

# Phase-Extraction Formula for Surface Profiling of Optical Flat using Wavelength-Tuning Interferometer

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Transparent parallel optical flats have been extensively used in production process of semiconductor industries. For improving the performance of semiconductor chips, the surface shape of the transparent parallel plate should be measured with nanoscale uncertainty. Atomic force microscopes (AFMs) have been widely used in the semiconductor industry to measure the surface profile of the optical devices precisely. However, the measurements using AFMs require a considerable amount of time to measure the entire surface profile distribution. To measure the surface shape of the transparent parallel plate, the phase-shifting interferometry using wavelength tuning has been widely used owing to its high resolution and noncontact measurement. In the phase-shifting technique, the phase difference between a target wave front and a reference wave front is changed linearly with wavelength tuning. The target phase can be calculated from the phase-extraction formula, the arctangent of the ratio between two combinations of the observed intensities. However, the measurement accuracy is degraded by not only phase-shift errors, but also by coupling errors between higher harmonic signals and phase-shift errors. The coupling errors can be the significant systematic errors when measuring the highly-reflective surface and optical thickness variation of the transparent plate. The phase-shift errors can occur because of the environmental uncertainties can be eliminated by using the special phase-extraction formula.

Systematic approaches for developing the phase-extraction formula have been proposed by several researchers based on the averaging method of successive samples, Fourier description of the sampling functions, and an analytical expansion using the linear equation group. In this study, the new 15-sample phase-extraction formula was developed that can suppress the coupling errors caused by harmonic signals up to the fourth-order and linear and nonlinear phase-shift errors. The 15-sample formula was derived using the linear equations consisting of the sampling amplitude of the phase-extraction formula. The characteristics of the proposed phase-extraction formula were visualized on the frequency domain and complex plane using the Fourier description and characteristic polynomial theory, respectively. In addition, the error-suppression capability of the proposed phase-extraction formula was confirmed by using the theory of the root mean square (RMS) error. The frequency response of the 15-sample phase-extraction formula was evaluated by using the concept of the signal-to-noise ratio (SNR) calculated using the frequency-transfer-function paradigm. Also, the sensitivity to the floor vibration of the newly developed 15-sample phase-extraction formula was evaluated quantitatively with comparing those of the other conventional phase-extraction formula. Finally, the surface profile of the transparent optical flat was measured by using the wavelength-tuning Fizeau interferometer and the newly developed 15-sample phase-extraction formula. The standard deviations and the systematic errors of the ripple distribution were discussed with comparing the other phase-extraction formula.

## 1. Introduction

In the production process of semiconductor industries, transparent parallel flats usage is increasing. For the high performance of the semiconductor, the surface shape of the transparent flats should be measured under nanoscale uncertainty. Wavelength-turning Fizeau interferometry has been widely used for the measurement of optical flat with nanoscale uncertainty. In this interferometry, the phase difference between a reference surface and transparent parallel flat changes linearly during the wavelength turning. However, various phase errors occur in this method. Phase-shift miscalibration and the effect of the second harmonics are the most influential errors when calculating the target phase. Phase-shift nonlinearity and coupling errors between the phase-shift miscalibration and the harmonics also affect the calculation. In this research, the new 15-sample phase-extraction formula was developed with compensation ability about the phase-shift miscalibration, the nonlinear phase-shift errors, and coupling errors caused by harmonic signals up to the fourth-order and linear phase-shift error. And the specificity of the proposed phase-extraction formula was visualized on the frequency domain and the complex plane. Furthermore, the error-suppression capability of the formula was confirmed by using the RMS error.



#### 2. Phase-extraction and correlated error

During wavelength-turning, the beam from the laser source is reflected from the two surfaces, and the reflected beams are combined with each other to generate an interference fringe pattern. (Fig. 1) And the interferogram intensity is expressed as follows:

$$I(x, y, \alpha_r) = s_0(x, y) + \sum_{k=1}^{j} s_k(x, y) \times \cos[k\alpha_r - \phi_k(x, y)]$$
(1)

where *r* is the frame number,  $\alpha_r$  is the phase-shift parameter,  $s_0$  is the DC component,  $s_k$  is the amplitude of the *k*th-order harmonic component, and  $\phi_k$  is the phase of the *k*th-order harmonic component.

