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A prescriptive maintenance policy in the presence of imperfect maintenance

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The objective of this paper is to provide preliminary insights to support maintenance decision-making in a context imperfect maintenance is present and "prescriptive actions" (i.e., actions which suppose that the performances of the system can be voluntarily altered to obtain a desired outcome) can be integrated.
The policy consists in performing an inspection at a predetermined time and, based on its outcome, deciding whether to immediately replace (i.e., perfect replacement) the unit or to postpone its replacement. If the replacement is postponed, an imperfect maintenance may be concurrently performed which reduces the degradation level and modifies (stochastically) the degradation behavior of the unit. After the first inspection, the imperfectly maintained systems are then inspected again and, based on the outcome of this second inspection, it is decided whether to immediately replace the unit or to further postpone its replacement to a future time where no inspection is performed and the unit is replaced systematically. In case of this second postponement, the degradation information gathered at the inspection times is also used to (possibly) adjust the usage rate of the unit. The idea here is to use the degradation measurements to quantify the uncertainty brought about by the imperfect action and intervene accordingly with the usage rate. Any change in the usage rate is assumed to affect both the future evolution of the degradation of the system and its operational costs. The driving idea behind the conception of this policy is to investigate if and how prescriptive maintenance actions (i.e., changing the usage rate) can assist in reducing the uncertainty brought about by the imperfect maintenance action. Failure is defined by the first passage time of the degradation process to a predefined threshold. It is also supposed to be not self-announcing and that failed units keep operating, albeit with reduced performances/additional costs.

Keywords: Prescriptive maintenance, imperfect maintenance, usage rate, uncertainty.

1. Introduction

In recent years, prescriptive maintenance has gained increasing interest in the literature. As proposed by Longhitano et al. (2021a), its basic premise is to develop an integrated framework where maintenance decisions are taken accounting also for operational and commercial aspects. This approach entails considering actions that go beyond the classical inspection, repair, and replacement of a system, such as adjusting some operational parameters that can provide additional degrees of freedom to reach a globally better tradeoff between preventive, corrective, and operational costs, especially when some external constraints are imposed on the maintenance intervention epochs.

Building on this general concept, Longhitano et al. (2021b) optimized the mission scheduling of a vehicle fleet to minimize brake pad maintenance. Longhitano et al. (2022) and Longhitano et al. (2024) broadened the decision space by optimizing routes and parameters for battery-powered vehicles through a physics-based approach. Esposito et al. (2022a), Esposito et al. (2022b), and Esposito et al. (2023a) proposed a dynamic framework incorporating constrained replacement schedules and adaptive usage rate adjustments. A common theme of these policies is also the use of "prescriptive action" (i.e., adjusting the usage rate, for example) based on real-time degradation measurement. In this way, the prescriptive action can, implicitly, account for and mitigate temporal variability (e.g., the natural randomness in the evolution of the degradation process over time).

In this paper, we follow the same driving principles as Longhitano et al. (2021b), Esposito et al. (2022a), and Esposito et al. (2023a), but aim to apply them in a framework where imperfect maintenance is present.

An imperfect maintenance action is defined as an action which, instead of restoring the system to the new state (i.e., perfect maintenance), restores it to some intermediate state between the one pre-maintenance and the new state. In the imperfect maintenance literature, it is often assumed that post-maintenance the degradation/failure behavior of the system does not change. However, this is not always the case, as the imperfect action can sometimes alter the degradation properties of the system (e.g., see Chen et al. 2024).

Implicitly, this introduces an additional form of variability, which we aim to account for and manage by using prescriptive actions.

In this paper, we propose a prescriptive policy for a degrading unit where an inspection is performed at a predetermined time and its outcome informs whether to immediately replace the unit, postpone its replacement, or perform an imperfect maintenance action. This latter action reduces degradation, but also randomly alters the degradation rate. In case the imperfect action is carried out, a second inspection is also scheduled and its outcome is then used to determine a new usage rate for the remainder of the operating life of the unit.

