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Sampling-Based Possibility Theory for Engineering Analysis Under Uncertainty: Inference, Prediction and Optimization

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This contribution is a response to the 2025 NASA and DNV challenge on optimization under uncertainty. Three typical engineering problems in the form of parameter identification, forward propagation of uncertainty, and optimization are addressed. The framework of possibility theory is outlined and applied to the problems of the challenge. Given the nature of possibility theory, the results provide a rigorous and deliberately cautious perspective on the challenge problems. Accordingly, this approach is expected to be among the more conservative responses, offering a robust and well-substantiated analysis of uncertainty. The analysis is implemented through a sampling-based approach, returning statistically valid confidence distributions with all prior information explicitly stated.

Keywords: uncertainty quantification, imprecise probabilities, possibility theory, sampling, statistical inference, confidence, forward propagation, optimization

1. Introduction

This is a response to the latest installment of a series of challenges by Crespo et al. (2014), Crespo and Kenny (2021), and Agrell et al. (2024) that aim to evaluate the state of the art in uncertainty quantification for engineering problems.

The significance of uncertainty quantification in engineering analysis is well established. With an increase in computational power and the development of advanced numerical methods, the demand for robust mathematical frameworks for uncertainty quantification continuously grows. While probabilistic methods dominate the field, alternative approaches under the collective term of imprecise probabilities have gained traction in recent years and have shown to provide a more general treatment of uncertainty in past challenges, e.g. by Bi et al. (2022) and Gray et al. (2022).

This contribution focuses on the application of possibility theory, a tool for handling imprecise probabilities that has recently seen advances in the theory of inference (Hose et al. (2022), Martin (2023)), prediction (Mäck and Hanss (2021)) and optimization (Hose et al. (2019)). A compelling aspect of the theory is the ability to provide statistically valid confidence distributions without the need for prior information, even in scenarios of limited data availability. Its sample-based implementation is particularly well suited for non-linear problems and complex relationships between input and output variables, while optimized sampling methods allow for relatively efficient exploration of high-dimensional parameter spaces.

The challenge's three distinct elements, parameter identification, forward uncertainty propagation, and optimization under uncertainty together shape the structure of this contribution.

While the presented methodology is tailored to the challenge at hand, the underlying principles are expected to be broadly applicable to a wide range of engineering problems.

2. The 2025 NASA and DNV Challenge

The following is a brief summary of the challenge problems. For a full description, see Agrell et al. (2024) and the supplementary material provided on the website of the challenge organizers[a].

Table 1 categorizes the nine parameters of the black-box simulation model that forms the central component of the challenge.

Table 1. Parameters and their ranges.

Parameter	Type	Value Range
a_1, a_2	Aleatory	[0, 1]
e_1, e_2, e_3	Epistemic	[0, 1]
c_1, c_2, c_3	Control	[0, 1]
ω	Aleatory	$\in \mathbb{N}_0$

The parameters are allocated to the parameter vector $\mathbf{X} = [\mathbf{X}_a, \mathbf{X}_e, \mathbf{X}_c] \in \mathbb{R}^8$ and the scalar seed ω. The system is deterministic in the sense that for a fixed set of parameters it will always produce the same response in the form of a multivariate time series $\mathbf{Y} \in \mathbb{R}^{6\times 60}$, which consists of $n_Y = 6$ features over 60 time steps.

Problem 1a: Parameter Identification

Given a globally available reference output $\mathbf{Y}_{\text{ref},1}$, consisting of 100 six-dimensional time series, as shown in Figure 1, the task is to extract information about five unknown quantities, i.e., an unknown probability distribution generating $\mathbf{X}_a = [a_1, a_2]$ and three fixed but unknown parameters $\mathbf{X}_e = [e_1, e_2, e_3]$. The (baseline) control parameters $\mathbf{X}_c^* = [0.533, 0.666, 0.5]$ for its creation are known. Registered participants can request up to ten additional data sets $\mathbf{Y}_{\text{ref},2\ldots11}$ of the same size for arbitrarily selectable control parameters through a web interface.

