Proceedings of the 35th European Safety and Reliability & the 33rd Society for Risk Analysis Europe Conference
Edited by Eirik Bjorheim Abrahamsen, Terje Aven, Frederic Bouder, Roger Flage, Marja Ylönen

doi: 10.3850/978-981-94-3281-3_ESREL-SRA-E2025-P1542-cd

The reliability evaluation and selective maintenance decision for lattice system

Cong Lin

China Aero-Polytechnology Establishment, China. E-mail: linc002@avic.com
Aviation Key Laboratory of Science and Technology on Aero Combined Environment, China

Zhaoyan Zeng

China Aero-Polytechnology Establishment, China. E-mail: zengzy007@avic.com
Aviation Key Laboratory of Science and Technology on Aero Combined Environment, China

Wensheng Peng

China Aero-Polytechnology Establishment, China. E-mail: pengws003@avic.com
Aviation Key Laboratory of Science and Technology on Aero Combined Environment, China

The phased array radar composed by the T/R units in a two-dimensional surface is a typical lattice system. The failure criteria can be described as the failure units within a subarea exceed a threshold. The exact reliability evaluation method of such system is not easy to be obtained. Thus, we proposed a reliability lower bound evaluation method. We provide an importance-based principle to identify the weakest area $r \times s$ to conduct a selective maintenance. Some numerical examples are provided to show how to use the proposed method.

Keywords: lattice system, two-dimensional system, k-within-$r \times s$-out-of-$m \times n$, system reliability, selective maintenance.

1. Introduction

The lattice system, characterized by components arranged in a regular two-dimensional array, was initially introduced by Salvia & Lasher in the early 1990s (Salvia and Lasher 1990). However, this system exhibited unreliability when consecutive components in the same row or column failed. Subsequently, Yamamoto and Akiba proposed the connected- k -out-of- n system to expand the failure criteria of the lattice system (Yamamoto and Akiba 2003). In recent years, a more general reliability model for the lattice system has been developed, namely the k-within-$r \times s$-out-of-$m \times n$ system. The failure criteria for this model are defined as the number of failed components within an arbitrary window $r \times s$, out of a system with m rows and n columns, has accumulated to at least k. This can lead to the appearance of a "blind area," causing the malfunction of the system, e.g. the phased array radar.

Fig. 1 illustrates a schematic diagram of a $m(5) \times n(6)$ lattice system, with $k(5)$ of them having failed (Lin et al. 2019). In Fig. 1-(a), the failure components are sparsely distributed, so the system is considered as functional. In contrast, Fig. 1-(b) shows that the 5 failed components are located within a window of $r(2) \times s(3)$, which may cause the malfunction of the subarea, so the system is regarded as malfunctional.

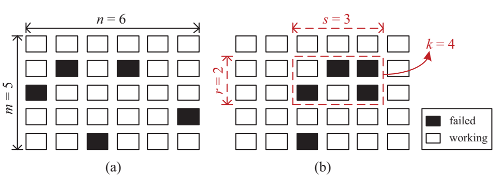

Fig. 1.Example of a lattice system.

The lattice system finds application in a diverse range of engineering systems, including phased array radar, X-ray diagnostics, design of electronic devices, and camera surveillance systems. In these applications, the overall performance of the system is contingent upon the functionality of the components within a specific block. Consequently, the system is deemed to have failed when a "blind area" emerges, indicating a loss of functionality in a particular region.

The selective maintenance is an imperfect maintenance strategy, particularly when spare parts are limited. The primary objective of selective maintenance is to identify and replace components which arise higher increment of the system reliability. For the lattice system, the central challenge lies in locating the window of $r \times s$ containing k failed components. In other words, it is essential to develop a reliability evaluation method for the k-within-$r \times s$-out-of-$m \times n$ system model.

However, obtaining an exact reliability evaluation method for the lattice system remains challenging. Previous research has primarily focused on providing upper or lower bounds for system reliability. Godbole et al. utilized Janson's exponential inequalities to derive improved upper bounds on the reliability of such systems (Godbole et al. 1998). Makri and Psillakis derived both lower and upper bounds on the reliability for the system model by employing improved Bonferroni inequalities (Makri and Psillakis 1996). Akiba and Yamamoto proposed a recursive algorithm for computing the reliability of the model, demonstrating its superiority over the enumeration method (Akiba and Yamamoto 2001) [5]. The limitation of these methods is that they are only applicable under the assumption of homogeneous components. Yet, this assumption is highly restrictive for the system model, as non-homogeneity is widely prevalent in real engineering applications due to differences in component types, remaining lifetimes, or working states. Lin et al. addressed this issue by employing the finite Markov chain imbedding approach (FMCIA) to develop a lower bound calculating method for the reliability of lattice systems with non-homogeneous components (Lin et al. 2019).

