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The decision of the product's performance margin and reliability: an evolutionary game perspective

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In product trading, the buyer generally wants a product with higher reliability, which will increase the seller's cost. As a result, there exists the buyer-seller game about the product's reliability. Nowadays, most studies just introduced a general concept of "reliability level" when modeling the buyer-seller game, but not further explored the meaning and origin of reliability. Therefore, they cannot effectively describe and forecast the seller's action and the buyer's appeal to influence the product's reliability. According to the basic definition, reliability essentially evaluates whether the function achievement of a product can meet its requirement, which can be further quantified by specific performance. Therefore, the product's reliability derives from the product's performance supplied by the seller and the buyer's requirement, and can be calculated by the probability that the product's performance margin is greater than 0. On this basis, in this work, a buyer-seller game model for the product's performance margin and reliability is proposed, aiming to predict and guide the buyer's and seller's behaviors. For this game, the buyer's strategy is the performance threshold representing the requirement for the product, and the seller's strategy is the product's performance. In the game model, the performance threshold profit, the performance cost, and the failure risks are introduced to quantify the payoffs of both sides. Evolutionary game theory is adopted to model the dynamics of strategy evolution and solve the stable strategy. Finally, the phase transition of different game results with the change in payoffs can be clarified. The case of a two-strategy game indicates that the proposed method can predict the both sides' actions under different payoffs; also, the guidance can be provided to promote the buyer-seller cooperation.

Keywords: Reliability, Buyer-seller game, Performance margin, Evolutionary game theory, Phase transition, Cooperation.

1. Introduction

In current business environments, the reliability of a product significantly influences on the profits and decisions of sellers and buyers. On the one hand, buyers expect to purchase the products with better reliability under the same price, thus propose requirements and incentives for sellers based on reliability; on the other hand, for sellers, improving the product's reliability inevitably increases the costs in the design, manufacture, and other processes, which will affect the sellers' profits. As a result, there is a game between the buyer and the seller about the product's reliability.

Nowadays, the buyer-seller game considering the product's reliability (or some related indices such as quality and availability) has been widely modeled and analyzed. Most studies concerned the behaviors of buyers and sellers with different levels of the product's reliability: (Wu et al. 2011; Kumar et al. 2016; Martin 2017; Mutha et al. 2019). However, these studies just adopted "reliability/quality level" to characterize the product's reliability and model the game, but do not answer how to evaluate and determine the "reliability level" of a product. In other words, these approaches are based on the hypothetical and predefined "reliability level", but not further discuss the origin of reliability. Therefore, they cannot forecast and guide the specific actions of buyers and sellers to determine the products' reliability.

For this point, some studies specifically calculated reliability based on reliability block diagram (Uvet et al. 2022) or the proportion of the working components (Li and Mishra 2022). Essentially, such approaches consider the component redundancy, which modularized components but ignored the specific attributes of products. However, the product's reliability is not only determined by the component redundancy, but also depends on the operational principles based on professional disciplines (Tao et al. 2024; Li et al. 2021). Meanwhile, in the real game, the buyer will concern whether the product can achieve its functions based on its operational principles, and the seller will specifically design the product's operational principles. However, the reliability calculated by redundancy cannot concern the issue of operational principles.

Recalling the basic definition of reliability, reliability measures whether the function achievement of a product can satisfy the buyer's requirement. Further, the function achievement is generally represented and quantified by specific performances. Since performances are determined by operational principles, when evaluating reliability based on performances, operational principles also can be considered and modeled within specific performances. For each performance, its performance margin can be established to measure the distance between the performance and the buyer's requirement, and the reliability can be evaluated as the probability that the performance margin is greater than 0 (Zhang et al. 2024), that is:

$$\begin{gathered} P = f_{\vec{t}}\left(\mathbf{X},\mathbf{Y}\right) \\ \tilde{M} = m_{\vec{t}}\left(\tilde{P},\tilde{P}_{\text{th}}\right) \\ R = c\left\{m_{\vec{t}}\left(\tilde{P},\tilde{P}_{\text{th}}\right) > 0\right\} \end{gathered} \tag{1}$$

