Geological Uncertainty: A Missing Element in Geotechnical Reliability Analysis

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Abstract: While the presence of geological uncertainty is often recognized by the engineer, it is seldom considered explicitly in current geotechnical reliability analyses. Indeed, it is largely a missing element in a geotechnical reliability analysis and its effect is generally not known or examined. In this paper, we present a case study to illustrate the characterization and consideration of the geological uncertainty in a geotechnical analysis. We illustrate that when combined with the random field theory, the soil behavior type index (I_c) derived from the cone penetration test (CPT) may be used to determine the composition of the ground at any given point, which forms a basis for characterization of the geological uncertainty at a site. Once the uncertainty associated with I_c is quantified, it may be used to assess the uncertainty associated with the soil stratigraphy and the impact of the geological uncertainty on the predicted geotechnical performance. Further, in the random field modeling the scale of fluctuation is an important parameter that defines the distance within which the information gathered through CPT sounding at a given location can affect the neighboring area. When the number of CPT soundings at a site is relatively small, the estimated scale of fluctuation may not be robust. The results obtained through the random field modeling can help guide decision on how to conduct additional site investigation work. Suggestions are also made for future studies regarding the identification and characterization of the geological uncertainty for use in the geotechnical analysis.

Keywords: Geological uncertainty; spatial variability; soil behavior type index; cone penetration test; random field.

1 Introduction

Three types of models are commonly involved, to varying extents, in a geotechnical analysis (e.g., Burland 1987; Morgenstern 2000; Knill 2003; Sullivan 2010; Keaton 2013). They are the geologic model, the ground model and the geotechnical model, as shown in Fig. 1. In this figure, the geologic model is a representation of the site geologic condition related to the project at hand. The ground model is the geologic model further detailed for the given project and expressed in terms of engineering parameters, and the geotechnical model is the ground model with specified design parameters used to establish predicted performance of the given project. Due to needed simplification and approximation involved, each model is associated with varying degree of uncertainty. The construction of a geologic model often requires the specification of the composition of the ground and the geological boundary conditions, which often involves high degree of uncertainty (e.g., Einstein and Baecher 1982; Fookes 1997; Hoek 1999; Bárdossy and Fodor 2001; Bock 2006; Parry et al. 2014). The ground model is derived from the geologic model and detailed through site investigation and field and laboratory testing. Due to the spatial variability of soil, the measurement error, and the statistical uncertainty caused by limited number of samples, the engineering parameters are difficult to determine with certainty (e.g., Lumb 1966; Tang 1984; Phoon and Kulhawy 1999; Christian 2004; Fenton and Griffiths 2008; Zhang et al. 2018). The geotechnical model, which intends to predict the performance of a geotechnical system, is always associated with some assumptions and limitations, thus, the uncertainty exists (e.g., Ang and Tang 1984; Juang et al. 2004; Zhang et al. 2009; Phoon and Tang 2019). Detailed discussion of the three types of uncertainties can be found in Juang et al. (2019).

In the past few decades, the importance of characterizing uncertainty and quantitatively assessing its impact on geotechnical analysis and design has been widely recognized, and methods for probabilistic analyses of geotechnical performance have been extensively studied (e.g., Alonso 1976; Tang et al. 1976; Whitman 1984; Wu et al. 1989; Honjo and Kuroda 1991; Low and Tang 1997; Juang et al. 2000; Zhang et al. 2001; Griffiths and Fenton 2004; Xue and Gavin 2007; Li et al. 2009; Huang et al. 2010; Ching and Phoon 2011, 2011; Stuedlein et al. 2012; Juang et al. 2013; Khoshnevisan et al. 2014; Dithinde et al. 2016). These studies mainly focus on the uncertainties associated with the ground model and/or geotechnical model (Juang et al. 2019). The relevant geological uncertainty, however, is seldom addressed and is considered a missing element in many current geotechnical probabilistic analyses. In geotechnical engineering, the late Prof. Wilson H. Tang pioneered methods of detecting geological anomalies in the ground (Tang 1986; Tang and Halim 1988; Halim and Tang 1993). In recent years, there has been an increasing interest in the characterization of the uncertainty associated

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with the soil stratigraphy (e.g., Phoon et al. 2003; Liao and Mayne 2007; Cao and Wang 2013; Li et al. 2013; Wang et al. 2014; Ching et al. 2015; Qi et al. 2016; Zheng et al. 2018). Studies have been carried out to assess the effect of uncertainty associated with the soil stratigraphy on slope stability analysis (e.g., Wang et al. 2018; Gong et al. 2019). Despite these studies, characterization of the geological uncertainty remains a challenging task and the effect of this uncertainty on a geotechnical reliability analysis has not been well examined.

