecting the best test to determine actual field use reliability efficient in cost and time, statistically accurate, and field-representative [3, 4]. To address these challenges, the Probability of Test Success (P_{ts}) was developed to assess test suitability [5]. It has been developed and shown to be beneficial for failure free tests (Success-Run tests) [6, 7], failure based tests [5, 8], censored tests [9], accelerated tests [4, 10, 11], tests for system reliability demonstration [4, 12–14], tests incorporating prior knowledge using bayes theorem [15–18], consideration of uncertainty in the prior knowledge [19] as well as combination of several tests [4, 14]. Although the load spectrum shape's influence has been analyzed [20, 21], it assumes a fixed spectrum. Actual field usage varies, resulting in different load spectra per user. Thus, a distribution of load spectra should

be considered for accurate field reliability predictions and test planning. The proposed approach accounts for such statistical load distributions.

2. Actual Field Load: Load Spectra and Their Distributions

The actual usage of the product in the field can be very divers. Considering cars for example the driving behavior but also the road condition are very important factors among other influencing parameters. The usage results in loads, which are triggering damage mechanisms of the products components. The resulting damage during the design life must result in a number of product failures not more than the maximum allowed failure probability [1]. Typical loads in the reliability context of technical products are forces, moments, temperature, humidity (corrosion), pressure, vibration etc. [22]. Since each user is using the product in a different manner, the loads will vary. These loads need to be measured or estimated using simulation models to derive load distributions. While the time series of a single user's loads result in a load distribution after counting (e.g., rainflow

algorithm [23, 24]), the distribution among different users is of interest here. This paper focuses on using distributions of load spectra of a single damaging mode in reliability demonstration. Gathering and deriving such load spectra is not the main topic. Two starting points can be considered:

- (i) *Option A: Known Load Spectra.* E.g. derived from measurements in the field or simulated using advanced statistical models. Correct coupling of duration and load.
- (ii) *Option B: Separate Distributions about Operational Hours/Mileage and Load.* Usually derived using separate sources. Usually decoupled duration and load.

While in Option A the load spectra can be used directly, Option B is in need of an algorithm in order to combine the two information. Such an algorithm also has to take into account the distribution of individual load cycles with regard to the load level as well as a distribution of occurring loads in a single load spectrum. These two pieces of information define the possible shapes of the load spectrum [20, 21]. Finally, the form of the field load distribution is the same for Option A and B: several arrays of load levels S_{spec} and cumulative cycle counts N_{spec} . These will be used as field load distributions and incorporated into the reliability demonstration and the planning of reliability demonstration tests. An exemplary field distribution of load spectra is shown in Fig. 1.

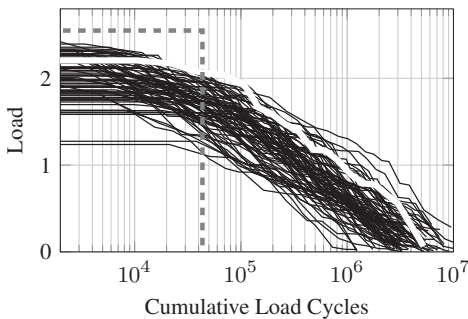


Fig. 1. Exemplary field load spectra. The white highlighted spectrum is the 95% damaging one and the equivalent damaging block profile is shown with a dashed gray line.

3. Efficient Reliability Demonstration

Reliability demonstration tests are typically designed as Success-Run tests because the binomial ansatz equations allow for straightforward calculation of required sample size [1]. However, these equations only consider type I statistical error, the required confidence. To assess the test's capability to successfully demonstrate the reliability requirement and to facilitate planning of failure-based tests, the Probability of Test Success was developed by Dazer et al. [3, 25] and Grundler et al. [4, 5, 12, 14, 15, 19, 26, 27]. It considers the reliability demonstration test as hypothesis test about reliability via the following null and alternative hypotheses [5]:

$$H_0 : t_R < t_r \quad (1)$$

$$H_1 : t_R \geq t_r \quad (2)$$

With t_R being the estimated lifetime quantile at required reliability R_r and t_r is the required service life. The calculation of the P_{ts} is then carried out by estimating the distribution of the estimated lifetime quantile of the test under validity of these two hypotheses and calculating the following integrals.