where P is the performance; $\mathbf{X}$ is the vector of internal variables; $\mathbf{Y}$ is the vector of external variables; $\vec{t}$ represent the degradation time; as a result, $f_{\vec{t}}\left(\cdot\right)$ is the function quantifying the operational principles; P_{th} is the threshold of P representing the buyer's requirement for P; $\tilde{P}$ and $\tilde{P}_{\text{th}}$ are the P and P_{th} considering uncertainty; $m_{\vec{t}}\left(\tilde{P},\tilde{P}_{\text{th}}\right)$ is the margin considering uncertainty, shorten as $\tilde{M}$; $c\left\{\cdot\right\}$ is the mathematical measure to quantify uncertainty, and in this work, the probability measure is adopted; R is reliability.

According to Eq. (1), $m_{\vec{t}}\left(\tilde{P},\tilde{P}_{\text{th}}\right)$ depends on the $f_{\vec{t}}\left(\cdot\right)$, which bridges the product's reliability and its operational principle. In addition, it can be noted that the game about reliability also reflects the relationship between the buyer's requirement and the performance of the seller's product. On the one hand, the buyer's order and pricing for the product suggest whether the buyer's requirement is satisfied: the better the requirement is satisfied, the more advantageous the order and pricing are for the seller. On the other hand, if the seller provides the product with better performance to satisfy the buyer's requirement, it will lead to a higher cost. Conclusively, the buyer-seller game about reliability is expected to be modeled by margins to integrate the product's operational principles and the buyer's requirement.

In this work, a buyer-seller game model for the product's performance margin and reliability is proposed. In the game, the buyer's strategy is the performance threshold representing the requirement for the product, and the seller's strategy is the product's performance. Combining the performance and its threshold, the margin and reliability can be calculated to construct the payoffs. For this game model, evolutionary game theory is adopted to solve equilibrium solutions. As a result, the expected requirements and performances under different profits, costs, and risks can be obtained, which can guide the buyer-seller cooperation about the product's performance margin and reliability.

The reminder of the paper is organized as follows. Section 2 briefly introduces the basic knowledge of evolutionary game theory. In section 3, a game model about the performance margin and reliability is first proposed, and the model solution and analysis are introduced based on a typical two-strategy game. Section 4 discusses the uncertainties in strategies. Section 5 concludes the work.

2. Preliminary: evolutionary game theory

Evolutionary game theory (EGT) is first proposed to study the behaviors of biosystems (Hofbauer and Sigmund 2003; Nowak 2006), and nowadays it has been widely applied to model the games from various fields including computer science, physics, and social science. Also, EGT has been adopted to solve the games related to reliability, such as software reliability (Zhao et al. 2021), the cascading failure in a network (Dui et al. 2020), and predictive maintenance (Meng et al. 2022).

Compared with the traditional game, EGT assumes that participants are bounded rational (Simon 1990). That is, participants will choose the strategies in their interests considering current situation, and constantly adjust strategies during the game process. For this point, EGT focuses on the evolutionary process of strategies. Generally, the strategies with high benefits will be preferred, while the ones with low benefits will diminish and disappear. For the dynamics to quantify the evolutionary process, replicator dynamics (Schuster and Sigmund 1983) is the most widely used evolutionary rule. In replicator dynamics, the gradient of the probability to choose a strategy is proportional to the value of that probability and the difference between the strategy's expected payoff and the averaged payoff. As a result, participants tend to choose the strategies with higher payoffs, which is applicable for the games in real-world scenarios. The replicator dynamics for the probability to choose one strategy ω can be expressed as:

$$\dot{x}_{\omega} = x_{\omega} \cdot \left(S_{\omega} - \bar{S}\right) \tag{2}$$

where x_{ω} is the probability to choose ω, S_{ω} is the expected payoff of ω, and $\bar{S}$ is the averaged payoff for all strategies.

Based on the evolutionary dynamics, EGT introduces the concept of evolutionary stable strategy (ESS), which has been proved in mathematics to be a more rigorous concept compared with the traditional Nash equilibrium (Smith and Price 1973). An ESS is the stable point of the evolutionary dynamics, which is robust to the disturbances due to slightly abrupted strategies and can represent stable game results. Since the ESS is determined by the evolutionary dynamics, when the parameters of dynamics change, the ESS will also change. Therefore, such a change process can be expressed as a phase transition of the equilibrium solution with the change of dynamics parameters.

