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Formalizing Testing of Collision Avoidance Systems using Signal Temporal Logic

Tom Arne Pedersen[1*], Chanjei Vasanthan[1], Minos Hemrich[2], Kristian B. Karolius[1], Stephanie Kemna[1]

[1] *Group Research and Development, DNV, Norway.*
[2] *Maritime Ship Classification, DNV, Norway.*
[*]*E-mail: tom.arne.pedersen@dnv.com*

The development of Maritime Autonomous Surface Ships (MASS) necessitates the integration of intelligent automated systems, which requires comprehensive testing methodologies to ensure their capability and reliability. Simulation-based testing is an important tool for collecting evidence for validating the safety and reliability of Collision Avoidance (COLAV) systems, ensuring safe navigation.

The International Regulations for Preventing Collisions at Sea (COLREG) provide a set of rules applicable to all ships to prevent collision at sea, including remotely operated or autonomous ships. A significant challenge lies in transforming these formal, highly human-centric, requirements into machine-readable acceptance criteria for efficient testing. This study explores formulating the COLREG rules using Signal Temporal Logic (STL) to create an evaluation framework tailored for simulation-based testing.

The paper presents STL formulations of COLREG rules 2, 6, 8, 13, 14, 15, 16 and 17, with a particular emphasis on rule 17, which outlines the actions for the stand-on vessel. Rule 17 mandates that the stand-on vessel maintains its course and speed unless a collision becomes imminent, necessitating evasive action. The proposed approach offers a structured method for the automatic verification of COLAV systems against established international standards. In a simulation of one situation, it is demonstrated how STL can be utilized to verify the compliance and effectiveness of collision avoidance algorithms in safety-critical situations.

Keywords: COLREG, collision avoidance, autonomy, evaluation, signal temporal logic, simulation-based testing.

1. Introduction

The maritime industry is advancing towards increased automation and remotely-operated functionalities in ships, driven by an ambition to reduce costs, increase safety, and address the shortage of qualified seafarers (Karolius, et al., 2024). The rules for navigating the open seas are primarily governed by COLREG (IMO, 1972), which clearly define the responsibilities and actions of vessels to avoid collisions. Adhering to these regulations ensures safe and efficient navigation. Automated collision avoidance (COLAV) systems should therefore be evaluated against these requirements. However, the COLREG are written for human interpretation and a direct translation to machine-readable requirements is not feasible, due to several ambiguous terms such as “safe speed”, “ample time”, “readily apparent”, and “safe distance”, which lack quantitative measures and are open to interpretation by human navigators.

In this paper we propose the use of Signal Temporal Logic (STL) for converting the COLREG rules into formal, quantifiable requirements, and we provide STL formulations for COLREG rules 2, 6, 8, 13, 14, 15, 16 and 17. We demonstrate the usefulness of this structured method for evaluating a multi-ship encounter situation.

2. Related work

A significant challenge lies in standardizing and formalizing COLREG requirements into machine-readable formats to assess the system’s performance. Woerner (2016) and Woerner et al. (2019) proposed a set of methods to evaluate COLREG rules 8, 13, 14, 15, 16 and 17 for ship-to-ship encounters in head-on, crossing and overtaking situations. Hagen (2022) extended this research by proposing enhancements and new protocols for evaluation. Both Woerner and Hagen utilized a scoring system ranging from 0 to 1 to assess ship encounters, but they did not define a threshold for what constitutes an acceptable score. They also did not cover COLREG rules 2 and 6, which we address in this paper.

Krasowski & Althoff (2021) introduced the use of temporal logic (TL) to model specific parts of COLREG to check rule compliance. By using TL, they could evaluate whether the ship navigated in compliance with the defined requirements. In addition to the rules already covered by Woerner and Hagen, Krasowski & Althoff also covered COLREG rule 6 regarding safe speed. Krasowski & Althoff (2024) improved their earlier formalization while also including a last-minute maneuver rule. They defined six evaluation rules, R_1 to R_6, where R_1 (emergency) has the highest priority, R_3 to R_5 (crossing, head-on and overtaking) have equal priority, and R_6 (stand-on) has the lowest priority. The argument for R_6 having the lowest priority was "whenever the other vessel changes its course so that the ego vessel becomes the give-way vessel, rules R_3 to R_5 are applied". However, this may constitute a breach of the COLREG, as rule 13(d) (IMO, 1972) states that "any subsequent alteration of the bearing between the two vessels shall not make the overtaking vessel a crossing vessel" … "until she is finally past and clear".

