

State modelling and prognostics of safety valves used in the oil and gas industry

Ewa Laskowska

Department of Mechanical and Industrial Engineering, Norwegian University of Science and Technology (NTNU), Norway. E-mail: ewa.m.laskowska@ntnu.no

Jørn Vatn

Department of Mechanical and Industrial Engineering, Norwegian University of Science and Technology (NTNU), Norway. E-mail: jorn.vatn@ntnu.no

A multiphase Markov degradation model is developed for safety valves. Real empirical data from oil and gas company is used to estimate the required reliability parameters in the model. Two situations are considered. In the first case negligible repair times is assumed. When repair times are negligible, it is rather straightforward to apply a repair matrix when integrating the Markov equation over the point of times of inspection. In the second case, where repair times are considered, virtual states are introduced in parallel to distinguish between degraded states not known to the operator, and states which are known based on inspection information. The objective of the modelling is to obtain average probability of failure on demand in order to verify that the inspection and maintenance strategy is sufficient to fulfil the required safety integrity level. The developed model is supported by a case study, and relevant calculation and sensitivity results are presented.

Keywords: Safety valve, degradation, empirical data, Markov process, multiphase Markov process, PFD_{avg}

1. Introduction

The ESREL 2018 Industrial panel session on future challenges for maintenance modelling and applications was dedicated to highlight the importance of cooperation and communication between academia and industry. On the threshold of the 4th industrial revolution it is especially important to notice benefits coming from the use of digitalization and big data. From the research point of view, the investigation of real problems is more attractive than purely theoretical considerations that may never find their applications. Although the topic of industry 4.0, big data and other related concepts exists already for some time, it is still unclear how to make use of them. By cooperating with universities, the industry can obtain better understanding of which data to collect and how raw data may be transformed into meaningful information.

The cooperation between industrial partners and academia is a core of the BRU 21 project from which this work has emerged. BRU 21 stands for Better Resources Utilization in the 21st century for the oil and gas industry. The objective of the project is to ‘increase efficiency, safety and environmental care in all operations in oil and gas activities’ by developing new digital methods and technologies.

The particular objective of the current work is to develop a degradation model for process safety valves based on the empirical data. Such a model can serve as a tool for maintenance optimization, and thus be used to find a balance

between process safety and cost effective predictive maintenance.

The basis for the analysis is a field data regarding reliability of safety valves, provided by an oil and gas exploration and production company.

A Multiphase Markov process will be used for model development. The Multiphase Markov is a stochastic process allowing to model changes in the system at fixed point of times. This feature enables to model repairable systems. In period between two inspections, the ongoing degradation is modelled by a Time Continuous Markov Chain (TCMC). At the fixed inspection times the system can change its state according to a repair decision transition matrix (Wu et al. 2018). The use of multiphase Markov is well known in modeling the impact of maintenance on the reliability of safety critical systems in process plants. The model was used for example by Lundteigen and Strand (2015) and Wu et al. (2018) to investigate the impact of repair delays on reliability of critical subsea equipment. Innal et al. (2016) applied the Multiphase Markov to consider the impact from repair times and partial testing on the reliability of Safety Instrumented Systems (SIS). Langeron et al. (2008) used the Multiphase Markovian approach to model stochastic behavior of SIS when the staggered testing policy was applied. The same method was employed by Srivastav et al. (2018) and Wu et al. (2018) to reflect the deteriorating impact of functional testing on safety critical systems. However, among the mentioned works only Lundteigen and Strand

(2015) used the real life data. An objective of the current work is therefore to develop a model that can utilize a real life data. In the study, the empirical data for 92 safety valves is used as a basis for the estimation of reliability parameter.

2. Safety valves

As commonly known, valves are mechanical devices used to control or regulate direction and flow of a medium (specific gas or fluid). The control function is performed by opening or closing valve to a desired position, which can be fully closed/open or partially open.

The object of the analysis are Emergency Shutdown Valves (ESV), which are safety valves. They are part of safety-instrumented systems that purpose is to execute safety function, i.e. maintain - or bring the process back to - a safe state (Lundteigen, Rausand, 2008). ESV are activated 'on demand', to stop the medium flow upon detection of a hazardous event. In addition, ESV are designed to be 'safe-fail', what means that in presence of faulty conditions the valves will go to the position causing no or minimal harm to the system/environment.