3. Modeling and analysis for the game

3.1. *Model assumption*

In the buyer-seller game about the product's performance and reliability, the buyer expects to propose a stricter requirement to ensure profit, but the seller hopes to degrade the performance to save cost. Therefore, there is a conflict of interest between the buyer and the seller. Meanwhile, with the necessity to use the product, the buyer has to cautiously consider the risk that the requirement is too strict for the seller to satisfy, which leads to an unreliable result; from another side, when reducing cost, the seller also needs to concern the risk and loss if the product's performance cannot meet the buyer's requirement. Therefore, there also exists the consistency of interest for both sides maintain the product reliable and achieve a cooperation.

On this basis, the following assumptions are proposed to construct the game model.

Assumption 1: The game players are the buyer and the seller. They are both bound rational: choose the strategies that are in their own interests based on current situation, and constantly adjust the strategies during the dynamic game. The seller's strategy is the product's performance, which is a random variable referred as $\left\{\tilde{P}_i\right\}, i = 1,\dots,m$, and m is the number of the seller's strategies. The buyer's strategy is the performance threshold, which is a random variable referred as $\left\{\tilde{P}_j^{\text{th}}\right\}, j = 1,\dots,n$, and n is the number of the buyer's strategies.

Assumption 2: The performance threshold reflects the buyer's requirement for the product's performance. The buyer expects to propose a stricter performance threshold, so that the buyer can obtain more profits if the product can meet the requirement. Therefore, a performance threshold profit function can be introduced for $\tilde{P}^{\text{th}}$, referred as $f_V\left(\tilde{P}^{\text{th}}\right)$, and the stricter $\tilde{P}^{\text{th}}$ correspond to the greater $f_V\left(\tilde{P}^{\text{th}}\right)$.

Assumption 3: There are distinguished costs when the seller provides different performances. In particular, a better performance corresponds to a higher cost of the seller. Therefore, a performance cost function for $\tilde{P}$ is proposed as $f_C\left(\tilde{P}\right)$. The better $\tilde{P}$ corresponds to the greater $f_C\left(\tilde{P}\right)$.

Assumption 4: Risk sharing. The buyer and the seller need to share the risks that the performance cannot meet the requirement (that is, the performance margin is less than 0). And such risks can be quantified by reliability. The less reliability indicates the higher risks owned by both sides. Thus, the seller failure risk function and the buyer failure risk function are introduced as $f_{W_P}\left[R\left(\tilde{P},\tilde{P}^{\text{th}}\right)\right]$ and $f_{W_{\text{th}}}\left[R\left(\tilde{P},\tilde{P}^{\text{th}}\right)\right]$ respectively. They are both functions of $R\left(\tilde{P},\tilde{P}^{\text{th}}\right)$, which is the reliability calculated by $\tilde{P}^{\text{th}}$ and $\tilde{P}$ according to Eq. (1). And the higher $R\left(\tilde{P},\tilde{P}^{\text{th}}\right)$ corresponds to the less $f_{W_P}\left[R\left(\tilde{P},\tilde{P}^{\text{th}}\right)\right]$ and $f_{W_{\text{th}}}\left[R\left(\tilde{P},\tilde{P}^{\text{th}}\right)\right]$.

Based on the above assumptions, when the buyer chooses $\tilde{P}_j^{\text{th}}$ and the seller chooses $\tilde{P}_i$, the buyer's payoff is $f_V\left(\tilde{P}_j^{\text{th}}\right)-f_{W_{\text{th}}}\left[R\left(\tilde{P}_i,\tilde{P}_j^{\text{th}}\right)\right]$ and the seller's payoff is $-f_C\left(\tilde{P}_i\right)-f_{W_P}\left[R\left(\tilde{P}_i,\tilde{P}_j^{\text{th}}\right)\right]$.