Problem 1b: Forward Propagation

Having identified the parameters in Problem 1a, Problem 1b requires propagating the identified distributions through the model for the baseline control parameters $\mathbf{X}_c^*$. The prediction intervals for the confidence levels $\alpha = 0.95$ and $\alpha = 0.999$ on each of the six outputs are to be determined.

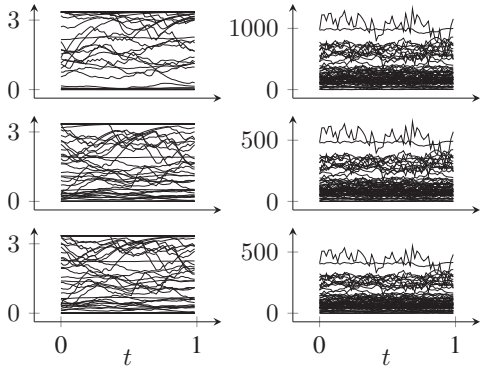

Fig. 1. The initial reference $\mathbf{Y}_{\text{ref},1} \in \mathbb{R}^{6\times 60\times 100}$.

Problem 2: Optimization Under Uncertainty

Finally, given the identified parameters, the task is to find three sets of control parameters, each optimizing a different objective function.

3. Possibility Theory

The posed challenge covers three key aspects of uncertainty quantification: imprecise parameter identification, forward uncertainty propagation, and optimization under uncertainty. Possibility theory—originally proposed by Dubois and Prade (1988) and in the flavor recently deepened by Balch (2020); Hose and Hanss (2021); Hose (2022); Martin (2023)—remains relatively little known in this context. Instead of providing a wide introduction to its facets, which could be argued necessary for the diverse topics covered in this challenge, the following outline provides only the most relevant background. Ample references are provided for further reading.

3.1. *Imprecise Probability Framework*

Possibility theory as a framework for the description of imprecise probabilities allows for a more general representation of uncertainty than classical probability theory. Instead of assigning a single probability, possibility theory assigns two measures, the possibility

$$\Pi(A) = \sup_{x\in A} \pi(x) \tag{1}$$

[a] available in March 2025 at `https://github.com/dnv-opensource/UQ-Challenge-2025`

and the necessity

$$\mathrm{N}(A) = 1 - \Pi(\bar{A}) = 1 - \sup_{x \notin A} \pi(x) \quad (2)$$

to an event A, where $\bar{A}$ denotes the complement of A. The possibility distribution π contains all information for constructing both measures, serving as the basic building block for all operations in possibility theory. Under the condition of normality, $\sup_{x \in \Omega} \pi(x) = 1$, the necessity and possibility measures bound a so-called credal set of probability measures of an event A by the consistency principle $\mathrm{N}(A) \leq \mathrm{P}(A) \leq \Pi(A)$.

3.2. *Possibilistic Calculus*

An imprecise variable V is a measurable function $V : \Omega \to \mathbb{V}$ from a sample space Ω to a value space $\mathbb{V}$. Unlike in probability theory, where a precise probability distribution is assigned to V, possibility theory describes it via the possibility distribution $\pi_V : \mathbb{V} \to [0, 1]$. The notation $V \sim \pi_V$ indicates that the true (potentially unknown) probability distribution of V is consistent with π_V—meaning that the possibility distribution bounds a credal set of all possible probability distributions that could describe V.

Three fundamental operations make up possibilistic calculus: forward propagation, marginalization, and combination of joint distributions. Forward propagation uses the extension principle

$$\pi_Z(z) = \sup_{z=\phi(u,v)} \pi_{U,V}(u, v), \quad (3)$$

proposed by Zadeh (1975), where a function ϕ transforms a—potentially multivariate—input distribution $\pi_{U,V}$ of the imprecise variables U and V into an output distribution π_Z of Z by taking the supremum over all input values mapping to each z. The marginal

$$\pi_V(v) = \sup_u \ \pi_{U,V}(u, v) \quad (4)$$

of a multivariate distribution $\pi_{U,V}$ is the supremum over all other variables. Joint distributions