$$P_{\text{ts}} = \int_{t_{\text{crit}}}^{\infty} f_1(t_R) dt_R \quad (3)$$

$$C_r \stackrel{!}{=} \int_0^{t_{\text{crit}}} f_0(t_R) dt_R \quad (4)$$

Since these lifetime quantiles scatter, the null distribution $f_0(t_R)$ under validity of H_0 as well as the alternative distribution $f_1(t_R)$ under validity of H_1 are holding the information about the scatter. The variable t_{crit} is to be calculated so that the desired significance level of the test according to the required confidence C_r is assured. Methods using a monte carlo simulation for the derivation of the null and alternative distribution and calculation of the integrals [5] as well as methods making use of the asymptotic properties of the likelihood estimation [5] have been developed for Success-Run tests, failure based tests, censored, accelerated tests [19, 27] and system tests [12–14, 26]. For a more detailed explanation of the involved equations and concepts, refer to [4, 5]

4. Considering Varying Field Loads in Reliability Demonstration

It is common practice to evaluate reliability demonstration tests using a single fixed load spectrum or a constant load value with a fixed duration in a block profile [1, 23]. Often, only this single load is considered during the development and validation process. However, if the field load spectra are known (see Fig. 1), the evaluation must be adapted accordingly.

4.1. Evaluating Success-Run tests

The general Success-Run (SR) equation for evaluating the reliability R – assuming no failures during the test – is:

$$R = (1 - C_r) \frac{1}{n \cdot L_V^b} \quad (5)$$

With n being the sample size, L_V the lifetime or acceleration ratio and b is the Weibull shape parameter [1, 28]. Since the field loads as in Fig. 1 shall be used, the reference lifetime of the product is not fixed anymore and we do not have a single number of L_V but instead several. Each load spectrum i is inducing a damage $D_{f,i}$ according to the damage model. It results in a distribution of damages $f_D(D_f)$. The test load profile on the other hand is also inducing a damage D_{test} . The ratio of the two is the resulting value $L_{V,i}$. Eq. (5) becomes

$$R = \int_0^{+\infty} f_D(D_f) \cdot (1 - C_r)^{n^{-1} \cdot \left(\frac{D_{test}}{D_f}\right)^{-b}} dD_f. \quad (6)$$

It can be regarded as the expectation of the minimum reliability R demonstrated for considering the known field damages. Since the available load spectra are finite, Eq. (6) is to be applied for each value of field damage $D_{f,i}$. The integral can be approximated by taking the empiric expectation as the mean of the m calculated values.

$$R = \frac{1}{m} \cdot \sum_{i=1}^m (1 - C_r)^{n^{-1} \cdot \left(\frac{D_{test}}{D_{f,i}}\right)^{-b}} \quad (7)$$

Using these equations, the reliability demonstration is done for the entire field distribution, repre-

sented by the field damages $D_{f,i}$. It no longer uses the single value of e.g. a 95 % customer. This is perfectly aligned with the fact that the reliability requirement is set for the field and not a single extreme customer.

4.2. Evaluating End-of-Life tests

Failure-based End-of-Life (EoL) tests can be conducted to demonstrate reliability [8]. The gathered failure times from these tests are used to estimate the failure distribution through methods like maximum likelihood estimation (MLE) [29]. The confidence bound for C_r is then evaluated at the lifetime of interest and compared to the reliability requirement. Since the field distribution lacks a fixed value, the evaluation is carried out using the damages D_{EoL} from the EoL test specimens. For this, a fixed parameter set of the damage model must be known. The same type of distribution as the failure distribution is then used to evaluate the damage distribution $f_{test}(D)$. By assessing this distribution for reliability R with confidence C_r for each value of field damage $D_{f,i}$ and taking the empirical expectation as the mean, the resulting value is the demonstrated reliability R for field usage with confidence C_r . This process is schematically shown in Fig. 2. As with the SR

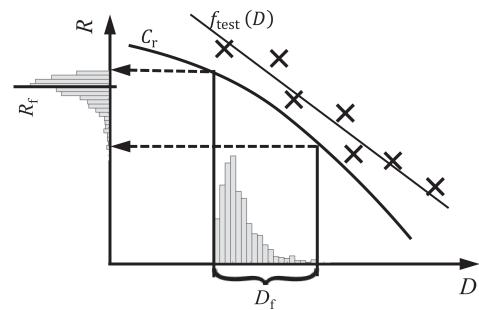


Fig. 2. Schematic overview of the evaluation of an EoL test for field loads.

test, which relies on the correct parameters of the damage model and failure distribution for reliability demonstration, the EoL test evaluation also depends on the correct damage model parameters but allows for estimating the failure distribution.