With the payoffs of both sides, the equilibrium solutions can be solved to identify the game results. Further, the game results can be divided into two types: cooperation and conflict. A cooperation means that with the joint effects of both sides, the calculated $R\left(\tilde{P},\tilde{P}^{\text{th}}\right)$ is high enough to meet the buyer's expectation. By contrast, a conflict means the final $R\left(\tilde{P},\tilde{P}^{\text{th}}\right)$ cannot meet the buyer's expectation and the cooperation fails. And in engineering practice, a cooperation is expected to be achieved, and a conflict should be avoided.

In addition, according to the strategies of both sides, a cooperation also can be categorized into two types: compromise and collaboration. A compromise means that in a cooperation, one side insists on the optimal strategy for its own interest, while the other side makes concessions. A collaboration means that both sides make some concessions to achieve a cooperation.

3.2. *Model solution and analysis for the case of a two-strategy game*

Based on the above general game model, in this section, the model solution and analysis are performed for a typical two-strategy game.

3.2.1. *Evolutionary dynamics modeling*

For a two-strategy game, the buyer has two strategies of performance threshold, $\tilde{P}_L^{\text{th}}$ and $\tilde{P}_U^{\text{th}}$, which represent the low requirement and high requirement respectively. The seller also has two strategies of performance, $\tilde{P}_L$ and $\tilde{P}_U$, which correspond to the low performance and high performance respectively. In particular, $f_V\left(\tilde{P}_U^{\text{th}}\right)>f_V\left(\tilde{P}_L^{\text{th}}\right)$, and $f_C\left(\tilde{P}_U\right)>f_C\left(\tilde{P}_L\right)$. For the simplification when describing the reliability with different performances and thresholds, the superscript of R means the threshold, and the subscript of R means the performance. For instance, $R_L^U=\Pr\left\{m_{\tilde{l}}\left(\tilde{P}_L,\tilde{P}_U^{\text{th}}\right)>0\right\}$.

For this two-strategy game, there are four possible pure strategy equilibrium solutions:

- High requirement, high performance ($\left\{\tilde{P}_U^{\text{th}},\tilde{P}_U\right\}$)

  It means the buyer insists on a strict requirement while the seller provides a high performance, thus it generally corresponds to the result of seller compromise.
- High requirement, low performance ($\left\{\tilde{P}_U^{\text{th}},\tilde{P}_L\right\}$)

  It means the buyer insists a high requirement, but the seller insists on a low performance. In this case, the product's reliability will be at a low level, which can correspond to the result of conflict.
- Low requirement, high performance ($\left\{\tilde{P}_L^{\text{th}},\tilde{P}_U\right\}$)

  It means that the buyer lowers the requirement and the seller improves the performance, thus it can reflect the result of collaboration.
- Low requirement, low performance ($\left\{\tilde{P}_L^{\text{th}},\tilde{P}_L\right\}$)

  It means the buyer lowers the requirement while the seller insists for its interest, which can represent the result of buyer compromise.

For the above two-strategy game, to solve its equilibrium solutions, in this work, the replicator dynamics in EGT is adopted to model the process for both sides to choose strategies, and the ESSs can be solved as the final equilibrium solutions.

Assume that the probability for the buyer chooses the strategy $\tilde{P}_L^{\text{th}}$ is x_L, thus the probability to choose $\tilde{P}_U^{\text{th}}$ is $1-x_L$. Similarly, suppose that the probability for the seller to choose $\tilde{P}_L$ is y_L, thus the probability to choose $\tilde{P}_U$ is $1-y_L$. On this basis, the expected payoffs for the buyer to choose $\tilde{P}_L^{\text{th}}$ and $\tilde{P}_U^{\text{th}}$ can be calculated as S_L^{th} and S_U^{th}, respectively:

$$\begin{aligned} S_L^{\text{th}} &= y_L \cdot \left[f_V\left(\tilde{P}_L^{\text{th}}\right) - f_{W_{\text{th}}}\left(R_L^L\right) \right] \\ &\quad + \left(1-y_L\right) \cdot \left[f_V\left(\tilde{P}_L^{\text{th}}\right) - f_{W_{\text{th}}}\left(R_U^L\right) \right] \\ S_U^{\text{th}} &= y_L \cdot \left[f_V\left(\tilde{P}_U^{\text{th}}\right) - f_{W_{\text{th}}}\left(R_L^U\right) \right] \\ &\quad + \left(1-y_L\right) \cdot \left[f_V\left(\tilde{P}_U^{\text{th}}\right) - f_{W_{\text{th}}}\left(R_U^U\right) \right] \end{aligned} \tag{3}$$