$$\pi_{U,V}(u, v) = \mathcal{J}\left(\pi_U(u), \pi_V(v)\right) \quad (5)$$

are calculated using copulas, with the minimum operator serving as the most basic copula $\mathcal{J}_1(\pi_1, \pi_2) = \min(\pi_1, \pi_2)$. However, Hose and Hanss (2021) show that the minimum-based copula does not always preserve consistency through propagation. For such cases, the copula

$$\mathcal{J}_2(\pi_1, \ldots, \pi_m) = 1 - (1 - \min_{i=1,\ldots,m} \pi_i)^m \quad (6)$$

is preferrable, as it preserves consistency with the underlying probability distributions as well as proving valid for the conjunction of confidence distributions of independent analyses. Refer to Hose (2022) for more details on copulas for modeling dependence in possibility theory.

The P-Π-Transform

When given a probabilistic distribution or statistical model in the form of an empirical probability density function f_P, the P-Π-transform allows for its representation in possibilistic terms. Baudrit and Dubois (2006) propose the optimal transform

$$\pi(x) = \mathrm{P}\left(\{\xi : f_\mathrm{P}(\xi) \leq f_\mathrm{P}(x)\}\right) \quad (7)$$

for two-sided bounds on the probability distribution. One-sided bounds can be obtained by the cumulative transform, see Hose (2022).

3.3. *Valid Prior-Free Inference*

Possibility theory not only allows for describing incomplete knowledge about probability distributions but also provides a framework for statistical inference about an unknown parameter θ in a frequentist sense. Central to possibilistic inference is the principle of validity

$$\mathrm{P}(C_\pi^\alpha) \geq 1 - \alpha \quad (8)$$

with the so-called α-cut $C_\pi^\alpha = \{\theta : \pi(\theta) \geq \alpha\}$. The resulting structures $\pi(\theta)$, labeled possibilistic confidence distributions by Hose and Hanss (2021), but also known as inferential models in Hose et al. (2022) and Martin (2023), and originally introduced as confidence curves by Birnbaum (1961), are stacks of nested confidence intervals, where the α-cut of the distribution forms a confidence interval to the level $1 - \alpha$.

The approach to possibilistic inference put forward by Hose et al. (2022) begins by defining a contour or a plausibility function ρ that orders the parameter space according to the fit of the

parameter θ to the data. While the likelihood function serves as a natural choice for this plausibility function, its selection is arbitrary, merely defining the shape of the result, not its correctness. Hose (2022) advocates using engineering-inspired cost functions as plausibilities. Additionally, the choice of plausibility function can be used to incorporate partial prior information about the parameters. The plausibility function $\rho(\theta)$ is subsequently transformed into a possibilistic confidence distribution $\pi(\theta)$ by solving the integral

$$\pi(\theta) = \mathrm{P}\left(\{\xi : \rho(\xi) \leq \rho(\theta)\}\right). \tag{9}$$

Note its similarity to the P-Π-transform in Eq. (7). Solving strategies are detailed by Hose (2022) and most recently by Martin (2025).

A modified version of this is the basis for the parameter identification in this contribution.

3.4. *Possibilistic Predictions*

Connecting the two presented interpretations of possibility theory, the prediction of future observations is a natural extension of the transformed parameter-dependent statistical model output $\pi_{Y|\theta}(y)$ obtained by Eq. (7) on the one hand and the previously identified confidence distribution $\pi_{\theta|q}(\theta)$ of the parameter θ on the other. Given an observation q, Hose (2022) proposes the prediction distribution

$$\pi_{Y|q}(y) = \sup_{\theta} \mathcal{J}(\pi_{Y|\theta}(y), \pi_{\theta|q}(\theta)) \tag{10}$$

that describes the next realization y of the statistical model. For clarity, $\pi_{Y|\theta}(y)$ is read as "$\pi_Y(y)$ given θ". This emphasizes the dependence of the possibilistic description of Y on θ and the confidence in θ being based on observation q.

3.5. *Numerical Strategies*

The results in this contribution are mainly based on two of the three numerical strategies in possibilistic calculus: sampling and optimization, with interval analysis not being employed.