The rest of the paper is organized as follows. Section 2 describes in detail the prescriptive policy. Section 3 illustrates the adopted degradation process. Section 4 describes the cost function, while Section 5 presents an example of application, and Section 6 concludes the paper.

2. Policy description

The assumptions which underpin the development of the policy are:

- Inspections are instantaneous, non-destructive, and reveal the exact degradation state of the unit;
- A maximum of two inspections can be carried out over the lifetime of the unit;
- Perfect maintenance actions (i.e., replacements) restore the unit to an "as good as new" state. Consequently, the time between successive replacements defines the cycles of a renewal process (i.e., the maintenance cycle);
- Imperfect maintenance can be performed a single time during the operational life of the unit, exclusively at the inspection time;
- Failures are not self-announcing. Consequently, units keep operating past their failure instant but an additional cost is incurred.

The proposed policy consists in performing an inspection at a predetermined time τ_1 which will return a measurement of the degradation level of the unit, hereinafter denoted by w_1. This value is compared with two thresholds, denoted by L_u and L_l (with $L_l \leq L_u$). Specifically, the unit can be replaced either immediately (if $w_1 > L_u$) or at a later time (if $w_1 \leq L_l$), denoted by τ_3. Otherwise, if $L_l < w_1 \leq L_u$, then an imperfect maintenance action will be performed. Table 1 summarizes the condition-based rule which informs decision-making used at τ_1.

Table 1. Condition-based rule which informs decision-making at the first inspection time.

Degradation at τ_1	Decision
$w_1 > L_u$	Replace immediately
$L_l < w_1 \leq L_u$	Perform imperfect maintenance
$w_1 \leq L_l$	Postpone replacement to τ_3

Unlike a replacement, which restores the unit to the new state, an imperfect maintenance action reduces the degradation level only by a certain factor (this is also referred to as the improvement factor model of imperfect maintenance, see Zhang et al. 2017). Hence, the degradation level after the action, denoted by w_1^+, is given by:

$$w_1^+ = w_1 \cdot (1 - \beta)$$

where β is the improvement factor which here is assumed to be a known deterministic number (Mercier and Castro 2013, Wang et al. 2020).

In addition, we assume that the imperfect maintenance action also randomly alters the future degradation behavior of the unit by modifying the parameters of its degradation process.

Anyhow, in case the imperfect maintenance action is carried out, then a second inspection is performed at time τ_2 ($\tau_2 > \tau_1$). Based on the outcome of this inspection, the unit is immediately replaced if it is found to be failed. Otherwise, its replacement is postponed to time τ_4 ($\tau_4 > \tau_2$). In this latter case, the usage rate of the unit can be adjusted. This action will have an influence both on the degradation evolution and on operating costs. The condition-based rule used at the second inspection time is summarized in Table 2, where w_2 is the measured degradation level at τ_2 and $g(w_1^+, w_2)$ is a function which adaptively determines the new usage rate based on the degradation level post-imperfect maintenance (i.e., w_1^+) and at the second inspection time (w_2).

Table 2. Condition-based rule which informs decision-making at the second inspection time.

Degradation at τ_2	Decision
$w_2 > w_M$	Replace immediately
$w_2 \leq w_M$	Postpone replacement to τ_4, adjust usage rate to $g(w_1^+, w_2)$.

We adopted the following functional form for $g(\cdot)$:

$$g(w_1^+, w_2) = \lambda_1 \cdot \left(1 - \mathbb{B}\left(\frac{w_2 - w_1^+}{w_M}; \lambda_2, \lambda_3\right)\right) \quad (1)$$

where λ_1, λ_2, and λ_3 are design parameters of the policy. We picked this functional form, inspired by the cdf of the Beta distribution, because it is a monotonic decreasing function which, by assigning adequate values to λ_1, λ_2, and λ_3, can take many different shapes (such as power-law shapes, S-shapes, and linear shapes). Additionally, it also has limited codomain, which is fitting in our case because it would be unrealistic to suppose that the usage rate might have no bounds.