Torben, et al. (2023) explored the use of STL for evaluating COLREG compliance for a subset of the COLREG rules, together with the use of a Gaussian Process (GP) model to enable automatic evaluation and guide the test case selection. This paper builds upon that work, extends the set of COLREG rules addressed, and improves the formulation of COLREG rule 6 over all prior works.

3. Background

3.1. Simulation-based testing

A COLAV system performs safety-critical decision-making without human intervention. To ensure the safety, reliability and effectiveness of the COLAV system, there is a need for simulation-based testing to complement real-world testing (Pedersen, et al., 2020). Real-world testing alone is inadequate due to its high cost, time-consuming nature, and potential risk to the actual vessel or personnel with respect to COLAV testing. Furthermore, the complexity and variability of marine environments present challenges that are difficult to replicate in real-world testing. In addition, simulation-based testing enables scalability and repeatability.

3.2. Signal temporal logic

STL makes it possible to evaluate a real-valued signal against a temporal logic formula using predicates (Maler & Nickovic, 2004), such as

$$\pi ::= f(y) \leq c, \quad (1)$$

where $f(y)$ is a scalar, real-valued function that maps the input signal y (an output from the simulation) to a real-valued scalar, and c is a real-valued scalar. In this work, we will be using the following operators:

- *Conjunction:* $\varphi_1 \wedge \varphi_2$ is true if both φ_1 and φ_2 are true.
- *Disjunction:* $\varphi_1 \vee \varphi_2$ is true if either φ_1 and φ_2 are true.
- *Negation*: $\neg\varphi$ is true if φ is false.
- *Always:* $\square_T \varphi$ is true if φ is true for all times $t \in T$.
- *Eventually:* $\diamond_T \varphi$ is true if φ is true for some time $t \in T$.

STL comes with an important feature introduced by Fainekos & Pappas (2009), known as robustness. Instead of providing a binary true or false outcome for a given requirement, STL robustness assesses how robustly a signal meets an STL formulation, resulting in a scalar value measuring how far the outcome is from breaking the requirement. A normalized and saturated robustness score is between -1 and 1. If the score is above 0, it means that the requirement holds, and how far the score is from 0 indicates how strong the result is. If it is close to 0, the requirement is soon to be broken. If the score is below 0, the requirement is not satisfied.

This feature is crucial for smart testing, where the results from previously conducted tests guide the selection of new test cases based on robustness (Torben, et al., 2023), and for evaluating safety-critical systems.

4. Using Signal Temporal Logic to express COLREG

4.1. COLREG rules

The COLREG consist of five parts in addition to four annexes. While not all rules can be tested using simulations, some can be evaluated as part of the verification and validating of a ship's complete navigation system. In this paper, STL requirements are defined for the following COLREG rules:

- Rule 2 – *Responsibility*
- Rule 6 – *Safe speed*
- Rule 8 – *Action to avoid collision*
- Rule 13 – *Overtaking*
- Rule 14 – *Head-on situation*
- Rule 15 – *Crossing situation*
- Rule 16 – *Action by give-way vessel*
- Rule 17 – *Action by stand-on vessel*

4.2. Formulating COLREG rules using STL

In this section, terms and concepts are briefly introduced, followed by the specification of the STL formulations.

4.2.1. Type of encounter

To evaluate the effectiveness of the COLAV system, the relevant COLREG rules need to be selected. Woerner (2016) proposed entry criteria for the COLREG rules 13 to 17, depending on a combination of relative geometry, relative speed and an assessed risk of collision. For the algorithm used to classify the type of encounters, reference is made to (Woerner, 2016).