The objective of this paper is to present a case study that showcases the characterization of the geological uncertainty at a project site through the cone penetration test (CPT). To this end, the role of the soil behavior type index (I_c), derived from CPT, in characterizing the geological uncertainty is explored, and its potential use in inferring the uncertainty associated with the soil stratigraphy is discussed. The effect of the geological uncertainty is then studied by evaluating soil liquefaction potential at the study site. Finally, the role of conducting more tests in the characterization and reduction of geological uncertainty is investigated. Suggestions are also made for future research towards better characterization of the geological uncertainty and better assessment of its impact on the predicted geotechnical performance.

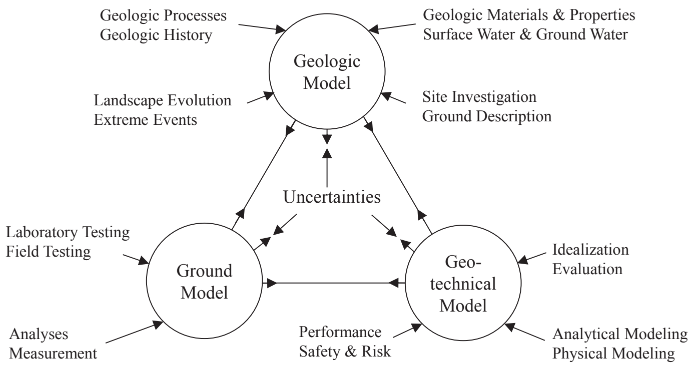


Figure. 1 Geological model, ground model and geotechnical model [Adapted from Keaton (2013)].

2 The Site of Case Study

Fig. 2 shows the site of Automobile Research and Testing Center (ARTC) located in the Lukang District of the Chang-Hwa Coastal Industrial Park (CHCIP) on the west coast of central Taiwan, which was created through a large-scale land reclamation project. As described in Lee et al. (2001), geologically, the CHCIP area was an extension of the recent alluvial plains (Q_a) of Changhua County, Taiwan. The area was reclaimed by hydraulic filling of dredged sediments (mainly consisting of silty sand to fine sand). The thickness of the hydraulic fill was approximately 4 m to 5 m. A backfill of gravel of approximately 0.2 m was placed over the hydraulic fill. According to Shen et al. (2018), this site mainly consisted of silty sands (SM or SP-SM) with thin layers of silts (ML) or silty clays (CL). The ARTC site, with an area of 2000 m × 800 m, was investigated with laboratory and field tests before the construction. In this study, we focus on the 27 CPTs that were conducted as part of the site investigation. Among these 27 CPT soundings, shown in Fig. 2, 14 of them reached the depth of 10 m, and the other 13 reached the depth of 20 m.

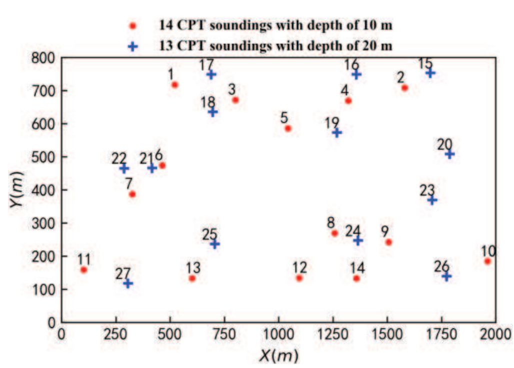


Figure 2. Layout of the 27 CPT soundings conducted at the study site.