Consequently, the vector of the design parameters of the proposed policy, denoted by $\boldsymbol{\xi}$, is $\boldsymbol{\xi} = \{L_l, L_u, \tau_1, \tau_2, \tau_3, \tau_4, \lambda_1, \lambda_2, \lambda_3\}$.

A possible application for this policy, which also helps justifying some of the assumptions we made, comes from the tire retreading (Qiang et al. 2020). Tire retreading involves removing a worn outer rubber layer and applying a new thread, resulting in a measurable and reproducible reduction in wear. However, depending on the state of the tire carcass and on the quality of the intervention, the retreaded tire might wear differently than a new one. For this reason, tire manufacturers recommend retreading tires a limited number of times (e.g., see Cardoso 2022).

3. The adopted gamma process

The gamma process (see Van Noortwijk 2009) $\{Y(t); t \geq 0\}$ is a monotonic increasing stochastic process characterized by gamma-distributed independent increments. Hence, it is fully defined by an initial condition (here $Y(0) = 0$) and the probability density function (pdf) of its generic increment $\Delta Y(t_1, t_2) = Y(t_2) - Y(t_1)$:

$$f_{\Delta Y(t_1,t_2)}(\delta) = \frac{\delta^{\Delta\eta(t_1,t_2)-1}}{\theta^{\Delta\eta(t_1,t_2)} \cdot \Gamma(\Delta\eta(t_1, t_2))} \cdot e^{-\frac{\delta}{\theta}}, \quad \delta > 0 \quad (2)$$

where $t_1 < t_2$, θ ($\theta > 0$) is the scale parameter, $\Gamma(\cdot)$ is the complete gamma function, $\Delta\eta(t_1, t_2) = \eta(t_2) - \eta(t_1)$, and $\eta(t)$ is a non-negative monotonic increasing function referred to as the age function.

In this paper, we assume that the degradation process $\{W(t); t \geq 0\}$ of the unit can be described by a gamma process and (following Tseng et al. 2009 and Esposito et al. 2023a) that an adjustment of the usage rate will impact the degradation behavior of the unit only through the age function. Similarly, we suppose that the age function captures also the effect of the imperfect maintenance action.

Consequently, the functional form of the shape function, and in turn of the degradation increment, will change multiple times during the operational life of the unit.

Specifically, under the policy described in Section 2, the increment $\Delta W_0(0, t) = W(t) - W(0) = W(t)$, at any time $t \leq \tau_1$ (i.e., at any time before the first inspection) has gamma pdf:

$$f_{\Delta W_0(0,t)}(\delta) = \frac{\delta^{\eta_0(t)-1} \cdot e^{-\frac{\delta}{\theta}}}{\theta^{\eta_0(t)} \cdot \Gamma(\eta_0(t))}, \quad \delta > 0 \quad (3)$$

with scale parameter θ and shape parameter determined by the age function $\eta_0(t) = a \cdot t$, which captures the degradation characteristics before the inspection time.

Then, at the inspection time τ_1, depending on the obtained measurement w_1 the degradation will evolve differently. Specifically, if and only if $w_1 \leq L_l$ (i.e., if the decision at τ_1 is "postpone the replacement to τ_3"), the pdf of the increment $\Delta W_0(\tau_1, t) = W(t) - W(\tau_1)$, with $\tau_1 \leq t < \tau_3$ is:

$$f_{\Delta W_0(\tau_1,t)}(\delta) = \frac{\delta^{\Delta\eta_0(\tau_1,t)-1} \cdot e^{-\frac{\delta}{\theta}}}{\theta^{\Delta\eta_0(\tau_1,t)} \cdot \Gamma\big(\Delta\eta_0(\tau_1,t)\big)}, \quad (4)$$

which is still gamma distributed, with scale parameter θ and shape parameter determined by $\Delta\eta_0(\tau_1,t) = \eta_0(t) - \eta_0(\tau_1)$.