Safety valves are therefore a critical equipment for two reasons:

- 1) Their failures can lead to dangerous events with severe consequences,
- 2) They can generate high costs related to production shutdowns.

Because ESVs operate 'on demand', the state of valve is unknown until a demand occurs. Therefore, periodic tests are performed in order to reveal the condition of an ESV and ensure its functionality in case of a demand. It is assumed that after a functional test, the valve is as good as new. The reliability measure of safety valves is expressed by an average probability that the item will not be able to perform its safety function if the demand occurs, and is denoted as Probability of Failure on Demand (PFD_{avg}) (Rausand 2014). The PFD_{avg} must fall within the limits determined by safety integrity requirements specified by IEC 61508. Of course, the more often periodic tests are performed, the lower PFD_{avg} can be achieved. However, tests are expensive, since they require temporary production shutdowns. Thus, the optimization of inspection intervals is of a great importance in order to find a balance between process safety and production assurance.

During functional test it is possible to check whether a valve can be fully open/closed, the time needed to perform safety function, and whether there is no an excessive leakage in closed position. Except the information related to the valve functionality, there is also a possibility to record the travel time and torque applied on valve stem. That information can be further used for a

prognostic purpose as indirect indicators of a valve condition.

3. Empirical data

The object of the analysis is the field data providing conditions of 92 ESV through 3 years in operation. The data includes type of equipment, a tag number, the system to which the valve belongs, the medium it is used for, the type of activity performed on the valve (function check, maintenance or other), the state of a valve before and after the performed activity, and description of what exactly has been done with a specific valve.

Because the observation interval is relatively short **in this study**, only few failures were observed. Therefore, all failures are considered together, although splitting them according to failure modes could provide a more meaningful information. Investigated valves operate in different process systems and with different types of medium (water, oil, gas) what can influence the speed of degradation. The impact from those differences is not considered in this analysis perhaps leading to lose of some information. However, mentioned factors can be incorporated in the model rather easily, via explanatory variables.

3.1 Condition monitoring

According to the provided data, there are five distinguishable states describing condition of ESV:

- 1) New
- 2) Good
- 3) Small degradation
- 4) Significant degradation
- 5) Function failure

The state New is assigned to all valves at their installation dates and to all replaced items. It is assumed that all valves have been installed 1.01.2016. The valve is repaired, when some degradation is revealed. The repair can be a renewal of valve or an imperfect repair of different kinds: replacement of a single subcomponent or some adjustments. Depending on the scope of the repair, the state of component can be improved to *new*, in case of the replacement or to *good* condition.

In the previous section, functional tests were mentioned as a way to reveal safety valve condition. However, according to the investigated data those tests are not the only ones kind of inspections performed on ESVs. Besides functional tests, the valves were checked during periodic testing, periodic inspection or similar. So far it has not been investigated, what is the

difference between the scope of those inspections and all of those actions were assumed equal. As a result, the frequency of valves testing was much higher than a typically assumed.

In addition, valves were the subject of many other activities such as small adjustments, lubrications etc. Those activities also helped to reveal states of valves, and they were often used to perform repairs. Thus, it could be reasonable to consider all interventions as inspections. The valves were always repaired when found in failed state and mostly repaired when small or significant degradation was revealed.

3.2 Data inconsistencies

Some repairs were performed between two inspections, but not registered in the system. Sometimes repair actions were described as renewals, but the assigned state after was *good* instead of *new*. For all such cases, the state after repair was set up as *new*. A similar inconsistency was observed in the description of failed items. The items were revealed as *significantly degraded* although according to the state description their states were already *functional failures*. For all such cases, the state before repair was set up as *failed*. There were also other minor data inconsistencies, which had to be removed manually or carefully treated in order to keep the relevant information.

4. Model assumptions. Parameters estimation.

Although degradation is a continuous process, it has been modelled by a Markov chain, which is a discrete event stochastic process. The discrete model has been chosen for two pragmatic reasons: to make work on the model easier, and because the provided data were already discretized, what reflects how are they considered and registered by the company.