Based on Eq. (3), the averaged payoff for the buyer, $\bar{S}^{\text{th}}$, can be calculated:

$$\bar{S}^{\text{th}} = x_L \cdot S_L^{\text{th}} + \left(1-x_L\right) \cdot S_U^{\text{th}} \tag{4}$$

According to the replicator dynamics, the evolutionary dynamics of x_L is:

$$\begin{aligned} \dot{x}_L &= x_L \cdot \left\{ S_L^{\text{th}} - \left[x_L \cdot S_L^{\text{th}} + \left(1-x_L\right) \cdot S_U^{\text{th}} \right] \right\} \\ &= x_L \cdot \left(1-x_L\right) \cdot \left(S_L^{\text{th}} - S_U^{\text{th}} \right) \end{aligned} \tag{5}$$

For the seller, the expected payoffs to choose $\tilde{P}_L$ and $\tilde{P}_U$, S_L and S_U, also can be provided:

$$\begin{aligned} S_L &= x_L \cdot \left[-f_C\left(\tilde{P}_L\right) - f_{W_P}\left(R_L^L\right) \right] \\ &\quad + \left(1-x_L\right) \cdot \left[-f_C\left(\tilde{P}_L\right) - f_{W_P}\left(R_L^U\right) \right] \\ S_U &= x_L \cdot \left[-f_C\left(\tilde{P}_U\right) - f_{W_P}\left(R_U^L\right) \right] \\ &\quad + \left(1-x_L\right) \cdot \left[-f_C\left(\tilde{P}_U\right) - f_{W_P}\left(R_U^U\right) \right] \end{aligned} \tag{6}$$

Similarly, the evolutionary dynamics of y_L can be expressed as:

$$\dot{y}_L = y_L \cdot \left(1-y_L\right) \cdot \left(S_L - S_U\right) \tag{7}$$

Combining Eq. (5) and (7), the evolutionary dynamics for the strategies of both sides during the game process can be concluded as:

$$\begin{cases} \dot{x}_L = x_L \cdot \left(1-x_L\right) \cdot \left(S_L^{\text{th}} - S_U^{\text{th}} \right) \\ \dot{y}_L = y_L \cdot \left(1-y_L\right) \cdot \left(S_L - S_U \right) \end{cases} \tag{8}$$

3.2.2. *Evolutionary stability analysis*

To solve the ESSs of Eq. (8), the fixed points where $\dot{x}_L = 0; \dot{y}_L = 0$ should be first identified, and the stability of such fixed points can be analyzed based on Lyapunov's first method. Lyapunov's first method states that for linear equations, if the real parts of all eigenvalues for the Jacobian matrix at a fixed point are negative, then this fixed point is stable. And in EGT, this point is an ESS.

For the evolutionary dynamics of Eq. (8), there exist 4 ESSs, particularly:

- (0,0), corresponding to $\left\{\tilde{P}_U^{\text{th}}, \tilde{P}_U\right\}$, is an ESS when:

$$\begin{aligned} f_V\left(\tilde{P}_U^{\text{th}}\right) - f_V\left(\tilde{P}_L^{\text{th}}\right) &> f_{W_{\text{th}}}\left(R_U^U\right) - f_{W_{\text{th}}}\left(R_U^L\right) \\ f_C\left(\tilde{P}_U\right) - f_C\left(\tilde{P}_L\right) &< f_{W_P}\left(R_L^U\right) - f_{W_P}\left(R_U^U\right) \end{aligned} \tag{9}$$

It means that the increased profit for the buyer to choose a higher requirement is greater than corresponding increased failure risk; while for the seller, the increased cost to improve performance is less than the increased failure risk to degrade performance. Therefore, the seller will compromise to the buyer.