On the other hand, if $L_l < w_1 \leq L_u$ (i.e., if the decision at τ_1 is "perform imperfect maintenance"), then the increment $\Delta W'(\tau_1, t) = W(t) - W(\tau_1)$, with $\tau_1 \leq t < \tau_2$ (i.e., at any time between the first and second inspection), has pdf:

$$f_{\Delta W'(\tau_1,t)}(\delta) = \frac{\delta^{\Delta\eta'(\tau_1,t)-1} \cdot e^{-\frac{\delta}{\theta}}}{\theta^{\Delta\eta'(\tau_1,t)} \cdot \Gamma\big(\Delta\eta'(\tau_1,t)\big)}, \quad (5)$$

where $\Delta\eta'(\tau_1,t) = \eta'(t) - \eta'(\tau_1)$ and $\eta'(t) = a^* \cdot t$.

Notably, this function, through the parameter a^*, incorporates the uncertainty introduced by the imperfect maintenance action. In fact, a^* should be interpreted as the (unobservable) realization of the random variable A^*, whose distribution captures the variability resulting from the imperfect maintenance action.

In this paper, A^* is assumed to be uniformly distributed:

$$A^* \sim Uniform(a_{min}, a_{max}).$$

where a_{min} and a_{max} are, respectively, the minimum and maximum possible values of a^*.

Finally, if and only if the second inspection is performed at τ_2 and $w_2 \leq w_M$ (i.e., the decision at τ_2 is "postpone the replacement, adjust usage rate"), then the subsequent adjustment in the usage rate again influences the degradation behavior and the increment $\Delta W''(\tau_2, t; u) = W(t) - W(\tau_2)$, with $\tau_2 \leq t < \tau_4$ (i.e., at any time between the second inspection and the eventual replacement time) has pdf:

$$f_{\Delta W''(\tau_2,t;u)}(\delta) = \frac{\delta^{\Delta\eta''(\tau_2,t;u)-1}}{\theta^{\Delta\eta''(\tau_2,t;u)} \cdot \Gamma\big(\Delta\eta''(\tau_2,t;u)\big)} \cdot e^{-\frac{\delta}{\theta}}, \quad (6)$$

Where $\Delta\eta''(\tau_2, t; u) = \eta''(t; u) - \eta''(\tau_2; u)$, $\eta''(t; u) = a^* \cdot \frac{u}{u_{max}} \cdot t$, and u_{max} is the maximum allowable usage rate. The presence of u in the expression of $\eta''(\cdot)$ explicitly indicates how the influence of the usage rate is accounted for.

In this paper, a unit is assumed to fail when its degradation process passes for the first (and sole) time a predetermined failure threshold, say w_M. Therefore, the useful life X of the unit is:

$$X = \inf\{x: W(x) > w_M\}.$$

In the remainder of this section some results which involve the lifetime X, and that are necessary to compute maintenance costs, will be illustrated.

It is possible (see Esposito et al. 2023b) to obtain the conditional cdf of X given $W(\tau_1) = w_1$ in the case where $w_1 > w_M$, and $0 < X \leq \tau_1$ as:

$$F_{X|W(\tau_1)}(x|w_1) = P(X \leq x|W(\tau_1) = w_1) = P(W(x) > w_M|W(\tau_1) = w_1) = 1 - \mathbb{B}\left(\frac{w_M}{w_1}; \eta_0(x), \Delta\eta_0(x,\tau_1)\right) \quad (7)$$

where the first equality is justified because the gamma process is monotonic increasing and (consequently) the event $\{X \leq x\}$ is equivalent to the event $\{W(x) > w_M\}$.