4.2.2. Ship region

The concept of using polygons to define a blocking area or *ship region*, dependent on the ship's maneuverability, was introduced by Bakdi, et al (2019) and has inspired the robustness metrics developed in this work. The concept is particularly relevant when considering the COLREG rule 17, which requires the stand-on vessel to maintain course and speed but also to make an evasive maneuver if the give-way vessel fails to act properly. Fig. 1 illustrates a situation where the own ship (OS) (black) must stand on to a crossing target ship (TS) (grey). The black and grey solid line polygons display the ship regions of the OS and TS, respectively, as suggested by Bakdi, et al (2019). Assuming a smaller *safety ship region* (dashed line), one could potentially use this to evaluate the OS behavior. The OS should maintain her course and speed while the distance between the black and grey ship solid line regions is greater than 0. If the distance becomes 0, the OS should make an evasive maneuver. If the TS ship region (solid grey) should always be outside the OS safety region (dashed black), Fig. 1 shows that the distance between the black solid and dashed regions of the OS is small, especially on port and aft side, limiting the time for the OS to react.

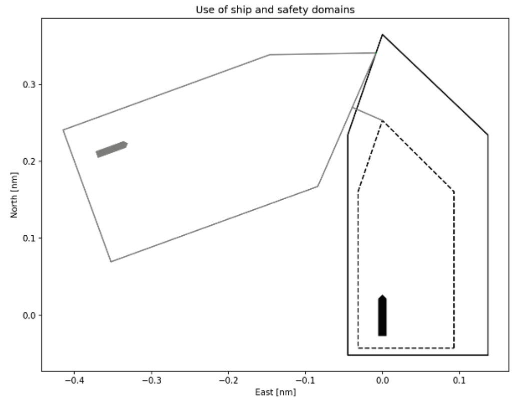

Fig. 1: Crossing situation. Ship regions are shown as black (OS) and grey (TS) polygons, and the OS safety region is the black dashed polygon. The grey line is shortest distance between OS safety region and TS ship region.

It is desirable to account for the relative speed between the OS and the TS when evaluating rule 17 a) ii) and 17b). Assuming both the OS and the TS are navigating at constant speed (14 and 16 knot, respectively) and courses, and setting the Closest Point of Approach (CPA) to 0, it is possible to draw circles around the OS, indicating where the TS would be located at the Time to Closest Point of Approach, TCPA = 7, 5 and 2 minutes. In Fig 2, these circles are drawn with a thin black edge color. These circles may now be used to determine if the OS, in a stand-on situation, should be allowed to perform an evasive maneuver. Rule 17 a) ii) states: "The latter vessel **may** take action to avoid collision…", while rule 17b) says "…she **shall** take such action.". As long as the TS is outside the dark grey circle, the OS as a stand-on vessel should maintain its course and speed. If the TS enters the dark grey circle, the OS is allowed to make a maneuver, but if the TS enters the grey circle, the OS is required to make an evasive maneuver as per rule 17b). The TS should, however, never enter the light grey circle. The limits used in Fig 2 (2, 5, 7 minutes) are just examples and can be set depending on the type of vessel, maneuverability, location, etc.

As observed in Fig 2, if the TS is approaching from the aft of the OS, the distances from the different circles (with dark edge color) to the OS are still small when the relative speed between the ships is low. This may be counteracted by adding additional circles around the OS. A circle with radius 2.0 times TS ship