The time to jump from state to state is assumed to be exponentially distributed with a constant transition rate. There is no guarantee however, whether this assumption is well founded in reality, as one could expect that with getting older valves deteriorate faster. Nevertheless, the assumption of exponentially distributed transition rates ensures memoryless property and enables the application of a straightforward Markov model. Since this is the first attempt to use presented herein data, the simplicity is desired, in order to check whether this type of model is applicable at all. Later the model can be further developed by application of the phase-type distribution, which allows modeling a degradation process in the lifetime of equipment. The phase-type distribution is constructed by a convolution of exponential

distributions (Harchol-Balter 2012). In a Markov model it can be applied by adding additional states to model for example a Weibull lifetime distribution.

The considered herein degradation model is a continuous time Markov chain because the transition from state to state is considered on the continuous time domain. Figure 1 shows assumed degradation path of ESV.

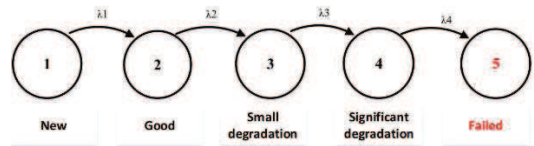


Fig. 1 Markov chain describing ESV degradation.

4.1. Transition rates estimation

It is assumed that a jump over two or more states is not physically possible, because the valve’s degradation is gradual (see Fig. 1). Therefore, when such a situation is observed in the data, it is assumed that the valve visited intermediate states in the period between two inspections.

4.1.1 Transition rates for degradation

Transition rates are estimated as the frequency of a change of state. A transition rate from state *i* to state *j* is calculated as the ratio between the number of jumps from *i* to *j* (performed by all valves) and the total number of hours spent in state *i*. In order to discuss how transition rates have been derived for simple jumps and in case of multiple jumps, the Table 1 has been introduced in the format of the data used in the analysis. According to this table, there is one jump from state 2 to state 3 performed between 03.03.2017 and 02.02.2018. In this case, the transition rate is equal one over the time difference (in hours) between two mentioned dates.

Table 1 Maintenance data for one safety valve

Date	State before	State after	Maintenance activity
03.03.2017	Small deg.	Good	Repair
10.08.2017	Good	Good	Periodic test
3.11.2017	Good	Good	Inspection
02.02.2018	Small deg.	Good	Repair

Let us consider now the situation when the last inspection reveals a *significant degradation*. Assuming that the system cannot skip intermediate states, they had to be visited without being explicitly revealed. There are two

approaches to consider ‘complex’ jump depending on the degradation propagation (based on the failure propagation concept (Vatn 2007)):

i) Sudden observable degradation propagation

Referring to Table 1, and assuming that on 02.02.2018 the valve’s condition was a *significant degradation* and taking into account assumption about gradual degradation, the investigated valve visited two states, between 3.11.2017, and 02.02.2018. Then according to the sudden observable degradation propagation, the time to

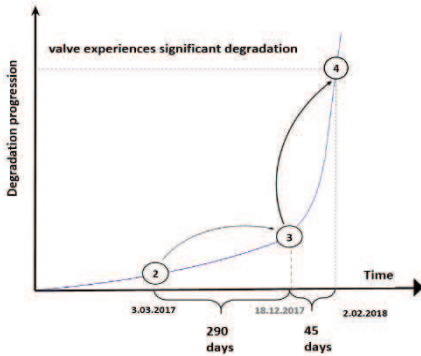


Fig. 2 Propagation of degradation from state 2 to 4, based on the data in Table 2, with stipulation that valve’s state 02.02.2018 was *significant degradation* (4). Adapted from (Vatn, 2007) with author’s permission.

go from *small* to *significant degradation* is short compare to the time necessary for a travel between *good* and *small degradation* states, see Fig. 1. It can be explained by the fact that already deteriorated valve will experience a faster degradation. Degradation transition rates based on this assumption are presented in Table 2. The transition from state i to state $i+1$ is denoted by λ_i

Table 2 Estimated transition rates between degraded states, according to assumption i)

Valve	Estimated value [1/hour]
λ_1	0.0001
λ_2	0.0001
λ_3	0.0034
λ_4	0.0023

ii) Gradual unobservable propagation

When a double or triple jump has been revealed, it means that the valve visited

intermediate states in the period from the last repair (03.03.2017) until a fault detection (02.02.2018). In this case it is assumed that the time spent in all visited states is equal, see Fig. 3. This is possible under assumption that the valve deteriorated in the way not possible to reveal during some inspections or just inspections were imperfect.