Similarly, we can obtain the conditional cdf of X given $W(\tau_1) = w_1$ and $W(\tau_3) = w_3$ in the case where $w_1 \leq L_l$, $w_3 > w_M$, and $\tau_1 < X \leq \tau_3$ as:

$$F_{X|W(\tau_1),W(\tau_3)}(x|w_1, w_3) = P(X \leq x|W(\tau_1) = w_1, W(\tau_3) = w_3) = P(W(x) > w_M|W(t_1) = w_1, W(\tau_3) = w_3) = 1 - \mathbb{B}\left(\frac{w_M - w_1}{w_3 - w_1}; \Delta\eta_0(\tau_1, x), \Delta\eta_0(x, \tau_3)\right) \quad (8)$$

Moreover, we can obtain the conditional cdf of X given $W(\tau_1) = w_1$ and $W(\tau_2) = w_2$ in the case where $L_l < w_1 \leq L_u$, $w_2 > w_M$, and $\tau_1 < X \leq \tau_2$ as:

$$F_{X|W(\tau_1),W(\tau_2)}(x|w_1, w_2) = P(X \leq x|W(\tau_1) = w_1, W(\tau_2) = w_2) = P(W(x) > w_M|W(t_1) = w_1, W(\tau_2) = w_2) = 1 - \mathbb{B}\left(\frac{w_M - w_1}{w_2 - w_1}; \Delta\eta'(\tau_1, x), \Delta\eta'(x, \tau_2)\right) \quad (9)$$

Finally, we can obtain the conditional cdf of X given $W(\tau_2) = w_2$ and $W(\tau_4) = w_4$ in the case where $w_2 < w_M$, $w_4 \geq w_M$, and $\tau_2 < X \leq \tau_4$ as:

$$F_{X|W(\tau_2),W(\tau_4)}(x|w_2, w_4; u) = P(X \leq x|W(\tau_2) = w_2, W(\tau_4) = w_4; u) = P(W(x) > w_M|W(t_2) = w_2, W(\tau_4) = w_4; u) = 1 - \mathbb{B}\left(\frac{w_M - w_2}{w_4 - w_2}; \Delta\eta''(\tau_2, x; u), \Delta\eta''(x, \tau_4; u)\right) \quad (10)$$

4. Formulation of the cost function and the long-run average maintenance cost rate

Maintenance costs are determined considering the cost of a preventive replacement c_p, the cost of a corrective replacement c_c $(c_c > c_p)$, the inspection cost c_i, a logistic cost c_l (incurred each time a maintenance action is performed), the cost of an imperfect maintenance action c_m, and a downtime cost (which captures the additional costs resulting from operating the unit past its failure point). This latter cost is determined as the product of a fixed cost rate c_d and the length of the downtime (i.e., the time elapsing from the failure of the unit until its eventual replacement). Moreover, the cost model also considers the impact of changes in the usage rate. Specifically, reducing the usage rate from its maximum value u_{max} incurs a penalty cost rate computed as:

$$c_{pen}(u) = c_u \cdot \left(1 - \frac{u}{u_{max}}\right)$$

Then, the actual penalty cost incurred by setting the usage rate u for a given time interval is obtained as the product of this cost rate and the length of the interval.

Table 3 lists all the possible scenarios along with the corresponding maintenance costs $C(w_1, w_2, X)$ and the length of the maintenance cycle $T(w_1, w_2)$. Coherently with the assumption of not self-announcing failure, in Table 3 the lifetime X is always denoted by a capital letter. The first column of Table 3 assigns a number to each scenario: this number will be needed to match the scenario with its corresponding cost and cycle length in the simulation algorithm presented in the next Section.