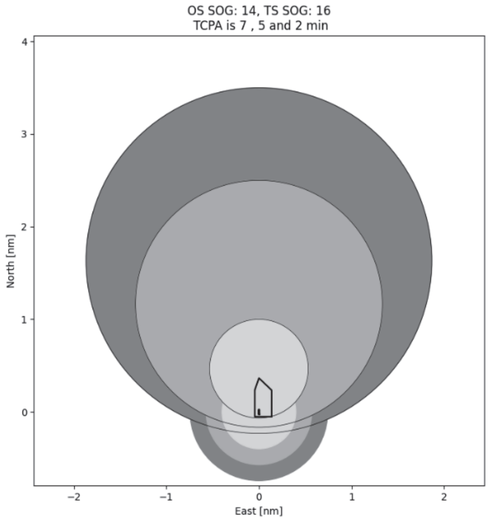

Fig 2: TCPA circles around OS with black edge color (dark grey: 7 min, grey: 5 min, light grey: 2 min). Additional circles around OS with radius 2, 1.5 and 1 x TS ship region length. Black solid line polygon indicates ship region, and black dot is position of OS.

region length is added for the dark grey circle, a circle with radius 1.5 times the TS ship region length is added for the grey circle, and lastly, a circle with radius 1.0 times the TS ship region length is added for the light grey circle, as shown in Fig 2.

4.2.3. Rule 2 - Responsibility

In navigating a ship, situations often arise where the OS encounters multiple TS, necessitating both stand-on and give-way responsibility simultaneously. Rule 2 allows deviations from the rules to avoid immediate danger. By utilizing the ship regions introduced in section 4.2.2 and ensuring that the ship regions for OS and TS never intersect, the robustness of Rule 2 can be evaluated by:

$$\varphi_2 \coloneqq \square \neg \left(\frac{d_{SR}}{d_{norm}} \leq 0 \right), \quad (2)$$

where d_{SR} is the shortest distance between the OS and TS ship regions and d_{norm} is a normalized distance parameter.

It should be noted that all robustness values in this paper are saturated and normalized to facilitate the assessment of how robustly the signal meets the STL formulation. The saturation range from -1 to 1 is however not shown in the formulas.

4.2.4. Rule 6 – Safe speed

Krasowski & Althoff (2021) proposed that safe speed should range between 0 and the maximum speed of the ship in open sea conditions. In this study, it is assumed that safe speed is bounded by a minimum maneuver speed and maximum speed, governed by the visibility (i.e. weather conditions), traffic density and distance to shore.

The following assumptions are made:

- $f_{visibility}$ ranges from 0 (no visibility) to 1 (full visibility).
- $f_{traffic\_density}$ ranges from 0 (complex traffic situations with multiple TSs) to 1 (low traffic or organized traffic in a traffic separation scheme (TSS)), and
- $f_{distance\_shore}$ ranges from 0 (short distance to shore) to 1 (long distance to shore).

In areas with limited maneuvering space, such as TSS, the safe speed should not necessarily be reduced if the distance to shore is low. The variation of $f_{visibility}$, $f_{traffic\_density}$ and $f_{distance\_shore}$ between 0 and 1, can be linear, quadratic or exponential, depending on the location, type of ship etc.

The STL robustness for rule 6 is then:

$$\varphi_6 \coloneqq \square \neg \left(\begin{array}{c} \frac{V - V_{safe\_min}}{V_{max}} \leq 0 \wedge \\ \frac{V_{safe\_max} - V}{V_{max}} \leq 0 \end{array} \right) \quad (3)$$

where V_{max} is the maximum speed of the ship, V_{safe_min} is the minimum maneuver speed and V_{safe_max} is given by

$$V_{safe\_max} = V_{max} \times \left(1 - \left(\begin{array}{c} c_{visibility}(1 - f_{visibility}) + \\ c_{traffic\_density}(1 - f_{traffic\_density}) + \\ c_{distance\_shore}(1 - f_{distance\_shore}) \end{array} \right) \right) \quad (4)$$

where $c_{visibility}$, $c_{traffic\_density}$ and $c_{distance\_shore}$ are coefficients chosen based on the contribution of each factor to the safe speed reduction. These factors should not be chosen such that V_{safe_max} becomes lower than V_{safe_min}.

4.2.5. Rule 8 – Action to avoid collision

Rule 8 of the COLREG outline the requirements for performing a maneuver to avoid collision. This section discusses specific aspects of Rule 8 proposed for evaluation.