- (0,1), corresponding to $\left\{\tilde{P}_U^{\text{th}}, \tilde{P}_L\right\}$, is an ESS when:

$$\begin{aligned} f_V\left(\tilde{P}_U^{\text{th}}\right) - f_V\left(\tilde{P}_L^{\text{th}}\right) &> f_{W_{\text{th}}}\left(R_L^U\right) - f_{W_{\text{th}}}\left(R_L^L\right) \\ f_C\left(\tilde{P}_U\right) - f_C\left(\tilde{P}_L\right) &> f_{W_P}\left(R_L^U\right) - f_{W_P}\left(R_U^U\right) \end{aligned} \tag{10}$$

It means that the increased profit for the buyer to choose a higher requirement is greater than corresponding increased failure risk; but for the seller, the increased cost to improve performance is also greater than the increased failure risk to degrade performance. Therefore, both sides will insist on the strategies for their own interests, which leads to a conflict.

- (1,0), corresponding to $\left\{\tilde{P}_L^{\text{th}}, \tilde{P}_U\right\}$, is an ESS when:

$$\begin{aligned} f_V\left(\tilde{P}_U^{\text{th}}\right) - f_V\left(\tilde{P}_L^{\text{th}}\right) &< f_{W_{\text{th}}}\left(R_U^U\right) - f_{W_{\text{th}}}\left(R_U^L\right) \\ f_C\left(\tilde{P}_U\right) - f_C\left(\tilde{P}_L\right) &< f_{W_P}\left(R_L^L\right) - f_{W_P}\left(R_U^L\right) \end{aligned} \tag{11}$$

It means that the increased profit for the buyer to choose a higher requirement is less than corresponding increased failure risk; also, for the seller, the increased cost to improve performance is less than the increased failure risk to degrade performance. In this case, the failure risks are relatively significant for both sides, thus both sides will achieve a collaboration to ensure the product's reliability.

- (1,1), corresponding to $\{\tilde{P}_L^{\text{th}}, \tilde{P}_L\}$, is an ESS when:

$$\begin{aligned} f_V\left(\tilde{P}_U^{\text{th}}\right) - f_V\left(\tilde{P}_L^{\text{th}}\right) &< f_{W_{\text{th}}}\left(R_L^U\right) - f_{W_{\text{th}}}\left(R_L^L\right) \\ f_C\left(\tilde{P}_U\right) - f_C\left(\tilde{P}_L\right) &> f_{W_P}\left(R_L^L\right) - f_{W_P}\left(R_U^L\right) \end{aligned} \quad (12)$$

It means that the increased profit for the buyer to choose a higher requirement is less than corresponding increased failure risk; but for the seller, the increased cost to improve performance is greater than the increased failure risk to degrade performance. Aiming to ensure the product's reliability, the buyer will lower the requirement and compromise to the seller.

3.2.3. *Phase transition analysis for game results*

In practice, which kind of ESS appears in a real game depends on the payoff parameter setting. And there will be the phase transition for ESSs and game results with the change of payoff parameters. In particular, according to section 3.2.2, with different values of $f_V\left(\tilde{P}_U^{\text{th}}\right) - f_V\left(\tilde{P}_L^{\text{th}}\right)$, $f_C\left(\tilde{P}_U\right) - f_C\left(\tilde{P}_L\right)$, $f_{W_{\text{th}}}\left(R_L^U\right) - f_{W_{\text{th}}}\left(R_L^L\right)$, $f_{W_{\text{th}}}\left(R_U^U\right) - f_{W_{\text{th}}}\left(R_U^L\right)$, $f_{W_P}\left(R_L^U\right) - f_{W_P}\left(R_U^U\right)$, and $f_{W_P}\left(R_L^L\right) - f_{W_P}\left(R_U^L\right)$, the phase transition of the buyer-seller game results can be illustrated as Fig. 1-3, in which the phases noted by "Collaboration / Conflict" and "Buyer compromise / Seller compromise" represents that there can be multiple ESSs under the parameter settings.