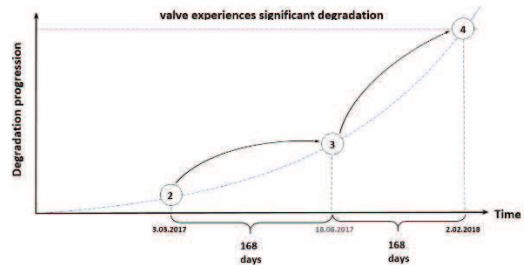


Fig. 3 Propagation of degradation from state 2 to 4, based on the data in Table 2, with stipulation that valve’s state 02.02.2018 was *significant degradation* (4). Dashed line denotes a not-observable degradation. Adapted from (Vatn, 2007) with author’s permission.

Degradation transition rates based on this assumption are presented in Table 3.

Table 3 Estimated transition rates between degraded states, according to assumption ii)

Transition rate	Estimated value [1/hour]
λ_1	0.0001
λ_2	0.0001
λ_3	0.0007
λ_4	0.0005

Above considerations are important, because jumps from state 3 to 4 and from state 4 to 5 were not observed directly, so transition rates between those states were derived only from complex jumps.

Another important aspect is the observation time. The data used herein comes from three-year observation interval, but there is a lack of information about the state of valves between the last inspection and the end of observation period, i.e. the data are censored. Herein, it is assumed that all valves remain in a *good* state from the last inspection to the end of observation period. This assumption seems to be realistic because valves are predominantly restored to a *good* state if during an inspection their state was worse than *good*. The time from the last inspection to the end of the observation interval is thus included in the estimation of transition rate between state 2 and 3. This is done by increase of cumulative hours

necessary to jump from state 2 to 3. This leads to a decrease of considered transition rate.

It is worth to notice that the transition from *new* to a *good* state is artificial in the meaning that the valve is new only when it is set in operation for the first time. Thus, the transition from *new* to a *good* state depends only on the time between checks, and does not reflect the real speed of degradation of ESVs.

4.1.2. Repair rates and probabilities of repair.

During a periodic test or other kind of function check, the state of a valve is revealed and the repair scope is decided. For degraded states, it can also be decided not improve the item.

Although the decision about maintenance activity is a deterministic variable, herein it is modelled as a probability to return to a better state. This probability reflects the frequency of making a specific decision with regard to repair. This approach should be later verified with the industry in order to apply a convenient deterministic function to model decision about maintenance. In addition, the uncertainty related to availability of spare parts or maintenance workload can be an argument to model the decision of repair as a stochastic variable.

The probability of a repair decision is estimated as the frequency of being repaired from degraded state to a *good* or *new* state. Table 4 presents probabilities of repairs, where P_{ij} denotes probability of performing repair from the state i to the state j after a fault detection. Situation when $i=j$ it reflects a decision about no making repair.

Although repair times are deterministic values, they are assumed to be exponentially distributed for the purpose of the model, i.e. to ensure they are stochastic variables with a memoryless property. As shown in the Table 1, the collected data does not contain any information about repair time. It is only known that they were performed on the day of the inspection.

Repair times are assumed different depending on the scope of a necessary maintenance work, where the scope refers to the number of steps by which the valve condition has been improved. Thus, one-step improvement lasts 8 hours; two-steps improvement lasts 12 hours, while 14 hours is needed for a three-step improvement. When it comes to a replacement of a valve, it is assumed to last 16 hours. In the data, there was only one occurrence, when the repair of the failed valve lasted few days, perhaps due to the waiting for spare parts. This occurrence is included in the estimated time to repair.

Table 4 Estimated probabilities of repair initialization and corresponding repair rates

Prob. of initiating repair	Estimated value	Repair rate	Estimated value [1/hour]
P_{31}	0.0093	μ_{31}	0.0833
P_{32}	0.9439	μ_{32}	0.1250
P_{33}	0.0468	μ_{33}	–
P_{42}	1	μ_{42}	0.0833
P_{51}	0.6667	μ_{51}	0.0714
P_{52}	0.3333	μ_{52}	0.0268

5. Model application.

Degradation of ESVs has been modelled by three Markov models with slightly different approaches. First, the model without any repair was applied, in order to investigate the Mean Time to Failure (MTTF) of a component if an item is not maintained. Next, the model was enhanced by the application of a repair possibility, under an assumption that the time to repair is negligible. Finally, in the last model maintenance activities are considered with repair rates described in the previous section. The results from first two models were compared by applying Monte Carlo simulations. For models considering the impact of maintenance, different inspection intervals were investigated.