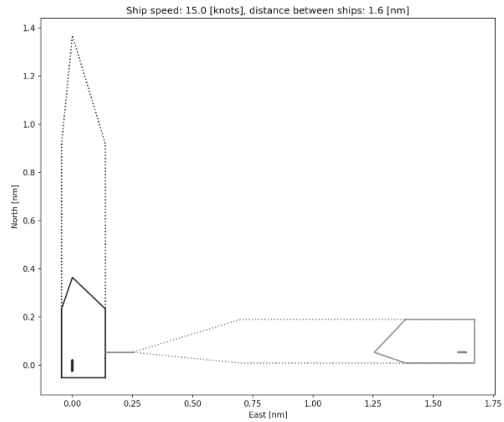

Fig 3: Crossing situation. OS (black at 0,0) with black solid polygon indicating ship region and dotted polygon indicating minimum distance region. TS (grey at 1.6,0.05) with grey solid ship region and grey dotted minimum distance region.

Rule 8a) requires that any action to avoid collision shall be taken "in ample time" and "with due regard". Inspired by the ship region used for Rule 2, and extending the length of the ship region by with a factor (e.g. 4), a *minimum distance region* (MDR) can be used to evaluate rule 8a), as illustrated in Fig 3. As the length of the ship regions is dependent on ship speed, this also applies for the MDR. The robustness of rule 8a) can be defined as:

$$\varphi_{8a} \coloneqq \Box \neg \left(\frac{d_{MDR}}{d_{norm}} \leq 0 \right) \tag{5}$$

where d_{MDR} is the minimum distance between the two MDRs, shown as a grey solid line in Fig 3 and d_{norm} is the normalized distance parameter.

Rule 8b) requires that any alteration in speed and/or course shall be "readily apparent" to the other ship. The rule also emphasizes avoiding several small alterations. By defining an absolute lower limit for the OS minimum course change ($\chi_{app_{min}}$) and OS relative speed change ($U^{rel}_{app_{min}}$), this can be used to calculate the robustness score. In narrow channels or other areas with restricted maneuverability, $\chi_{app_{min}}$ should be decreased. The robustness for rule 8b) is then:

$$\varphi_{8b} \coloneqq \varphi_{8b_\chi} \vee \varphi_{8b_U} \tag{6}$$

where:

$$\varphi_{8b_\chi} \coloneqq \Diamond_{\left[t_{\chi_{start}},\ t_{\chi_{end}}\right]} \neg \frac{|\Delta\chi| - \chi_{app_{min}}}{\chi_{norm}} \leq 0 \quad ,$$

$$\varphi_{8b_U} \coloneqq \Diamond_{\left[t_{U_{start}},\ t_{U_{end}}\right]} \neg \left(\left|\Delta U^{rel}\right| - U^{rel}_{app_{min}} \leq 0 \right).$$

$\Delta\chi$ and ΔU^{rel} represents the total course change and relative speed change for the OS maneuver. If the maneuver involves multiple smaller changes, $\Delta\chi$ and ΔU^{rel} represent the first change only. The course and speed changes are calculated independently by determining (via the derivative of speed and course change) the start and end times of each change ($t_{\chi_{start}},\ t_{\chi_{end}}$ for course and $t_{U_{start}},\ t_{U_{end}}$ for speed).

Rule 8d) requires that any actions taken to avoid collision result in passing at a safe distance, similar to rule 2:

$$\varphi_{8d} \coloneqq \Box \neg \left(\frac{d_{SR}}{d_{norm}} \leq 0 \right) \tag{7}$$

The total STL robustness for rule 8 is the conjunction of all robustness scores:

$$\varphi_8 \coloneqq \varphi_{8a} \wedge \varphi_{8b} \wedge \varphi_{8d} \tag{8}$$