The main advantage of sampling-based procedures is their ability to handle non-linear problems. Additionally, unlike probabilistic sampling, possibilistic sampling does not sample from the volume (the density) of a distribution, but rather along its surface—where the information in a sample is carried by its value rather than its position relative to other samples. This enables arbitrary sampling densities, enabling major computational savings. Novel strategies employed here follow ideas from Mäck and Hanss (2021), and will be published separately. Their main idea is to carry out an analysis iteratively, tracking a contribution metric for each sample to the overall result and adaptively refining the parameter space in high-impact regions.

Furthermore, particle swarm optimization is repeatedly employed, as it is a simple and effective method for global optimization, integrating well with sampling-based procedures. See the original paper by Shi and Eberhart (1998) for an introduction to the method.

4. Solution Strategy

This section presents the methodology used for addressing the challenge. Though tailored to this specific challenge, the principles underlying the approach are expected to apply to a wide range of problems. For clarity and conciseness, the methodology is presented alongside corresponding results, as they are inherently linked.

4.1. *Parameter Identification*

Representation of Aleatory Variables

The methodology behind Eq. (9) focuses on the estimation of epistemic parameters. This necessitates quantifying the aleatory parameters using parametric distributions. Two independent beta-distributions are chosen for their flexibility in representing a wide range of shapes and returning values in the interval $[0, 1]$, which has motivated their use in previous challenges (Gray et al., 2022). The aleatory parameters $\mathbf{X}_a = [a_1, a_2]$ are modeled with parameters $\boldsymbol{\theta}_a = [\theta_1, \theta_2, \theta_3, \theta_4]$, expanding the parameter vector $\boldsymbol{\theta} = [\boldsymbol{\theta}_a \in \mathbb{R}^4, \boldsymbol{\theta}_e \in \mathbb{R}^3]$ to seven parameters describing the uncertainty about $\mathbf{X}_a$ and $\mathbf{X}_e$.

Dimensionality Reduction

To compare model responses on a scalar level, the output data is reduced to a d-dimensional subspace by Principal Component Analysis (PCA),

which retains as much of its variance as possible. Given the centered feature-wise responses $\mathbf{Y}_i \in \mathbb{R}^{60\times k}$ of k sampled time series, PCA computes the eigenvectors of the covariance matrix $\mathbf{C}_i = \frac{1}{k-1}\mathbf{Y}_i\mathbf{Y}_i^\mathsf{T}$. By selecting the $d < \min(60, k)$ eigenvectors corresponding to the largest eigenvalues as columns of the matrix $\mathbf{W}_i \in \mathbb{R}^{60\times d}$, the reduced representation

$$\mathbf{S}_i = \mathbf{W}_i^\mathsf{T}\mathbf{Y}_i \in \mathbb{R}^{d\times k} \tag{11}$$

is obtained. For the reference data $\mathbf{Y}_{\text{ref},1}$, the first principal component captures more than 98% of its variance, justifying $d = 1$ and enabling a one-dimensional comparison of model responses.

Confidence Distribution via Hypothesis Testing

In the inference approach proposed by Hose et al. (2022), a candidate parameter's fit is evaluated according to the costs of the observations generated from the statistical model for this parameter and the reference observation. This approach has been tailored to scenarios where one observation is available. In the present challenge, however, the reference data available consists of 100 time series. This allows a shift to a more classical statistical setting, where the cost distributions from the reference and candidate data are compared using a hypothesis test. The Kolmogorov-Smirnov test provides a p-value that quantifies the statistical evidence against the null hypothesis that both cost distributions are drawn from the same underlying model. A p-value is the probability of obtaining a test result at least as extreme as the observation. This aligns with the confidence value in a valid possibilistic confidence distribution in Hose (2022), measuring agreement between candidate parameter and reference observation. The confidence in a candidate parameter set is then calculated by the minimum p-value over all outputs:

$$\pi_{\boldsymbol{\theta}}(\boldsymbol{\theta}) = \min_{i=1,\dots,n_Y} \mathrm{P}\left(d_{\mathrm{KS}}(\mathbf{S}_{\text{ref},i}, \mathbf{S}_{\boldsymbol{\theta},i})\right), \tag{12}$$

where $\mathrm{P}(d_{\mathrm{KS}}(\dots))$ is the Kolmogorov-Smirnov test statistic's p-value, and $\mathbf{S}_{\text{ref},i}$ and $\mathbf{S}_{\boldsymbol{\theta},i}$ are the reduced representations of the reference and candidate data, respectively. Figure 2 aims at generating an understanding of this procedure by contrasting it to the one put forward in literature.