Table 3. Possible scenarios and corresponding maintenance costs and cycle length

Scenario N°	Degradation at τ_1	Degradation at τ_2	Lifetime	Cost $C(w_1, w_2, X)$	Cycle length $T(w_1, w_2)$
1	$w_1 > w_M$		$X < \tau_1$	$c_l + c_i + c_c + c_d \cdot (\tau_1 - x)$	τ_1
2	$L_u < w_1 \leq w_M$		$X > \tau_1$	$c_l + c_i + c_p$	τ_1
3	$L_l < w_1 \leq L_u$	$w_2 > w_M$	$\tau_1 < x \leq \tau_2$	$2 \cdot (c_l + c_i) + c_m$ $+ c_c + c_d \cdot (\tau_2 - x)$	τ_2
4	$L_l < w_1 \leq L_u$	$w_2 \leq w_M$	$\tau_2 < X \leq \tau_4$	$3c_l + 2c_i + c_m + c_c$ $+ c_d \cdot (\tau_4 - x)$ $+ c_{pen}(u) \cdot (\tau_4 - \tau_2)$	τ_4
5	$L_l < w_1 \leq L_u$	$w_2 \leq w_M$	$X > \tau_4$	$3c_l + 2c_i + c_m + c_p$ $+ c_{pen}(u) \cdot (\tau_4 - \tau_2)$	τ_4
6	$w_1 \leq L_l$		$\tau_1 < X \leq \tau_3$	$2c_l + c_i + c_c + c_d \cdot (\tau_3 - x)$	τ_3
7	$w_1 \leq L_l$		$X > \tau_3$	$2c_l + c_i + c_p$	τ_3

By using the renewal-reward theorem (see Ross 1983), the long-run average maintenance cost rate $C_\infty(\boldsymbol{\xi})$ can be formulated as:

$$C_\infty(\boldsymbol{\xi}) = \frac{E\{C(w_1, w_2, X)\}}{E\{T(w_1, w_2)\}} \tag{11}$$

Under the proposed policy, the expectations included in this expression cannot be computed in closed form but can be obtained through Monte Carlo simulation. The pseudocode used for this simulation is illustrated in Table 4. In Table 4, $C[i]$ and $T[i]$ denote the cost and the cycle length obtained in the ith simulated run, where $i = 1, \dots, N$ and N is the total number of simulated runs. Of course, the accuracy of the results will increase with N, which should be determined by balancing accuracy and computational time (which also increases with N).

The notation $C[i], T[i] \leftarrow scenario\ k$ indicates that the cost and the cycle length of the ith simulated run coincide with the corresponding values from the kth scenario (see Table 3).

Table 4. Pseudocode used to compute $C_\infty(\boldsymbol{\xi})$.

```
SIMULATOR
For i = 1 to N
 | w_1 ← sample from ΔW_0(0, τ_1)
 | If w_1 > w_M then
 |  | x ← sample from X|W(τ_1)
 |  | C[i], T[i] ← scenario 1
 | Elseif L_u < w_1 ≤ w_M
 |  | C[i], T[i] ← scenario 2
 | Elseif L_l < w_1 ≤ L_u then
 |  | w_1^+ ← w_1 · (1 − β)
 |  | a* ← sample from A*
 |  | Δw ← sample from ΔW'(τ_1, τ_2)
 |  | w_2 ← w_1^+ + Δw
 |  | If w_2 > w_M then
 |  |  | x ← sample from
 |  |  |        X|W(τ_1), W(τ_2)
 |  |  | C[i], T[i] ← scenario 3
 |  | Else
 |  |  | u ← g(w_1^+, w_2)
 |  |  | Δw ← sample from
 |  |  |        ΔW''(τ_2, τ_4; u)
 |  |  | w_4 ← w_2 + Δw
 |  |  | If w_4 > w_M then
 |  |  |  | x ← sample from
 |  |  |  | X|W(τ_2), W(τ_4)
 |  |  |  | C[i], T[i] ← scenario 4
 |  |  | Else
 |  |  |  | C[i], T[i] ← scenario 5
 | Else
 |  | Δw ← sample from ΔW_0(τ_1, τ_3)
 |  | w_3 ← w_1 + Δw
 |  | If w_3 > w_M then
 |  |  | x ← sample from
 |  |  |        X|W(τ_1), W(τ_3)
 |  |  | C[i], T[i] ← scenario 6
 |  | Else
 |  |  | C[i], T[i] ← scenario 7
C_∞(ξ) ← mean(C)/mean(T)
```