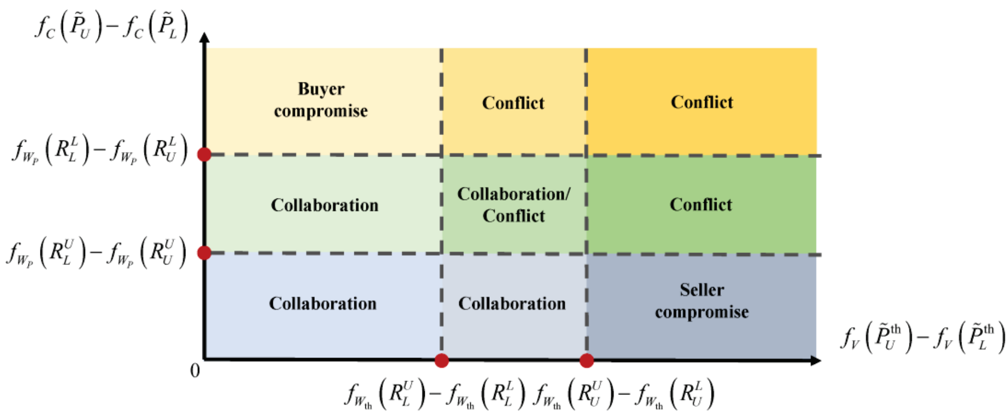

Fig. 1. Phase transition of game results when $f_{W_{\text{th}}}\left(R_L^U\right) - f_{W_{\text{th}}}\left(R_L^L\right) < f_{W_{\text{th}}}\left(R_U^U\right) - f_{W_{\text{th}}}\left(R_U^L\right)$ and $f_{W_P}\left(R_L^U\right) - f_{W_P}\left(R_U^U\right) < f_{W_P}\left(R_L^L\right) - f_{W_P}\left(R_U^L\right)$.

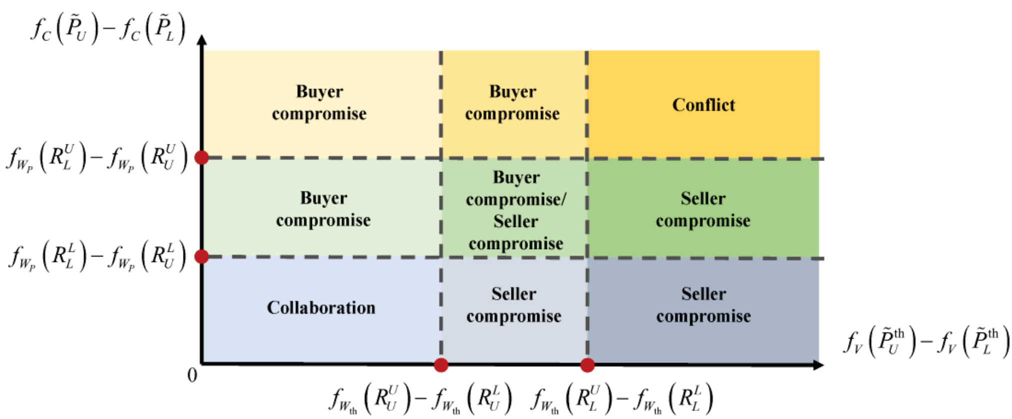

Fig. 2. Phase transition of game results when $f_{W_{th}}\left(R_L^U\right)-f_{W_{th}}\left(R_L^L\right)>f_{W_{th}}\left(R_U^U\right)-f_{W_{th}}\left(R_U^L\right)$ and $f_{W_P}\left(R_L^U\right)-f_{W_P}\left(R_U^U\right)>f_{W_P}\left(R_L^L\right)-f_{W_P}\left(R_U^L\right)$.