5.1. Model application

5.1.1 Model without repair

Fig. 1 shows the Markov graph of the degradation model without consideration of maintenance activities. The observation of a valve starts at time $t = 0$ when the component is new, so it is in state 1. We are interested of what is the state of the valve at different times t , and what is its MTTF. The probability of being in one of five considered states $P_i(t)$ is derived by the numerical integration of standard Markov differential equations:

$$P_i(t + dt) \cong P_i(t)(1 - \lambda_i dt) + P_{i-1}(t)\lambda_{i-1}dt \quad (1)$$

Where dt is a very small time interval. The MTTF is calculate as following integral:

$$MTTF = \int_{t=0}^{\infty} (1 - P_5(t))dt \quad (2)$$

5.1.2 Model with repairs

The Markov graph for the degradation model with consideration of maintenance activities is shown on Fig. 4:

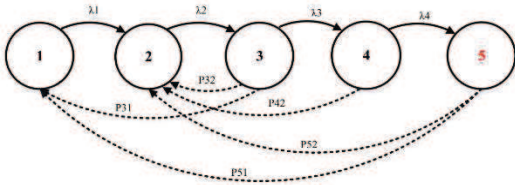


Fig. 4 The Markov model including maintenance actions. Dashed lines indicate repair after functional test, where time to repair is zero.

In the model it is assumed that the valve can be repaired when found in state 3, 4 or 5. The probabilities of initiating repair were shown in Table 4. The model was created in the same way as previously, but at times of inspection $t = \tau, 2\tau, \dots, n\tau$, the additional equation (eq. 3) is considered:

$$P_i(t^+) \cong P_i(t) + P_j(t)P_{ji} \quad (3)$$

Where $j > i$ and $j \geq 3$, and notation $P_i(t^+)$ is used to express that repairs are instantaneous.

During normal operation, the valve undergoes a degradation according to a time continuous Markov process. Next, at inspections times the repair is decided. Because the transitions to better states are instantaneous, they happen according to a discrete time Markov model with assigned probabilities instead of rates. Such a model is multiphase in the meaning that transition probabilities change with time.

5.1.3 Model with repair rates

In the last model, a valve is the subject of physical degradation according to CTMC during normal operation. The states probabilities were derived using the numerical integration of following equations in a matrix form:

$$\mathbf{P}(t + dt) = \mathbf{P}(t) \cdot (\mathbf{A}dt + \mathbf{I}) \quad (4)$$

Where \mathbf{P} is a probability vector for considered states, \mathbf{A} is a transition matrix for 10 states in Fig. 4, and \mathbf{I} is an identity matrix. \mathbf{A} matrix contains only failure and repair rates. At each inspection a new information about the valve state is achieved and based on that, the decision about planned repair is made according to the transition matrix \mathbf{A}_2 , which is a repair decision matrix and contains only probabilities of jump to repair states (Table 4). This transition happens instantaneous and allows the valve to make a transition to new logical states, without changing its physical state. The logical states, described on Fig. 5 by double digits, denote changes of states due to decision about repair without a physical change of the

valve condition. The change of the valve state at inspection is described by equation 5:

$$\mathbf{P}(t^+) = \mathbf{P}(t) \cdot (\mathbf{A}t + \mathbf{I}) \cdot \mathbf{A}_2 \quad (5)$$

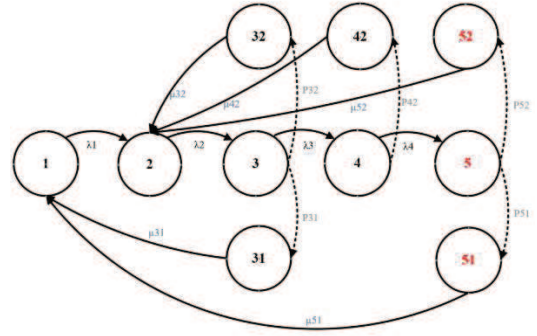


Fig. 5 Markov model describing maintenance actions. Dashed lines indicate repair decisions. For states denoted by double digits, the first number denotes actual physical state of the component and the second number tells to which state the valve will be improved by repair.

The matrix \mathbf{A}_2 from eq. 5 is similar P_{ij} in eq. 3 as they both express the instantaneous change of the valve state at inspection time.