The optimal value of $\boldsymbol{\xi}$, denoted by $\boldsymbol{\xi}^*$, is obtained by minimizing the long-run average maintenance cost rate in Eq. (11). The corresponding optimal value $C_\infty(\boldsymbol{\xi}^*)$ is denoted as C_∞^*.

5. Numerical example

In this section, we show the utility of the proposed policy via a numerical example.

As already mentioned, the main novelty of the policy comes from the idea of exploiting the information gathered through a dedicated inspection to adjust the usage rate of the unit with the aim of managing the uncertainty introduced by the imperfect maintenance action (hereinafter, for the sake of brevity, this particular kind of uncertainty will be referred to as "heterogeneity").

To highlight the effectiveness of this prescriptive action, we compare the proposed policy, hereinafter denoted as P_0, with a similar policy derived from P_0 where the second inspection (and hence the prescriptive action) is not allowed, hereinafter denoted as P_1.

Indeed, under P_1 the first inspection and (possibly) the imperfect maintenance action are still performed at τ_1. However, the second inspection at τ_2 is not performed, but the units are directly replaced at τ_4.

It is worth remarking that P_1 cannot be obtained as a direct special case of P_0 (that is, there is no combination of design parameters that allows P_0 to exactly reduce to P_1).

The rationale for this comparison is that under P_0 the heterogeneity is directly accounted for and managed via the prescriptive action, while under P_1 it is not. Therefore, we compared the performances of these two policies in terms of the long-run average maintenance cost rate across various setups which differ in the value of c_u (which modulates the cost of the prescriptive action) and the magnitude of the heterogeneity. All other parameters of the cost model, as well as the parameters of the degradation process, are kept constant. The values of these constant parameters are reported in Table 5.

Table 5. Values of the parameters of the cost model and of the degradation process.

c_l	c_i	c_p	c_c	c_d	c_m	a	θ	w_M	k
0.2	0.1	2	6	0.2	0.5	0.85	0.47	35	0.5

The magnitude of the heterogeneity is modulated by varying the bounds of the uniform distribution that we assumed for the variable A^*. Let φ be a real number such that $0 \le \varphi \le 1$. Then, the distribution of A^* is:

$$A^* \sim Uniform(a_{min}, a_{max})$$

where:

$$a_{min} = a \cdot (1 - 0.05 \cdot \varphi)$$
$$a_{max} = a \cdot (1 + 0.4 \cdot \varphi)$$

Roughly speaking, as φ increases ~~the~~ a_{min} and a_{max} get progressively further from a, thus increasing the magnitude of the heterogeneity.

Figure 1 reports the results of this sensitivity analysis. It depicts the optimal long-run average maintenance cost rate C_∞^* obtained under both P_A (red and orange solid lines) and P_B (blue dashed line) as a function of φ, for different values of c_u. Of course, the performances of P_B are not affected by changing c_u.

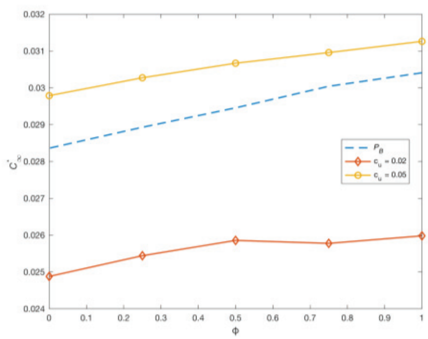

Fig. 1. Optimal long-run average maintenance cost rate under P_A and P_B as a function of φ, for different values of c_u.