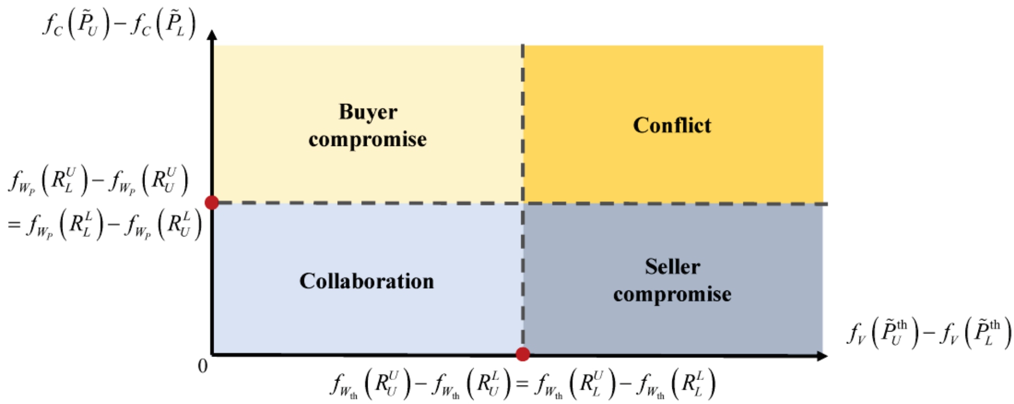

Fig. 3. Phase transition of game results when $f_{W_{th}}\left(R_L^U\right)-f_{W_{th}}\left(R_L^L\right)=f_{W_{th}}\left(R_U^U\right)-f_{W_{th}}\left(R_U^L\right)$ and $f_{W_P}\left(R_L^U\right)-f_{W_P}\left(R_U^U\right)=f_{W_P}\left(R_L^L\right)-f_{W_P}\left(R_U^L\right)$.

According to the phase transitions in Fig. 1-3, three aspects of guidance can be concluded to ensure cooperation and avoid conflict:

- For the seller, $f_C\left(\tilde{P}_U\right)-f_C\left(\tilde{P}_L\right)$ should be reduced to ensure cooperation. In engineering practice, it is mainly achieved by reducing the cost of high performance, i.e. $f_C\left(\tilde{P}_U\right)$, through technology improvement or other approaches. In other words, the seller mainly focuses on saving the cost when the performance is prospectively high.
- For the buyer, $f_V\left(\tilde{P}_U^{th}\right)-f_V\left(\tilde{P}_L^{th}\right)$ should be reduced to ensure cooperation. In practice, it can be achieved by increasing the profit of low requirement, i.e. $f_V\left(\tilde{P}_L^{th}\right)$. That is, the buyer mainly focuses on how to ensure own profit when the performance is prospectively low.
- From the aspect of risk sharing, the failure risks for both sides can be properly increased to ensure cooperation. A typical approach is to assign agreements or contracts related to the product's reliability, which identifies the penalties for both sides if the performance cannot meet the requirement.

4. Discussion: uncertainties in strategies

For the model assumptions in section 3.1, the both sides' strategies are random variables with uncertainties. Therefore, although in this work a general model is proposed without the limitation of the payoffs' forms, for specific scenarios, if the payoffs can be refined considering the uncertainties in performances and thresholds, it is expected to distinguish the effects of uncertainties.

Take the uncertainty in the product's performance as an instance. In practice, the lower uncertainty in the performance requires the higher cost of quality control. Assume that the seller's cost is inversely proportional to the performance's standard deviation, that is: $f_C\left(\tilde{P}\right)=\mathrm{C}/\sigma\left(\tilde{P}\right)$, where C reflects the cost of quality control; and for $\tilde{P}_L$ and $\tilde{P}_U$, there is $\sigma\left(\tilde{P}_L\right)>\sigma\left(\tilde{P}_U\right)$. According to Fig. 1-3, it can be inferred that with the increase of C, the seller may be reluctant to provide the product with high consistency, which leads to the buyer-seller conflict. Conclusively, it is necessary to include the strategies' uncertainties in the game, which can model the actual consideration of quality control.

5. Conclusion

The product's reliability is essentially determined by the function and performance of the seller's product and the buyer's requirement. In this work, the buyer-seller game is studied based on the performance margin, which is expected to integrate the product's operational principles and the buyer's requirement into reliability. In the game model, the performance threshold profit, performance cost, and failure risks are proposed to quantify the interest relationship between the buyer and the seller. A typical two-strategy game is modeled and solved by evolutionary game theory. As a result, the seller's behavior to develop product and the buyer's strategy to propose requirements can be forecasted. Also, three approaches are concluded to facilitate cooperation: for the seller, reduce cost when providing a high performance; for the buyer, ensure profit when the performance is prospectively low; and for both sides, increase the penalties of failure.

For the future work, the buyer-seller game for the product's performance margin and reliability will be further fulfilled, including the game solving and the consideration of multiple scenarios like incomplete information. Based on the proposed method, it is expected to provide guidance for the decision making about reliability in practice.

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