3 Soil Behavior Type Index and Geological Uncertainty

The geological uncertainty may be defined as the uncertainty associated with the specification of the composition of the ground and the geological boundary conditions (Bock 2006). The in-situ behavior of a soil may be described with the soil behavior type index, I_c, obtained through CPT (e.g., Robertson 1998). In recent years, the CPT-based classification systems based on I_c is increasingly popular in geotechnical engineering due to its cost-effectiveness and efficiency (Robertson 2016). Note the soil type defined in the CPT-based soil classification system is not the same as the traditional soil classification system based on the physical (textural) characteristics of soil such as grain size and plasticity. The CPT-based soil classification is related to the in-situ behavior of the soil, and the traditional textural-based soil classification (such as USCS) is based on disturbed and remolded samples. Assume that the composition of the ground at each point is adequately reflected by the soil behavior type index I_c, which is illustrated later by the features of the composition, such as the thickness of different layers. Then at each point in the ground where I_c is known, there is no uncertainty associated with the composition of the ground and hence no geological uncertainty. *Therefore, the uncertainty associated with I_c at a location may be viewed as an indication of the geological uncertainty at that location.*

Figures 3(a), 3(b), and 3(c) show the values of I_c derived from CPTs at locations No. 10, 19, and 23, respectively. As can be seen from these figures, the distributions of I_c along the depth of the three CPT soundings are quite different. For example, at the depth of 3 m, the I_c values at the three CPT soundings are 2.06, 2.15, and 1.92, respectively. According to Robertson (1998), the soils at these three locations are Type 5 soil (silty sand to sandy silt), Type 6 soil (clean sand to silty sand), and Type 5 soil, respectively. At each location with CPT sounding, the I_c value is known with certainty (assuming no measurement error). At locations without CPT, the I_c value has to be inferred from the known values at other locations and thus is subjected to uncertainty. If such uncertainty can be quantified, so can the geological uncertainty at each point in the ground. In the following, we will illustrate how the uncertainty associated with the soil behavior type, and thus the geological uncertainty, can be quantified with the aid of random field theory.

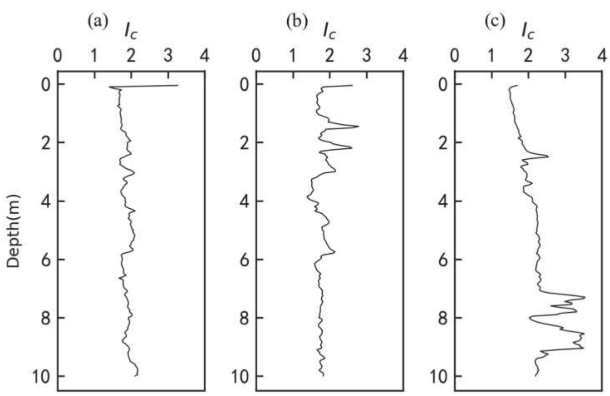


Figure 3. Soil Behavior Index (I_c) measured at deferent locations: (a) No. 10; (b) No. 19; and (c) No. 23.

4 Random Field Modeling

It is well recognized that geotechnical properties vary spatially; the closer the two points are, the more similar the properties at the two points will be. The random field theory is an effective tool to model the spatial variability of soil (e.g., Vanmarcke 1977; Honjo and Kuroda 1991; Lacasse and Nadim 1996; Fenton 1999; Elkateb et al. 2003; Uzielli et al. 2005, 2006; Cho 2007; Baker and Faber 2008; Fenton and Griffiths 2008; Stuedlein et al. 2012; Zhu and Zhang 2013; Gong et al. 2017). The random field model can be used to infer soil properties (such as I_c values) at locations without CPT sounding. As an illustration, the stationary Gaussian random field model (e.g., Vanmarcke 1977; Fenton and Griffiths 2008) is used in this paper. Suppose that the study site can be discretized into n points. Let x_i denote the soil property of interest at the i^{th} point. Let $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}^T$. Let $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ denote the mean and the covariance matrix of $\mathbf{x}$. Suppose the correlation coefficient between points i and j can be modeled through the following exponential correlation function (e.g., Vanmarcke 1977; Fenton and Griffiths 2008).

$$\rho\left(\Delta_{ij}\right) = \exp\left(-2\left|\Delta_{ij}\right| / \lambda\right) \tag{1}$$

where Δ_{ij} is the distance between point i and j and λ is the scale of fluctuation. The scale of fluctuation measures the intensity of the spatial variability. Within the scale of fluctuation, the soil properties are strongly correlated. Beyond the scale of fluctuation, the correlation is considered weak. The greater the scale of fluctuation is, the more spatially correlated the soil properties are. Based on Gaussian random field assumption, $\mathbf{x}$ follows the multivariate normal distribution with a mean of $\boldsymbol{\mu}$ and a covariance matrix of $\boldsymbol{\Sigma}$.