The organization of this paper is as follows. Section 2 elucidates the process of modeling the lower bound of system reliability utilizing the methodology introduced by Lin et al. (Lin et al. 2019). Section 3 presents an importance-based method for determining the failure probability of a specific area, which is essential for supporting selective maintenance decisions. Subsequently, Section 4 illustrates the application of the proposed method through several numerical examples, demonstrating how it can be employed to facilitate selective maintenance decisions. Finally, Section 5 summarizes the key findings and conclusions of the study.

2. Method to Evaluate the Lower Bound of System Reliability

To facilitate a more concise description of the k-within-$r \times s$-out-of-$m \times n$ model, we denote it as $k - r \times s/m \times n$. This model primarily aims to ascertain whether the quantity of failed components within a window of size $r \times s$ out of the entire area of $m \times n$ surpasses the threshold k. To verify whether the failure criterion is accomplished, a sliding window of size $r \times s$ is employed to systematically scan the entire area from the top-left corner to the bottom-right corner. We denote $R^{xy}_{k/r\times s}$ as the reliability index of the $r \times s$ sliding window expanding from component a_{xy}, seen in Fig. 2.

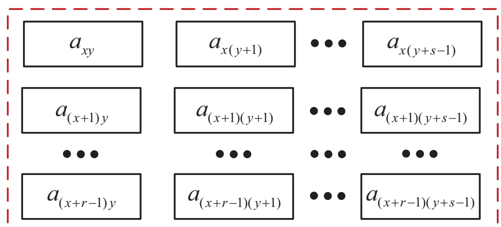

Fig. 2. A sliding window expanding from a_{xy}.

By employing FMCIA, the reliability of the sliding window equals to the k-out-of-n model and can be obtained with Eq. (1), where $\boldsymbol{\Lambda}_{ij}$ is the system state transition matrix when component a_{ij} with working probability p_{ij} is embedded; $\boldsymbol{\pi}_0 = (1,0,0,\dots,0)_{1\times(k+1)}$ is the initial state probability vector which represents all components in the system are working at beginning; $\mathbf{U}_0 = (1,1,\dots,1,0)^T_{1\times(k+1)}$ is to add up all the probability values of the reliable states and T is a vector transpose operator.

$$R^{xy}_{k/r\times s} = \boldsymbol{\pi}_0 \prod_{i=x}^{x+r-1} \prod_{j=y}^{y+s-1} \boldsymbol{\Lambda}_{ij} \mathbf{U}_0. \qquad (1)$$

$$\mathbf{\Lambda}_{ij}=\begin{bmatrix} p_{ij} & 1-p_{ij} & 0 & \cdots & 0 \\ 0 & p_{ij} & 1-p_{ij} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & p_{ij} & 1-p_{ij} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}_{(k+1)\times(k+1)}$$

Then, we denote the reliability lower bound of the "k-within-$r\times s$-out-of-$m\times n$" lattice system as $R_{k-r\times s/m\times n}$, which can be presented as a series model composed by the sliding window with $(n-s+1)\times(m-r+1)$ possible situations. Eq. (2) reveals how to obtain the system reliability lower bound, where we assume all components are working by the time we conduct the reliability evaluation.

$$R_{k-r\times s/m\times n}=\prod_{x=1}^{m-r+1}\prod_{y=1}^{n-s+1}\left[\boldsymbol{\pi}_0\prod_{i=x}^{x+r-1}\prod_{j=y}^{y+s-1}\mathbf{\Lambda}_{ij}\mathbf{U}_0\right]. \quad (2)$$

In some situations, there may have been several failed components by the time we conduct the reliability evaluation. It will influence the initial state of the Markov chain so that we need to update the $\boldsymbol{\pi}_0\rightarrow\boldsymbol{\pi}_1$. We suppose there have been w failed components in the lattice system and they are denoted as $a_{i_\theta j_\theta}$, ($\theta\in\{1,2,\dots,w\}$), where (i_θ,j_θ) represents the coordinate at which the failed component placed.

For an arbitrary sliding window expanding from the component a_{xy}, we can get the number of failed components within this window, $N_{r\times s}^{xy}$, by Eq. (3), where $I\{\cdot\}$ represents the index function, i.e. $I=1$ if the conditions are satisfied and $I=0$ otherwise. The reason that we use the index function is that we need to see whether the failed component $a_{i_\theta j_\theta}$ is inside the sliding window.

$$N_{r\times s}^{xy}=\sum_{\theta=1}^{w}I\left\{\begin{matrix}[x\le i_\theta\le(x+r-1)]\\ \&[y\le j_\theta\le(y+s-1)]\end{matrix}\right\}. \quad (3)$$

Then, we can update the initial state probability vector to $\boldsymbol{\pi}_1$ according to following rules, where the symbol of $\mathbf{0}$ represents a zero vector with corresponding length.