4.2.6. Rule 13 - Overtaking

Rule 13 requires that any vessel overtaking another shall keep out of the way of the vessel being overtaken. This can be evaluated using the MDR from rule 8a):

$$\varphi_{13} \coloneqq \Box(d_{MDR} > 0) \tag{9}$$

4.2.7. Rule 14 – Head-on situation

The COLREG Rule 14 states that ships in a head-on encounter shall (i) alter course to starboard so that (ii) each passes on the port side of the other. Inspired by Woerner (2016), it is suggested to use the relative bearing β and contact angle α at CPA for evaluating the first part of rule 14. β is the relative bearing between the OS and the TS as seen from the OS, while α is the relative bearing between the TS and the OS as seen from the TS. The STL robustness is then:

$$\varphi_{14_i} \coloneqq \varphi_{14_{i_\beta}} \wedge \varphi_{14_{i_\alpha}} \tag{10}$$

where:

$$\varphi_{14_{i_\beta}} \coloneqq \Box_{CPA} \neg \left(\frac{c_{lim\_p2p} - |\beta - 270°|}{c_{lim\_p2p}} \leq 0 \right)$$

$$\varphi_{14_{i_\alpha}} \coloneqq \Box_{CPA} \neg \left(\frac{c_{lim\_p2p} - |\alpha + 90°|}{c_{lim\_p2p}} \leq 0 \right)$$

c_{lim_p2p} is the maximum deviation in degrees from a clear port-to-port passing (each passing strictly on the port side of each other).

The second part of rule 14 requires a starboard turn, meaning that maintaining course or turning to portside breaks this part of the rule. The STL robustness is then:

$$\varphi_{14_{ii}} \coloneqq \Diamond_{[t_0,\, t_{cpa}]} \neg \left(\frac{\Delta\chi - \chi_{app_{min}}}{\chi_{norm}} \leq 0 \right) \tag{11}$$

where t_0 is the time at the start of the situation, t_{cpa} is the time at CPA, $\Delta\chi$ is the total course change, and $\chi_{app_{min}}$ is the minimum allowed course change.

The total robustness score for Rule 14 is then:

$$\varphi_{14} \coloneqq \varphi_{14_i} \wedge \varphi_{14_{ii}} \quad (12)$$

4.2.8. Rule 15 – Crossing situation

Rule 15 defines the responsibilities of vessels in a crossing situation, specifying which vessel shall give-way and which shall stand-on. It also advises against crossing ahead of another vessel unless the distance between them is sufficient, in which case the MDR of rule 8 may be applied for evaluation:

$$\varphi_{15} \coloneqq \square \neg \left(\frac{d_{MDR}}{d_{norm}} \leq 0 \right) \quad (13)$$

4.2.9. Rule 16 – Action by give-way vessel

Rule 16 outlines the actions required of the give-way vessel, emphasizing three key requirements: early action, substantial action and maintaining a safe distance. The STL robustness formulation for rule 16 is similar to that of rule 8.

4.2.10. Rule 17 – Action by stand-on vessel

Rule 17 specifies the actions required of the stand-on vessel. The first part of 17 a) requires that vessel maintains its *course and speed,* while the second part of 17 a) and 17 b) states that the stand-on vessel may or shall take action to best avoid collision. Using the regions proposed in section 4.2.2 (see Fig 2), the dark grey region is designated as the Maneuvering Stand-on Region (MSR) and the light grey region is designated as the Prohibited Stand-On Region (PSR). The OS, as stand-on vessel, shall maintain its course and speed as long as the TS is outside of the MSR. If the TS enters the MSR, the OS is allowed to change speed or course, but the TS should never enter the PSR.

Assuming "maintain speed and course" implies a constant course and speed, and setting an upper limit on permissible changes, the STL robustness formulation for rule 17a/b is:

$$\varphi_{17_{a/b}} \coloneqq \square_{[t_0,\ t_{cpa}]} \Big(\big(d_{MSR} > 0 \rightarrow \big(|\Delta\chi| - \Delta\chi_{allowed} \leq 0 \wedge \big(U^{rel}_{allowed} - |\Delta U^{rel}| \big) \geq 0 \big) \big) \vee \big(d_{MSR} \leq 0 \rightarrow \neg d_{PSR} \leq 0 \big) \Big) \quad (14)$$