The probability of being in any of degraded states is the sum of probabilities of being in the considered physical state and assigned (dashed lines on Fig. 5) logical states. After an inspection, the system again undergoes degradation according to a TCMC.

5.2. Choice of inspection intervals

Three different τ values were applied in the model in order to perform a sensitivity analysis and investigate the impact of an inspection interval on the average probability of failure of safety valves. Commonly, functional tests of safety equipment condition are performed around once per year. Therefore, one choice for τ was to consider 8 months' interval (5844 h). However, according to the provided data there were also others kinds of valves' checks (section 3.2). Based on that information, the frequency of checks for one valve was calculated to be 4392 hours, which is 6 months. The last value of τ was derived based on the frequency of all interventions performed on valves. In this case, each valve was checked on average every 64 days (1546 h).

6. Results

Fig. 6 presents the probability of a failure of an ESV as a function of time.

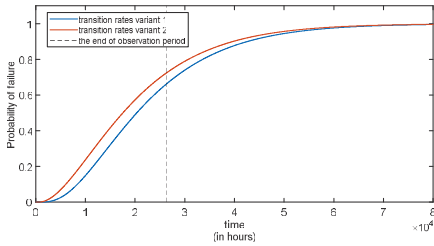


Fig. 6 The plot of a failure probability as a function of time, based on the model without repair with transition rates according to Table 3 and Table 4, for variants 1 and 2 respectively.

The PFD_{avg} is calculated as the integral over the failure probability function shown on Fig. 6. For 3 years in operation, it is equal: 0.347 and 0.2783 for two considered variants. The PFD_{avg} is so high because repairs were not included in the model. This also explains why PFD_{avg} increases with the length of the considered interval. The calculated MTTF is equal 2.4 years for two considered options.

Next, the PFD_{avg} function was derived for two models considering repairs (model 2 and 3). Three different inspection intervals were used. Fig. 7 shows the probability of an ESV failure as a function of time. Table 5 presents the PFD_{avg} according to transition rates in Table 2 and Table 3. In the Table 6 the number of ESVs' failures was derived as realizations of the model 2 by Monte Carlo simulation. The results obtained by the application of model 2 and 3 are almost the same in all considered cases. It can be explained by the fact that repair times are very short compare to deterioration times. The obtained PFD_{avg} differ twice depending on applied transition rates for $\tau = 6$ and 8 months. For the shortest inspection interval, the results differ by one range of magnitude. The PFD_{avg} decreases with a shortening of inspection interval, dropping almost 10 times when changing the inspection interval from 8 to 2 months.

Table 6 shows the results obtained from the Monte Carlo simulation for model 2. The number of failures differs twice between variant 1 and 2 for both $\tau = 6$ and 8 months. For 2 months' inspection interval, the number of observed failures for variant 1 is equal 8, what is eight times

lower than for variant 2. In addition, that is almost the same as the actually observed number of failures - 6. This indicates that the assumption of gradual non-observable degradation propagation and the 2 months' inspection interval reflects the reality better.

Table 5 PFD_{avg} values derived from model 2 and 3 according to transition rates in Table 3 (variant 1) and Table 4 (variant 2).

τ [days]	PFD _{avg} variant 1		PFD _{avg} variant 2	
	Model 2	Model 3	Model 2	Model 3
65	0.0194	0.0213	0.0034	0.0039
183	0.0838	0.0855	0.0333	0.0345
240	0.1131	0.1147	0.0538	0.0552

When it comes to degradation propagation, the result can be explained by imperfectness of inspections, as different checks are predicted to reveal different faults. In addition, it can be that some faults are impossible to discover in an early development. Based on Rausand (2014) and Hauge (2009) considerations for PFD_{avg} calculations, the typical ESV reaches PFD in range of 10^{-3} for $\tau = 8$ months. Here, such the magnitude of PFD was obtained only for $\tau = 2$ months. However, mentioned studies assume that after each functional test, the valve's state is as good as new, what is not a case herein.

Table 6 Number of failures from 100 realizations of Markov process based on model 2.

τ	Number of failures	
	variant 1	Variant 2
65 days	8	64
183 days	38	82
240 days	43	87

7. Summary/Conclusions

A degradation process of safety valves used in a process facility was investigated based on the empirical data. The Multiphase Markov approach has been presented as a tool for modelling of a degradation propagation. The proposed method enables incorporating a repair decision and time to repair in the model, and to evaluate their impact on the propagation of deterioration.