Several observations can be made from Figure 1.

Firstly, as expected, as the magnitude of the heterogeneity increases, the optimal long-run average maintenance cost rate obtained under both P_A and P_B increases.

Secondly, we can observe that, under the policy P_B, C_∞^* increases with φ roughly linearly, while under P_A the trend appears to be slightly less than linear, indicating that P_A can adapt to higher magnitudes of the heterogeneity marginally better than P_B.

Thirdly, we see that even in the absence of heterogeneity, when c_u is low, P_A can still comfortably outperform P_B. In fact, as observed in Esposito et al. (2023a), adjusting the usage rate can be an effective action even when (as it is when $\varphi = 0$) the only source of uncertainty is temporal variability.

However, the cost associated with this action can hinder its effectiveness. Figure 2 delves deeper into this issue. It reports the optimal long-run average maintenance cost rate obtained under P_A (blue solid line) and P_B (red dashed line) as a function of the unitary cost rate of the usage rate adjustment c_u, when $\varphi = 1$.

From Figure 2 we can observe that, even in the most favorable scenario for P_A (i.e., when the heterogeneity is high) if the value of c_u is sufficiently high (in the considered setup, for $c_u > 0.03$) it is not economically convenient to perform the second inspection and adjust the usage rate.

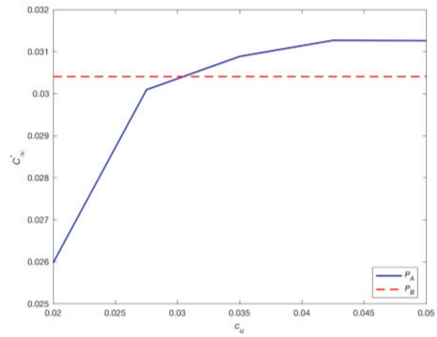

Fig. 2. Optimal long-run average maintenance cost rate under P_A and P_B as a function of c_u, with φ set to 1.

Another interesting observation can be made regarding the functional form we adopted to model the function $g(w_1^+, w_2)$, which is used to assign the usage rate at τ_2 as a function of the measured degradation level at the two inspection times.

In fact, the role of this function is to assign an appropriate usage rate that can account for the heterogeneity introduced by the imperfect maintenance action, which alters the parameter a of the underlying gamma degradation process.

Ideally, the usage rate should be calibrated by directly measuring, or estimating, this parameter. However, direct measurements are not available and estimating it from degradation measurements can be challenging and time intensive.

Therefore, we had to rely on a proxy measurement, which, in this paper, was the difference between the degradation level at the second inspection and after the imperfect maintenance action, i.e., $w_2 - w_1^+$. The rationale being that high values of this difference would suggest an accelerated degradation rate, and vice versa. However, natural temporal variability can partially mask this effect, which makes $w_2 - w_1^+$ and imperfect proxy, which can be improved upon.

6. Conclusions

In this paper, we have proposed a prescriptive maintenance policy for degrading units in the presence of imperfect maintenance. The policy consists in performing an inspection at a predetermined time and, based on its outcome, deciding whether to immediately replace the unit, postpone its replacement, or perform, at the same inspection time, an imperfect maintenance action.

This action is supposed to deterministically reduce the degradation level of the unit, but also randomly alter its degradation rate. In case the imperfect maintenance action is performed, a second inspection is carried out and, depending on both gathered measurements, the usage rate of the unit can possibly be adjusted.
The driving idea behind this policy is to use the prescriptive action (i.e., the adjustment of the usage rate) to account for the uncertainty introduced by the maintenance action and moderate its consequences.
The performances of the policy, in terms of long-run average cost rate, have been compared with those of a similar policy where the second inspection, and hence the prescriptive action, are not considered.
Obtained results show that, depending on the setup of the cost model, introducing the prescriptive action can provide noticeable benefits in economic terms.

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