4.3. Evaluating accelerated tests

The accelerated test additionally allows for the estimation of the parameters of the damage model in order to estimate the reliability [30]. Several specimen are usually placed on two constant load levels and tested until failure. The failure times are used for the MLE of the damage model, e.g. a Wöhler-Weibull model [5, 31]. Using those parameters to evaluate the reliability of the field load spectra according to the confidence level C_r , the mean of the resulting reliability values is the demonstrated reliability of field use with confidence. The calculation is similar to Fig. 2, using the estimated damage model parameters. The advantage of accelerated tests is that all reliability demonstration parameters (damage model and failure distribution) are estimated using test results. These tests can be highly efficient if suitable, as higher test loads result in shorter testing times.

5. Probability of Test Success for Efficient Reliability Demonstration Taking into Account the Distribution of Field Loads

The planning of reliability demonstration tests using the Probability of Test Success P_{ts} is beneficial in terms of identifying the most efficient and promising test with desired statistical accuracy and high probability of achieving the reliability demonstration [4, 5, 8]. Here, it is developed to make use of information about the distribution of load spectra in field usage.

5.1. P_{ts} of Success-Run tests

The calculation of the P_{ts} of SR tests does not change for the consideration of field load distributions compared to the case of a fixed load [5]. This is because the P_{ts} is calculated using the Reliability of prior knowledge [19] according to the testing load. It is independent of the loads in the field. The equation therefore is [5]

$$P_{ts} = R_p^n. \quad (8)$$

Here, R_p has to be calculated using the damage model and its parameters including the failure distribution from prior knowledge, so that R_p is

valid for the damage during testing D_{test} of the n specimen. However, due to the fact that with Eq. (7) the demonstrated reliability is usually higher than what one would obtain using e.g. a 95 % customer load, the sample size or the induced damage during testing could be reduced which would in turn result in a higher value of P_{ts} . For a Wöhler-Weibull model [14], the reliability of one specimen for the load profile during testing can be calculated to

$$R_p = \exp \left(\ln(0.5) \cdot \left(\sum_{l=1}^o \frac{N_{test,l}}{N_A \left(\frac{S_{test,l}}{S_A} \right)^{-k}} \right)^b \right). \quad (9)$$

The number of cycles N_{test} of the test for the o respective load levels S_{test} result in the damage. The parameters of the Wöhler-Weibull model with support point (N_A, S_A) and slope k are used here together with the weibull shape parameter b . It has to be noted, that the calculated values and the reliability demonstration are only valid for the assumed parameters of the failure distribution and damage model. They should best be derived from applicable prior knowledge [14, 32].

5.2. P_{ts} of Accelerated tests

The accelerated test estimates all parameters involved for the reliability estimation. A bootstrap procedure for calculating the P_{ts} of accelerated tests has already been developed in [27] for Wöhler and Arrhenius models using either Weibull or Lognormal distributions. In order to take into account the distribution of the field load spectra, the involved equations have to be altered accordingly. First, samples have to be drawn from the failure distribution of the damage model according to the respective load levels of the test S_{test} , sample size and boundary conditions. For a Wöhler-Weibull model, the failure distribution is [27]

$$f(N) = -\ln(0.5) \frac{b}{N} \left(\frac{N}{N_A} \left(\frac{S}{S_A} \right)^k \right)^b \cdot \exp \left(\ln(0.5) \left(\frac{N}{N_A} \left(\frac{S}{S_A} \right)^k \right)^b \right). \quad (10)$$

The failure times are used with e.g. an MLE for estimation of the model parameters N_A , S_A , k and b [10, 27, 33]. Those parameters are then used to estimate the lifetime in the field in terms of load cycles $N_{f,i}$ for spectrum i, \dots, m .

$$N_{f,i} = \frac{N_i}{\left(\frac{\ln(R_r)}{\ln(0.5)}\right)^{1/b_v} \cdot \sum_{j=1}^{w_i} \frac{N_{A,v}}{N_j} \left(\frac{S_j}{S_{A,v}}\right)^{-k_v}} \quad (11)$$

Spectrum i has w_i load levels S_j with cycle counts N_j . While v is the iteration variable of the monte carlo and N_i is the sum of all cycles of load spectrum i . To get the estimate of \hat{N}_f for the entire field, the expected value of it can be calculated using the mean of the m values:

$$\hat{N}_{f,v} = \frac{1}{m} \sum_{i=1}^m N_{f,i} \quad (12)$$

By iterating this multiple times, e.g. MC = 10.000 times (iteration variable v) several candidates of the alternative distribution $f_{H_1}(N)$ according to Eq. (2) can be obtained in the form of load cycles N instead of t . Since the estimation methods tend to be biased [5, 34, 35], a correction of the form