A deterioration speed is incorporated in the model via estimated transition rates. Two variants of the degradation propagation were considered providing different results. The repair and inspection policy in the real study case differs from the theoretical one.

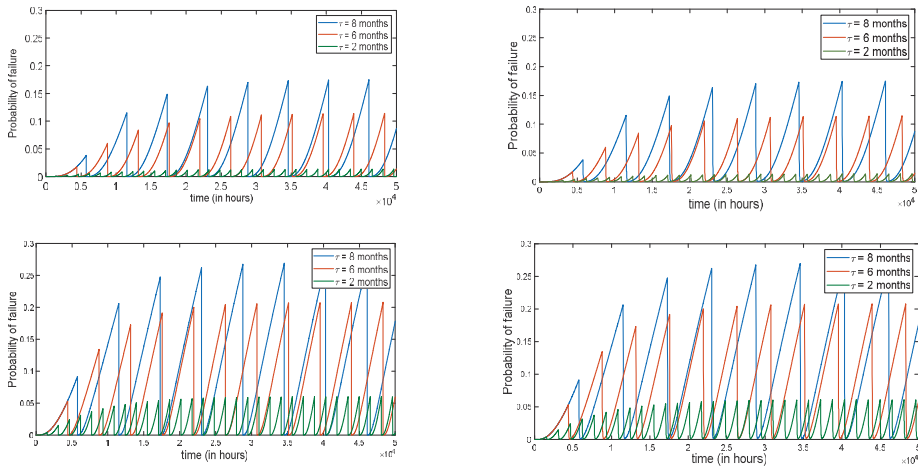


Fig. 7 Probability of a valve failure as a function of time, based on model 2 (left) and 3 (right) for variants 1 (down figures) and 2 (top figures) respectively.

The obtained results depend heavily on the consideration of condition monitoring and maintenance activities, and on the model assumptions regarding degradation propagation. Therefore, the model assumptions as well as the understanding of the collected data proved to be important factors influencing the probability of equipment failure.

The model requires further development by consideration failures with regard to different failure modes. This would allow obtaining more realistic transition rates and inspections intervals. The applied model can be also improved by modeling non-constant failure rates according to the phase-type distribution theory. In addition, the incorporation of explanatory variables related to valve condition could help to reflect the degradation process better.

Acknowledgements

We thank Lundin Company for providing the data for the analysis and funding the PhD position as a part of project BRU 21.

References

- Commission, I. E. 2005. Functional Safety of Electrical/Electronic/Programmable Electronic Safety-related Systems (E/E/PE, or E/E/PES) 61508.
- Harchol-Balter, M. 2012. Real-World Workloads: High Variability and Heavy Tails. Cambridge University Press.
- Innal, F., M. A. Lundteigen, Y. Liu, and A. Barros. 2016. "PFDavg generalized formulas for SIS subject to partial and full periodic tests based on multi-phase Markov models." Reliability Engineering & System Safety 150:160-170.
- Langeron, Y., A. Barros, A. Grall, and C. Bérenguer. 2008. "Combination of safety integrity levels (SILs): A study of IEC61508 merging rules." Journal of Loss Prevention in the Process Industries 21 (4):437-449.
- Lundteigen, M. A., and G. O. Strand. 2015. "Risk control in the well drilling phase BOP system reliability assessment." European Safety and Reliability Conference
- Rausand, M. 2014. Reliability of safety-critical systems: theory and applications: John Wiley & sons.
- Srivastav, H., A. V. Guilherme, A. Barros, M. A. Lundteigen, F. B. Pedersen, A. Hafver, and F. L. Oliveira. 2018. Optimization of periodic inspection time of sis subject to a regular proof testing.
- Vatn, J. 2007. "Veien frem til "World Class Maintenance": Maintenance Optimisation." In: Norwegian University of Science and Technology (NTNU). <http://folk.ntnu.no/jvatn/pdf/MaintenanceOptimisationWCM.pdf>.
- Wu, S., L. Zhang, A. Barros, W. Zheng, and Y. Liu. 2018. "Performance analysis for subsea blind shear ram preventers subject to testing strategies." Reliability Engineering & System Safety 169:281-298.