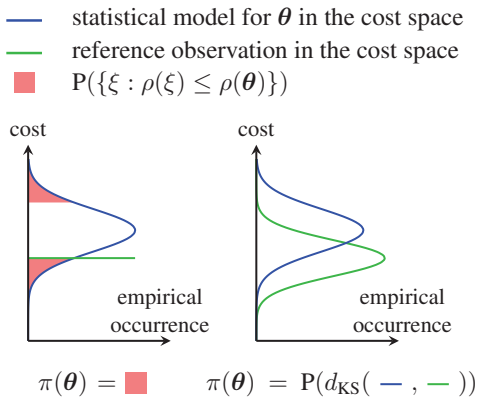

Fig. 2. The comparison of model outputs and observation in the cost space enables statistically valid inference. The left panel shows the core idea as presented in Hose et al. (2022). The right panel illustrates the approach taken in this contribution.

Fixing the Seed

Using a fixed set of seed values $\boldsymbol{\omega} \in {\mathbb{N}_0}^{1000}$ to generate the candidate data $\mathbf{S}_{\boldsymbol{\theta},i} \in \mathbb{R}^{1000}$ for all analyses ensures smooth surfaces for all sampling and optimization schemes, isolating the effects of variations between target and nuisance parameters. This methodological choice does not impede the validity of the analysis; while it fixes the stochasticity of the model outputs, every candidate parameter is evaluated in 1000 varied model runs, providing a good representation of the model's behavior, even for low-probability boundary cases.

Inclusion of additional data

In addition to the globallly available reference data $\mathbf{Y}_{\text{ref},1}$, up to ten additional data sets $\mathbf{Y}_{\text{ref},2\dots11}$ can be requested. Details on the selection of the control parameters for their creation are provided in Section 4.3.

The inference procedure is carried out for all reference solutions seperately and subsequently combined according to Eq. (6) to the final confidence distribution $\pi_{\boldsymbol{\theta}|\text{ref}} = \mathcal{J}_2(\pi_{\boldsymbol{\theta},1}, \dots, \pi_{\boldsymbol{\theta},11})$, where $\pi_{\boldsymbol{\theta},j}$ is the confidence distribution obtained on the basis of the jth reference data set.

The final seven-dimensional confidence distribution is shown in Figure 3. Its subnormality can be attributed to the approximation of the aleatory distribution by a beta-distribution and the analysis

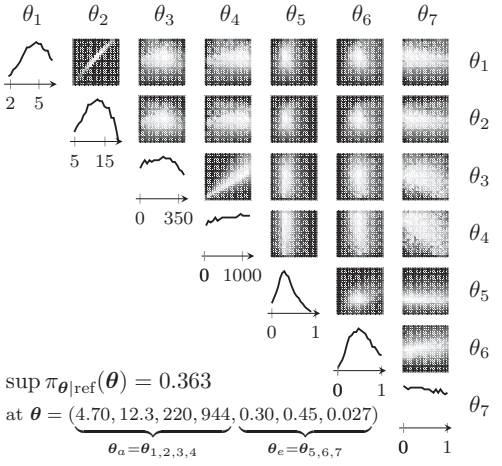

Fig. 3. A full representation of the identified seven-dimensional confidence distribution $\pi_{\boldsymbol{\theta}|\text{ref}}$. The diagonal shows one-dimensional marginals, while the off-diagonals display pair-wise two-dimensional marginals.

with globally fixed seed values. However, as Hose (2022) argues, subnormal confidence distributions do not pose a problem for possibilistic inference.

Extraction of Deliverables

The challenge organizers require submission of a probability distribution f_a satisfying $\mathbf{X}_a \sim f_a$ and a set E complying with $\mathbf{X}_e \in E$.

For f_a, one of infinitely many consistent probability densities to the conjunction π_a of the P-Π-transformed joint beta-distribution and the marginal confidence distribution $\pi_{\theta_1,\ldots,\theta_4}$ is selected by an algorithmic inversion[b] of Eq. (7).