$$N_{f,v,H_1} = \frac{1}{m} \sum_{i=1}^m \frac{\hat{N}_{f,v}}{\frac{1}{\text{MC}} \sum_{v=1}^{\text{MC}} \hat{N}_{f,v}} \frac{N_i}{D_{f,i}} \quad (13)$$

is advised so that the resulting distribution $f_{H_1}(N)$ is in accordance with the hypotheses of Eq. (1) and (2). Here $D_{f,i}$ is the damage of the spectrum i for the parameter set stemming from prior knowledge. In order to get candidates of the null distribution $f_{H_0}(N)$ the same values of Eq. (12) can be used as follows:

$$N_{f,v,H_0} = \frac{1}{m} \sum_{i=1}^m \frac{\hat{N}_{f,v} \cdot N_i}{\frac{1}{\text{MC}} \sum_{v=1}^{\text{MC}} \hat{N}_{f,v}} \quad (14)$$

Here the limit case of the product just having the required reliability for the field loads is considered. The P_{ts} can finally be calculated using a percentile bootstrap approach [5, 14, 27]:

$$C_r = \frac{\text{count}(N_{f,v,H_0} < N_{\text{crit}})}{\text{MC}} \quad (15)$$

$$P_{ts} = \frac{\text{count}(N_{f,v,H_1} \geq N_{\text{crit}})}{\text{MC}} \quad (16)$$

The reason quantiles of field load cycles N are used here instead of the lifetime quantiles of [14] is due to the fact that the field loads are for the requirement operating hours, lifetime (e.g. 3e5 km) and age (e.g. 15 years), whatever comes first.

5.3. P_{ts} of End-of-Life tests

The calculation of the P_{ts} for an EoL test is similar to the accelerated test procedure. However, the damage model parameters, e.g., (N_A, S_A) , k , are fixed and not estimated by the test. Here, Eq. (10) is used for sampling failure cycle counts for the test load. Since the damage model is fixed, the damages of the failure times of the sample D_{test} can be evaluated for any load level height. An MLE estimates the parameters of the failure distribution, using the Weibull distribution for the Wöhler-Weibull model. The parameter estimate of one Monte Carlo iteration of the shape parameter is b_v . This shape parameter is then used for Eq. (11) onward to calculate the P_{ts} as with the accelerated test.

6. Exemplary Study

In this example, we demonstrate the reliability of a traction motor of an electric vehicle, focusing on shaft breakage due to torque cycle fatigue. The requirement is $R_r = 0.9$, $C_r = 0.9$ for a lifetime of 300,000 km, 9,000 hours, or 15 years. The load spectra of field torque load are derived from field measurements combined with simulations to transfer the measured data to the product of interest and the load cycle domain. The resulting distribution of field load spectra can be seen in Fig. 3. Each load spectrum corresponds to the lifetime requirement: either km, hours or years are reached. As prior knowledge a Wöhler-Weibull model is used with the parameters $N_A = 1e8$, $S_A = 90$ Nm, $k = 6$ and $b = 2.5$. The respective model can also be seen in Fig. 3. In order to calculate damages, the torque needs to be translated into stresses at the correct failure location of the shaft by e.g. an appropriate Finite Element model [3, 36, 37]. However, since the stress is proportional to torque, the load is considered as

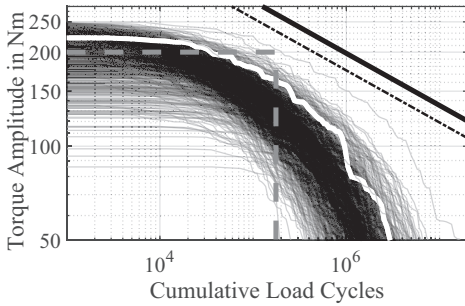


Fig. 3. Field load spectra of exemplary study. The damage model is the straight line in the upper right, also for R_r as a dash-dotted line.

torque for exemplary purposes here. The 95 % damaging load spectrum (depicted white in Fig. 3) used as a block profile for 200 Nm (grey dashed line) in testing is compared to the consideration of the entire distribution of field load spectra. The distribution of damages is depicted in Fig. 4.

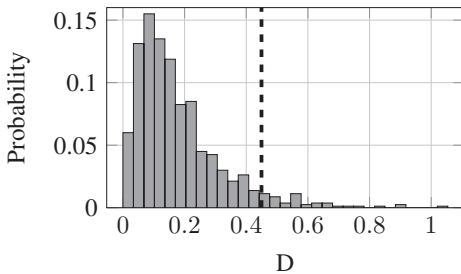


Fig. 4. Field load damages. The 95 % damage corresponds to $R = 0.91$ (dashed line).