- If $N_{r\times s}^{xy}<k$, then $\boldsymbol{\pi}_1=(\mathbf{0},\overset{(N_{r\times s}^{xy}+1)^{\text{th}}}{1},\mathbf{0})_{1\times(k+1)}$;
- If $N_{r\times s}^{xy}\ge k$, then $\boldsymbol{\pi}_1=(\mathbf{0},\overset{(k+1)^{\text{th}}}{1})_{1\times(k+1)}$.

3. Importance-based Selective Maintenance Policy

When spare components are limited, it is infeasible to replace all components perfectly. Therefore, it becomes necessary to identify the weakest part of the lattice system. Given the failure criteria of the lattice system, even the failure of the most unreliable single component will not necessarily lead to system malfunction. Typically, to determine the weakest part of a system, importance analysis can be applied to assess the significance of each component to the overall system reliability. However, for the lattice system, we are more interested in identifying the weakest area that could potentially cause system failure, rather than just the weakest individual component. Thus, the principle to do the selective maintenance is shown in Fig. 3. First, we need to identify the weakest $r\times s$ area. Second, we determine the importance of components in the weakest area. Finally, we can selectively conduct the maintenance according to their importance.

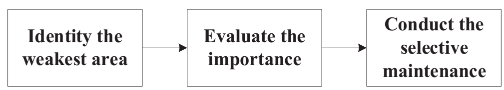

Fig. 3. The principle for selective maintenance.

We define a term, "causing probability", to measure how likely the system is failed because of a specific sliding window. As $R_{k/r\times s}^{xy}$ stands for the probability that the number of failed components within the sliding window $r\times s$, expanded from component a_{xy}, is less than k, then the "causing probability" can be obtained by Eq. (4).

$$P^{xy}=1-R_{k/r\times s}^{xy}. \quad (4)$$

The importance of a_{ij} in the area of $r\times s$, expanding from a_{xy}, is calculated by equation (5), where the $R_{k/r\times s}^{xy}(p_{ij}=1)$ represents the reliability that component a_{ij} is replaced as new. Finally, we can select the component to be replaced descending by their importance value.

$$I_{ij}^{xy}=R_{k/r\times s}^{xy}(p_{ij}=1)-R_{k/r\times s}^{xy}. \quad (5)$$

4. Numerical Examples

4.1. *Reliability evaluation for lattice system with homogeneous components*

In this example, we present how the system reliability evaluation result will be affected if changing the parameters. We assume the components in the lattice system are the same with working probability p. We fix the parameters $s = 3$ and $n = 8$ so that we can compare the system reliability by change one parameter of $\{k, r, m\}$ at a time.

Fig. 4 shows the system reliability lower bound evaluation result, given $k = 6$ and $r = 4$, with different value of $m = \{10,20,30\}$. The comparison reveals that the larger m is, the less the system is reliable. It is because as m grows, the number of the possible sliding window $r \times s$ increases, which leads to more multiple terms in the system reliability calculation equation.

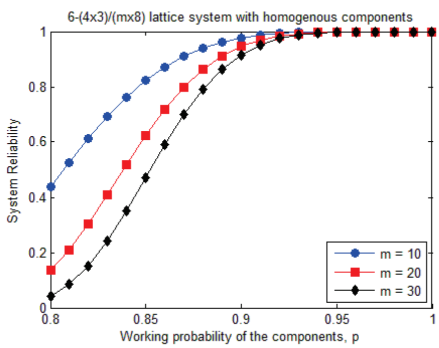

Fig. 4. Reliability comparison for $6 - 4 \times 3/m \times 8$.

4.2. *Reliability evaluation for lattice system with non-homogeneous components*

In this example we fix the parameters value and assume that the lifetime of component a_{ij} in the $6 - 4 \times 3/10 \times 8$ lattice system is subjected to an exponential distribution with different failure rate φ_{ij}, seen in Table I. Thus, we have the working probability of component a_{ij} at time t equals to $p_{ij} = e^{-\varphi_{ij} \cdot t}$. In case some components are out of work, we set their failure rate $\varphi_{ij} = \infty$, e.g. $a_{i_\theta j_\theta} \in \{a_{12}, a_{36}, a_{43}, a_{64}, a_{71}, a_{81}, a_{86}, a_{98}\}$, in Table 1, which leads the working probability to be $p_{i_\theta j_\theta} = 0$.

Table 1. The failure rate φ_{ij} of component a_{ij}.