Consider an *m*-sample phase-shifting algorithm, the reference phases are separated by *m*-1 equal intervals  $2\pi/n$  rad, where *n* is an integer. The target phase, the phase of the fundamental signal, can be calculated using the following phase-extraction formula:

$$\phi_k(x, y) = \arctan\left[\frac{\sum_{r=1}^m b_r I(x, y, \alpha_r)}{\sum_{r=1}^m a_r I(x, y, \alpha_r)}\right]$$
(2)

where  $a_r$  and  $b_r$  are the sampling amplitudes,  $I(\alpha_r)$  is the interferogram intensity defined by Eq. (1), and m is the number of total images of the phase-extraction formular. When the phase shift is nonlinear, the phase-shift parameter,  $\alpha_r$ , can be expressed as:

$$\alpha_r = \alpha_{0r} \left( 1 + \varepsilon_0 + \varepsilon_1 \frac{\alpha_{0r}}{\pi} + \varepsilon_2 \left( \frac{\alpha_{0r}}{\pi} \right)^2 + \cdots \right)$$
(3)

where  $\varepsilon_0$  is the factor of the phase-shift miscalibration,  $\varepsilon_k$  ( $k \ge 1$ ) is the factor of the *k*th nonlinearity phase error, and  $\alpha_{0r}$  is the ideal phase-shift parameter, defined as:

$$\alpha_{0r} = \frac{2\pi}{N} \left( r - \frac{M+1}{2} \right) \tag{4}$$

The error in the calculated phase can be expanded into a Taylor series as follows:

$$\Delta \phi = \phi - \phi_{1} = o(s_{k}) + o(\varepsilon_{p}) + o(\varepsilon_{p}s_{k})$$
(5)

for k = 2, 3, ..., j and  $q = 1, 2, ..., p. o(s_k), o(\varepsilon_q)$ , and  $o(\varepsilon_q s_k)$  denote, respectively, the errors in the harmonics, phase-shift error, and the coupling error.



Fig. 1 Laser interferometer for surface profiling of the transparent parallel plate

#### 3. 15-sample phase-extraction formula

*M*-sample phase-extraction formula is composed of the intensity of signal and sampling amplitudes. The sampling amplitudes of the formula determined the error-suppression ability of the formula. The  $a_r$  is a symmetric vector, and the  $b_r$  is an asymmetric vector. For derivation of a new 15-sample formula, the sampling function of the sampling amplitudes,  $F_1$  and  $F_2$  are defined as follows:

$$F_1(v) = \sum_{r=1}^m b_r \exp(-iv\alpha_r)$$
(6)

$$F_2(v) = \sum_{r=1}^{m} a_r \exp(-iv\alpha_r)$$
<sup>(7)</sup>

For measurement of the surface shape of transparent parallel plates while suppressing phase error, the sampling functions satisfies the several conditions as follows:

$$iF_1(v_1) = F_2(v_1)$$
 (8)

$$iF_1(mv_1) = F_2(mv_1) = 0 \quad (m = 0, 2, 3, 4)$$
 (9)

$$\frac{d}{dv}\left[iF_1(v_1)\right] = \frac{d}{dv}\left[F_2(v_1)\right] \tag{10}$$

$$\frac{d}{dv} \left[ iF_1(mv_1) \right] = \frac{d}{dv} \left[ F_2(mv_1) \right] \quad (m = 2, 3, 4)$$
(11)

$$\frac{d^2}{dv^2} \Big[ iF_1(mv_1) \Big] = \frac{d^2}{dv^2} \Big[ F_2(mv_1) \Big] \quad (m = 1, 2, 3, 4)$$
(12)

where Eq. (8) is the condition for calculating of the target phase  $\phi_{l}$ , Eq. (9) is the condition for suppressing of the harmonic signals up to the fourth-order, Eq. (10) is the condition for suppressing of the phase-shift miscalibration, Eq. (11) is the condition for suppressing of the coupling errors, and Eq. (12) is the condition for suppressing of the phase-shift nonlinearity.