First, an SR test is planned, which results in $n_{SR} = 22$ specimen for the reliability requirement. However, since the field load is lower in 95 % of the cases, the test duration can be reduced to 49.5 % of the testing time of the block profile according to the damage model. In this way not only the test is finished earlier, but also the P_{ts} increases from 12.7 % to 70 % using Eq. (8). Alternatively, the same block profile and test duration could be used but the sample size reduced to $n = 4$ which would result in a value of $P_{ts} = 68.7$ %. The combination of reduced test time and reduced sample size is also possible. Additionally, if a proper Design for Reliability

[38] approach is implemented, overdise of the shaft could possibly be reduced (smaller Wöhler parameters) to the point where an equilibrium of test expenditure and technically reasonable design changes is met. From these numbers, it can be seen that there is a clear benefit in taking into account the field distribution of load spectra in SR test planning. Not only does it render the test to be a feasible one with $P_{ts} > 50$ % but also the expenditure is reduced significantly at the same time.

Secondly, an accelerated test is planned. Two test load levels are used $S_{test} = [255, 150]$ Nm with a distribution of specimen of $n = [0.15, 0.85] \cdot n_{tot}$ for the consideration of field loads to get a good estimate about the model parameters according to [31, 33, 39]. For the block profile an equal distribution is used, since its a sole interpolating case. The results for the P_{ts} with $MC = 1e4$ over the total sample size n_{tot} for both the reliability demonstration using the block profile of 95 % user as well as the entire field distribution of load spectra can be seen in Fig. 5. It

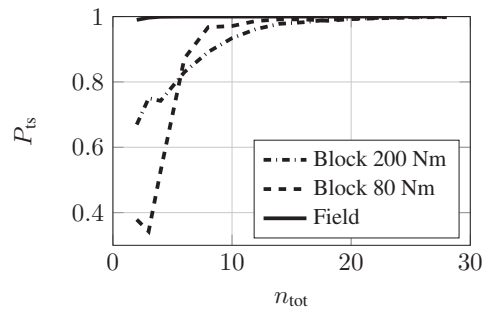


Fig. 5. Values of P_{ts} for accelerated tests with two load levels for demonstrating 95 % user load using block profiles and for the entire field load.

shows that the consideration of field loads yields much higher values of P_{ts} than the 95 % load. This is to be expected due to the lower load. However, due to the consideration of the field distribution, the reliability requirement is correctly demonstrated in the sense, that as an expected value, a proportion R_r of the products in the field will not fail as opposed to much less when using the

95 % damage load. To see the effect of the block load level height, the values of P_{ts} of a damaging equivalent block profile of the for 80 Nm is also shown in Fig. 5. Here, 71 % of the total sample size was used for the lower load level. Due to the block profile now representing a different load level than the initial block profile with 200 Nm, the P_{ts} values are lower because the extrapolation has to be done to a greater extend. This highlights a clear shortcoming of using a block profile in general: Due to extrapolation, the P_{ts} is dependent on the chosen block profiles load level. Using the field load distribution, this is not an issue since all occurring load levels are considered in the demonstration and calculation of the P_{ts} . The demonstrated reliability using the field load distribution is therefore not only beneficial in terms of expenditure, but also more representative both in terms of reliability estimation and P_{ts} . Additionally, Fig. 5 shows that the shaft is overdesigned for field use. It can be adapted to reduce dimensions, material, and possibly manufacturing costs. Using the methods and equations presented here, the optimal design can be determined regarding the required reliability, while also demonstrating said reliability within set expenditure constraints.

7. Summary and Conclusion

Demonstrating reliability is challenging, as a theoretically feasible test doesn't guarantee success. However, using the Probability of Test Success as a planning metric helps identify the best test type and design. Methods incorporating this metric and field load distribution were developed for Success-Run tests, End-of-Life tests, and accelerated tests. An exemplary study shows that incorporating field load distribution makes the Success-Run test feasible while reducing expenditure: shorter testing time and/or smaller sample size. Similarly, considering field load distribution in accelerated tests results in higher Probability of Test Success and reduced sample size. Using a damage-equivalent block profile for testing has a significant drawback, as the confidence bounds of the damage model differ for various load levels. This must be considered in test planning by using field load distribution. The presented approach contributes

to a holistic reliability demonstration by considering all available information. If field distributions can be estimated, actual reliability of field usage can be demonstrated instead of focusing on an extreme customer and unnecessarily over-fulfilling the requirement. The equations are worked out for a Wöhler-Weibull model but could also be extended to models like Arrhenius or Coffin-Manson using other failure distributions such as the log-normal distribution [14].

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