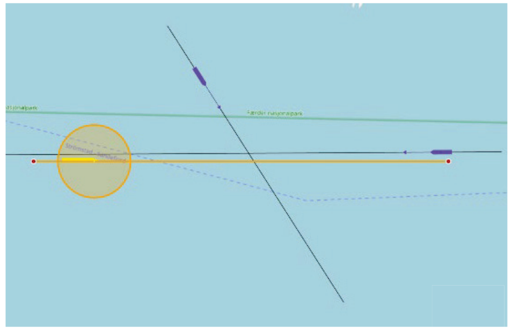

Fig 4: Multi-ship situation where OS (length: 150m) is head-on (W→E) towards TS1 (length: 100m) (E→W) and crossing stand-on towards TS2 (length: 100m) (NW→SE).

t_0 is the time at the start of the situation, t_{cpa} is the time at the CPA, $\Delta\chi$ and ΔU^{rel} are the total course change and relative speed change, while $\Delta\chi_{allowed}$ and $\Delta U^{rel}_{allowed}$ are the permissible changes for a stand-on vessel. d_{MSD} is the shortest distance between the TS and MSR, while d_{PSR} is the shortest distance between the TS and the PSR.

Rule 17c) requires that any course change by the stand-on vessel to avoid a potential collision should not alter the course to port for a TS on its own port side:

$$\varphi_{17_c} \coloneqq \diamond_{[t_{MSR\_enters}, t_{CPA}]} \left(\Delta\chi_{sb} \geq \chi_{app_{min}} \right) \quad (15)$$

for $\alpha_{t_{MSR\_enters}} > 0°$ and $\beta_{t_{MSR\_enter}} > 180°$. t_{MSR_enters} is the timestamp when the TS enters the MSR, and $\beta_{t_{MSR\_enter}}$ and $\alpha_{t_{MSR\_enters}}$ are the relative bearing and contact angle at that time. $\Delta\chi_{sb}$ is the total course change to starboard, and $\chi_{app_{min}}$ is the minimum apparent course change.

5. Testing the method using simulation

5.1. Robustness evaluation

The robustness-based evaluation algorithm is demonstrated in an open sea scenario involving three ships; OS, TS1 and TS2, as illustrated in Fig 4. OS is the yellow ship navigating from west to east. TS1 approaches head-on from east to west, while TS2 crosses from northwest to southeast. The ships are not shown to scale; their sizes have been enlarged for better visualization. In this scenario, the OS is both give-way and stand-on simultaneously. OS should turn starboard to avoid the head-on collision with TS1 while maintaining course and speed for TS2. Both TSs navigate in a straight line without performing evasive

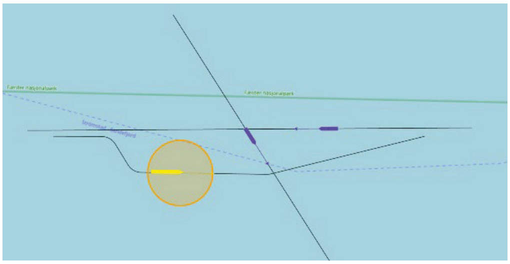

Fig. 6: OS (W→E) making a starboard turn to pass the TGT1 (E→W) on its port side.

maneuvers, leaving the OS to handle both situations.

Several parameters must be set to evaluate the COLAV system. The parameters for evaluating the head-on and the crossing stand-on encounters are:

- α_{crit}^{13}: 67.5 [deg]
- α_{crit}^{14}: 5 [deg]
- α_{crit}^{15}: 5 [deg]
- α^{15}, $[112.5, 247.5]$[deg]
- $[V_{safe\_min}, V_{safe\_max}]$: $[2, 20.0]$ [knots]
- $U_{app_{min}}^{rel}$: 0.25 [-]
- $\chi_{app_{min}}$: 10 [deg]
- $U_{allowed}^{rel}$: 0.1 [-]
- $\Delta\chi_{allowed}$: 5 [deg]
- MDD length factor: 3.5 [-]
- Evasive maneuver: MSR: 7 [min], PSR: 2 [min]