φ_{ij}	Column $j \in \{1,2,\ldots,n\}$							
$\times 10^{-4}$	1	2	3	4	5	6	7	8
Row $i \in \{1,2,\ldots,m\}$ 1	1	7/∞	30	9	10	10	40	90
2	50	7	10	30	70	70	10	80
3	40	6	4	30	9	6/∞	80	7
4	20	20	10/∞	10	50	50	40	10
5	20	60	90	30	10	7	6	10
6	90	30	20	10/∞	8	70	70	10
7	7/∞	20	10	50	40	80	10	80
8	50/∞	20	30	70	30	30/∞	20	90
9	9	9	50	60	70	70	50	70/∞
10	10	10	10	1	1	60	50	20

Then we can employ Eq. (2) to compute the reliability of $6 - 4 \times 3/10 \times 8$ lattice system reliability given component $a_{i_\theta j_\theta}$ failed. The comparison of the reliability index over operation time t, is shown in Fig. 5, which indicates that the system reliability decrease dramatically than that of a system with all components are working.

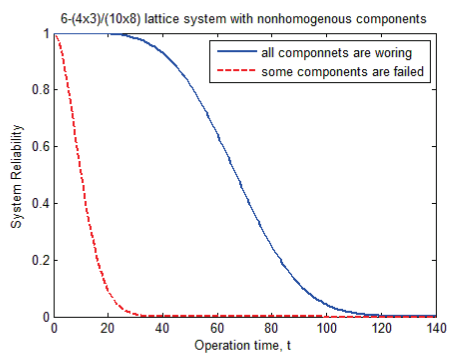

Fig. 5. Reliability comparison for $6 - 4 \times 3/10 \times 8$ with all working components and some failed components.

4.3. *Selective maintenance for lattice system*

In this example, we first employ Eq. (4) to calculate the causing probability which can be used to determine the weakest area. The same parameters in Table 1 are used for this example.

Fig. 6 shows the system reliability and the causing probability of all the sliding windows according to operation time t. It is shown that the area expanding from a_{76} at time $t = 140$ is weakest area which may cause the failure of the system.

Thus, if the spares are limited, we further use Eq. (5) to calculate the importance of each component in the sliding window expanding from a_{76} to identify the components with higher importance to $R^{76}_{6/4\times3}$. Eventually, we can arrange the maintenance action to the selected components.

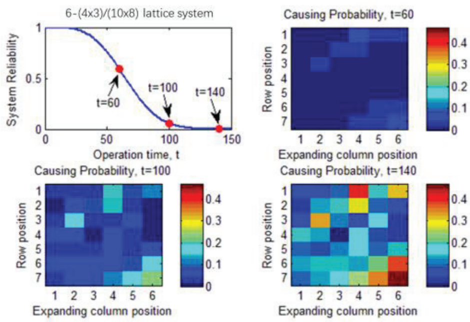

Fig. 6. The weakest area of lattice system at different time t.

5. Conclusion

The lattice system model finds practical applications in real-world scenarios, such as phased array radar systems. We utilize the *k*-within-*r*×*s*-out-of-*m*×*n* model to characterize the lattice system, wherein components are arranged within a two-dimensional area. The system is deemed unreliable if any window of size *r*×*s* contains more than *k* failed components. Additionally, we propose a principle for conducting selective maintenance. A method for identifying the weakest area that could lead to system failure is presented. Several numerical examples are provided to illustrate the application of the proposed method.

Acknowledgement

This research is supported by Aeronautical Science Foundation of China (No. 20240046041001).

References

Salvia AA, Lasher WC. 2-dimensional consecutive-k-out-of-n:F models. *IEEE Trans Reliab* 1990, 39: 382–5.

Yamamoto H, Akiba T. Survey of reliability studies of multi-dimensional consecutive-k-out-of-n:F system. *Reliab Eng Assoc Jpn* 2003, 25(8): 783–96.

Godbole AP, Potter LK, Sklar JK. Improved upper bounds for the reliability of Ddimensional consecutive-k-out-of-n:F systems. *Nav Res Logist* 1998, 45: 219–30.

Makri FS, Psillakis ZM. Bounds for reliability of k-within two-dimensional consecutive-r-out-of-n:F system failure systems. *Microelectron Reliab* 1996, 36: 341–5.

Akiba T, Yamamoto H. Reliability of a 2-dimensional k-within-consecutive-r×sout-of-m×n:F system. *Naval Reserch Logist* 2001, 48: 625–37.

Lin C, Zeng Z, Zhou Y, Xu M, Ren Z. A lower bound of reliability calculating method for lattice system with nonhomogeneous components. *Reliability Engineering and System Safety* 2019, 188: 36-46.