For an arbitrarily chosen $\alpha = 0.001$, the set

$$E = \left\{\xi \in [0,1]^3 : \ \pi_{\theta_5,\theta_6,\theta_7}(\xi) \geq 0.001\right\} \quad (13)$$

is approximated using rejection sampling from the superlevel set of the marginal $\pi_{\theta_5,\theta_6,\theta_7}$.

Figure 4 shows representations of f_a and E.

[b] A valid option is the probability distribution

$$f_a(\xi) = \int_0^1 \frac{\mathbf{1}_{C^\alpha_{\pi_a}}(\xi)}{m(C^\alpha_{\pi_a})}\,\mathrm{d}\alpha,$$

with the area $m(C^\alpha_{\pi_a})$ of the α-cut $C^\alpha_{\pi_a}$ and the indicator function $\mathbf{1}_{C^\alpha_{\pi_a}}$ indicating membership of the same set. This option is chosen for comparability to other approaches due to its alignment with the expected values of the interval-valued moments of the underlying credal set. Other options include the upper and lower cumulative distribution functions.

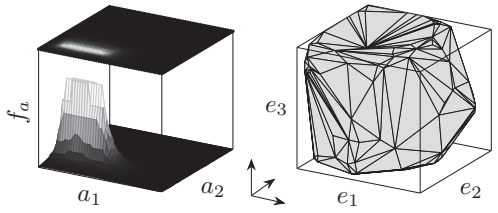

Fig. 4. Probability distribution f_a and set E.

4.2. *Forward Propagation*

The identified parameter distribution propagates through the model by application of Eq. (10). The statistical models $\overline{y_i}(\mathbf{X}_c^*, \alpha)$ and $\underline{y_i}(\mathbf{X}_c^*, \alpha)$, defined in Agrell et al. (2024), bound the response for the baseline design $\mathbf{X}_c^*$ and confidence level α. They are jointly transformed into the possibilistic description $\pi_{Y_i|\boldsymbol{\theta}}(y)$ for every feature $i = 1, \ldots, n_Y$ according to Eq. (7). Equation (10) then eliminates the imprecise parameter $\boldsymbol{\theta}$, yielding the prediction distributions $\pi_{Y_i|\,\text{ref}}(y)$, from which prediction intervals for arbitrary confidence levels can be read off, as shown in Figure 5 and Table 2.

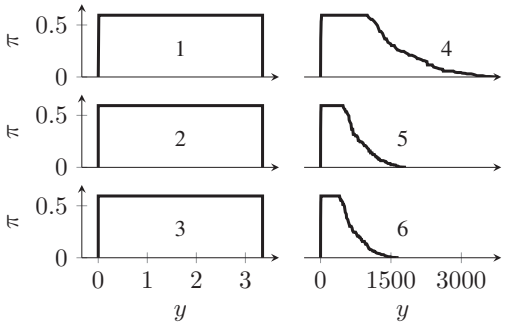

Fig. 5. Possibilistic prediction distributions $\pi_{Y_i|\,\text{ref}}(y)$ for all six features $i = 1, \ldots, 6$.

Table 2. Prediction intervals.

i	$\alpha = 0.95$		$\alpha = 0.999$	
	$y_{\min}$	$y_{\max}$	$y_{\min}$	$y_{\max}$
1,2,3	0.0	3.35	0.0	3.35
4	0.0	2459.1	0.0	3712.4
5	0.0	1225.8	0.0	1755.3
6	0.0	1036.1	0.0	1590.9