These parameter values are suggestions and may vary depending on ship type, maneuverability, and location etc. Fig. 5 and Fig. 6 illustrate different navigation strategies for OS to manage the situation, encountering TS1 head-on and crossing towards TS2. In Fig. 5 OS makes a starboard turn to pass on the port side of TS1, increasing the CPA towards TS1 and passing aft of TS2 with a low CPA. In Fig. 6 OS first makes a 360° turn before a starboard turn to handle both situations, increasing the CPA between OS and TS2 but decreasing the distance between OS and TS1 before the starboard maneuver.

The STL robustness results for the two scenarios are shown in Tab 1. The robustness score indicates how close a signal is to violating a requirement. A score above 0 means the requirement holds, while a score below 0 indicates a violation. The value ranges between -1 and 1, as it is both normalized and saturated.

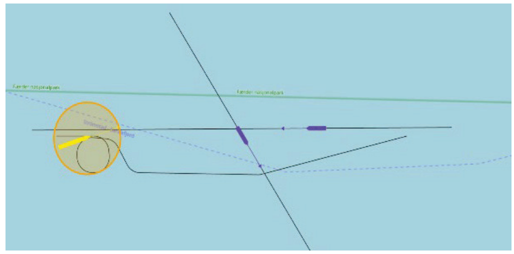

Fig. 5: OS (W→E) first making a circle maneuver, then a starboard turn to pass TGT1 (E→W) on its port side and aft the TGT2 (NW→SE).

Tab 1: Robustness evaluation

	Scenario 1		Scenario 2	
	TS1 (HO)	TS2 (CR-SO)	TS1 (HO)	TS2 (CR-SO)
Rule 2	1.00	-0.02	1.00	1.00
Rule 6	0.50	0.5	0.50	0.50
Rule 8	1.00	-0.13	0.40	0.01
Rule 14	0.86	-	0.80	-
Rule 15	-	-0.13	-	0.01
Rule 17	-	-1.0	-	1.00

5.2. Results

In scenario 1, the robustness score towards TS2 in Tab 1 shows that OS violates most rules concerning TS2 due to proximity. The score for Rule 17 is significantly worse than for Rule 2, as the evaluation region for Rule 2 is smaller than the PSR for Rule 17. Rule 17 requires OS to maintain speed and course, but OS must maneuver due to proximity to TS2.

In scenario 2, all scores are above 0, indicating satisfactory compliance. Rule 8 and 15 (which is similar to 8) are just above 0, close to being violated. Some robustness scores towards TS1 are lower in scenario 2 compared to scenario 1.

5.3. Discussions & Future Work

This paper proposes using STL robustness to formalize the COLREG rules 2, 6, 8, and 13 – 17. The results in Sections 5.1 - 5.2 demonstrate the approach's effectiveness in evaluating multi-ship situations where the OS is in both give-way and stand-on positions.

However, the results depend on the size and shape of regions around the OS and TS vessels. Further work is needed to find suitable parameters for different types of ship encounters. As already described, the COLREG are written by humans for humans, and a direct translation to machine-readable code is not possible. To understand how the COLREG are applied in different situations and also to adjust the parameters used for robustness evaluation, analyzing historical Automatic Identification Systems (AIS) data, interviewing experienced navigators, and studying known incidents and accidents, will be done as part of further work.

Additionally, the impact of the grounding risk on region sizes should be explored.

6. Conclusion

To ensure the safety and reliability of automatic collision avoidance systems operating without human intervention, rigorous testing is essential. This paper demonstrates how signal temporal logic robustness can formalize COLREG requirements into evaluation criteria for multi-ship encounters, with a focus on Rule 17, which was under-addressed in former works.

One significant advantage of STL robustness is that it provides a score indicating how close requirements are to be violated, in addition to a pass/fail result. This feature is crucial for implementing smart testing, where previous tests outcomes guide the selection of new tests to be conducted.

Acknowledgement

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