From Eq. (8)-(12), the sampling amplitudes of the new 15-frame phase-extraction algorithm can be expressed as



$$a_r = \begin{pmatrix} 0.0182, \ 0.0455, \ -0.0091, \ -0.1913, \ -0.3279, \\ -0.1366, \ 0.3188, \ 0.5647, 0.3188, \ -0.1366, \\ -0.3279, \ -0.1913, \ -0.0091, \ 0.0455, \ 0.0182 \end{pmatrix}$$
(13)

 $b_r = \begin{pmatrix} 0, \ 0.0473, \ 0.1420, \ 0.1420, \ -0.0946, \\ -0.4259, \ -0.4259, \ 0, \ 0.4259, \ 0.4259, \\ 0.0946, \ -0.1420, \ -0.1420, \ -0.0473, \ 0 \end{pmatrix}$ (14)

#### 4. Visualization of the 15-sample algorithm

For evaluating the characteristics of the new 15-sample algorithm, the algorithm is visualized in frequency domain using the sampling functions. In Fig. 2, the sampling functions have same magnitude and gradient at the fundamental signal. It can be seen that the new formula can calculate the target phase and suppress the phase-shift error. When the relative frequency is 2, 3, 4, the sampling functions have same magnitude and gradient at the fundamental signal, too. It can be seen that the new algorithm can suppress the harmonics up to the fourth-order, and the coupling errors. However, the error-suppression ability for phase-shift nonlinearity cannot be confirmed in frequency domain.

In order to verify the total error-compensation ability of the formula, the algorithm is visualized in complex plane using characteristic polynomial. In complex plane, the single roots are located on the diagram to suppress the *m*th harmonics, expect at m = 1, and the double root is located on the diagram to suppress the phase-shift error at m = -1. To suppress the coupling errors, the double roots are located on the diagram at m = 0, 2 (= -4), 3 (= -3), 4 (= -2). Finally, the triple roots are located on the diagram to suppress the 1st-order nonlinearity, at m = -1, 2 (= -4), 3 (= -3), 4 (= -2).



Fig. 2 Sampling functions of the proposed 15-sample formula



Fig. 3 Diagram of the 15-sample algorithm on the complex plane.

#### 5. Numerical analysis

The correlated-error suppression ability of the proposed formula was confirmed by using the theory of root-mean-square (RMS) error. The RMS error  $\sigma_1$  resulted from the phase-shift miscalibration is given by

$$\sigma_1 = \frac{1}{2\sqrt{2}} \left| \frac{iF_1(v)}{F_2(v)} - 1 \right|$$
(15)

The RMS error  $\sigma_2$  due to the correlated error between the *m*th harmonics and phase-shift miscalibration  $\varepsilon 0$  is given by

$$\sigma_{2} = \frac{1}{2} \sum_{m=2}^{\infty} \frac{\gamma_{m}}{\gamma_{1}} \sqrt{\left[\frac{iF_{1}(mv)}{iF_{1}(v)}\right]^{2} + \left[\frac{F_{2}(mv)}{F_{2}(v)}\right]^{2}}$$
(16)

where  $\gamma_m$  is the visibility of the *m*th harmonics. Consequently, the total RMS phase error can be calculated as follows:

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} \tag{17}$$

Fig. 4 and Fig. 5 show the solutions to Eq. (17) with comparing other algorithms. From Fig. 4 and 5, the developed 15-sample phase-shifting algorithm (red line) has the best suppression ability among the several phase-shift algorithms. The new 15-sample formula exhibits the smallest phase error among the several phase-shift algorithms as shown in Fig. 4 and 5.





Fig. 4 RMS phase-shift miscalibration of the algorithms.



Fig. 5 RMS phase-shift nonlinearity of the algorithms.

### 6. Conclusion

This research has presented the derivation of the new 15-frame phase-shifting formula that can suppress for the linear miscalibration, 1st-order nonlinearity of the phase shift, and coupling errors between the higher harmonics and phase-shift miscalibration. The properties of the formula were confirmed in frequency domain and complex plane. Lastly, the 15-sample algorithm was shown the smallest phase error among the other phase-shifting algorithms.

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