4.3. *Optimization*

The final part of the challenge is the optimization of the control parameters with regard to three different design goals across two performance indicators. The first indicator

$$J_{\min} = \min_{\boldsymbol{\theta} \in T} \sum_{i=1}^{3} \sum_{t=1}^{60} y_{i,\text{mean}}(\boldsymbol{\theta}, t) \qquad (14)$$

for the confidence set $T = C^{\alpha=0.001}_{\pi_{\boldsymbol{\theta}|\text{ref}}}$ represents the minimal sum of the responses $y_{i,\text{mean}}(\boldsymbol{\theta}, t)$, where each response is averaged across samples k. This sum is evaluated for the first three features over all time steps. The confidence level $\alpha = 0.001$ is chosen in accordance with reliability requirements elsewhere in this challenge. The second indicator pof_{sys} is the system failure probability, i.e. the probability of one of the latter three features exceeding their critical thresholds of $c_4 = 2750$, $c_5 = 2000$, and $c_6 = 1000$, respectively, at any point in time. The necessary data for both indicators are precalculated during the inference procedure across all candidate parameters and ultimately updated according to the combined confidence $\pi_{\boldsymbol{\theta}|\text{ref}}$.

Three designs are sought, a *performance-based* design maximizing $J_{\min}$, a *reliability-based* design minimizing pof_{sys}, and a *constrained performance* design maximizing $J_{\min}$ under the constraint of $\text{pof}_{\text{sys}} \leq \varepsilon$ with $\varepsilon = 10^{-3}$ and $\varepsilon = 10^{-4}$.

Surrogate Modeling and Reference Selection

Evaluating a candidate control parameter set requires a full evaluation of the model for all samples of $\pi_{\boldsymbol{\theta}|\text{ref}}$, exceeding 100 days of processor time. This makes optimization on the full uncertainty model computationally infeasible. Instead, a surrogate model is built on the full evaluations—representing the complete set of possible parameter values—from the inference procedure to interpolate between values in the control parameter space as inputs and performance indicators as outputs. The adopted meta-modeling approach uses Gaussian Process Regression (GPR) with a radial basis function kernel. This approach is chosen for its ability to provide predictions as well as uncertainty estimates, both of which guide the selection of subsequent evaluation points. New points are manually chosen in regions of highest prediction uncertainty of the GPR model, maximal predicted $J_{\min}$, or minimal predicted pof_{sys}. The training data for the GPR model and the optimized control parameters are shown in Tables 3 and 4. The lowest attainable probability of system failure is $2.87\,\%$, violating both constraints for the constrained performance design. A design for $\varepsilon = 0.05$ is provided instead.

Table 3. Training data for the GPR model.

c_1	c_2	c_3	$J_{\min}$	pof_{sys}
0.533	0.5	0.666	368.4	$5.60\,\%$
1	0	1	317.5	$30.30\,\%$
0	0	0	283.9	$11.72\,\%$
0	1	1	362.0	$2.87\,\%$
1	1	0	306.2	$15.59\,\%$
0.8	1	0.8	360.0	$13.93\,\%$
0.1	1	0.5	366.5	$4.03\,\%$
0	0.83	0	316.5	$9.24\,\%$
0	1	0.73	365.4	$9.82\,\%$
0	1	0.31	357.4	$8.70\,\%$
0.35	0	0.52	329.8	$9.82\,\%$

Table 4. Optimized control parameters.

	Performance	Reliability	$\text{pof}_{\text{sys}} \leq 5\,\%$
c_1	0.237	0	0.210
c_2	0.780	1	0.791
c_3	0.573	1	0.539
$J_{\min}$	370.8	362.0	365.1
$J_{\max}$	486.2	482.6	482.2
pof_{sys}	$5.80\,\%$	$2.87\,\%$	$5.00\,\%$

5. Conclusions

The presented results are—owing to the conservative nature of possibility theory—expected to be among the broadest of all challenge responses. Further factors driving conservatism are the dimensionality reduction and the choice of test statistics. However, within the potential suboptimality in undersampled regions and the—admittedly spirited—choice of beta-distributions as aleatory marginals, the results are expected to be robust/valid: any derived set of confidence α

covers the true parameter value in $(1-\alpha)\cdot 100\,\%$ of repetitions of this challenge. All results are available for arbitrary confidence levels, giving engineers the freedom to balance reliability against performance requirements in their design.

Numerical solution strategies for possibilistic calculus continue to evolve, as evidenced by recent developments in Martin (2025). A key theoretical challenge remains in developing a general possibilistic framework for polymorphic parameter identification. Given their ability to characterize both credal sets of probabilities and confidence distributions, possibility distributions suggest a promising avenue forward.

Acknowledgments

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