Let $\mathbf{x}_1$ be the soil properties at the locations with CPT soundings, and let $\mathbf{x}_2$ denote the soil properties at the locations without CPT sounding. Let $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ denote the mean values of $\mathbf{x}_1$ and $\mathbf{x}_2$, and let $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ denote the covariance matrix of $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. Let $\mathbf{a}$ denote the values of $\mathbf{x}_1$ measured through the CPT soundings. The above information can be used to calculate the mean and covariance matrix of $\mathbf{x}_2$ at locations without CPT tests. Let $\boldsymbol{\mu}_{2|1}$ and $\boldsymbol{\Sigma}_{2|1}$ denote the mean and covariance matrix of $\mathbf{x}_2$ given the CPT sounding results at $\mathbf{x}_1$, i.e., $\mathbf{x}_1 = \mathbf{a}$. Based on the property of a multivariate normal distribution (e.g., Eaton 1983), $\boldsymbol{\mu}_{2|1}$ and $\boldsymbol{\Sigma}_{2|1}$ can be calculated using the following equations:

$$\boldsymbol{\mu}_{2|1} = \boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\left(\mathbf{a} - \boldsymbol{\mu}_2\right) \tag{2}$$

$$\boldsymbol{\Sigma}_{2|1} = \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21} \tag{3}$$

Equation (2) gives the best-estimated soil properties at the locations without CPT, and Eq. (3) measures the uncertainty associated with the estimated soil properties. In the context of geological uncertainty characterization, Eqs. (2) and (3) can be used to estimate the soil behavior type index at each location in the ground, and the geological uncertainty at each location. To apply the random field theory, one can first calibrate the random field model based on the measurements at the locations with CPT soundings (i.e., $\mathbf{x}_1 = \mathbf{a}$) using methods like the maximum likelihood method (e.g., Juang and Zhang 2017). Then, Eqs. (2) and (3) can be used to estimate the soil properties at location without CPT sounding.

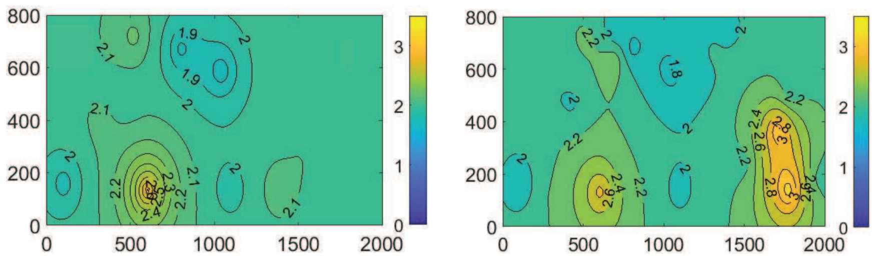
Equation (3) can be used to assess the uncertainty associated with the soil property at each point in the study site. In practice, one may also be interested in the overall uncertainty associated with the site. In information theory, the differential entropy is often used to measure the total amount of uncertainty associated with the random variables (Cover and Thomas 2012). For the case study presented here, the uncertainty of the study site is represented by the uncertainty associated with $\mathbf{x}_2$, which follows the multivariate normal distribution. According to the property of multivariate normal distribution, the entropy of $\mathbf{x}_2$ can be calculated as follows (e.g., Ahmed and Gokhale 1989):

$$h\left(\mathbf{x}_2\right) = \frac{k}{2}\ln(2\pi e) + \frac{1}{2}\ln\left(\left|\boldsymbol{\Sigma}_{2|1}\right|\right) \tag{4}$$

where e is the base of the natural logarithm, and k is the dimension of $\mathbf{x}_2$. The greater the entropy, the greater the uncertainty is involved.

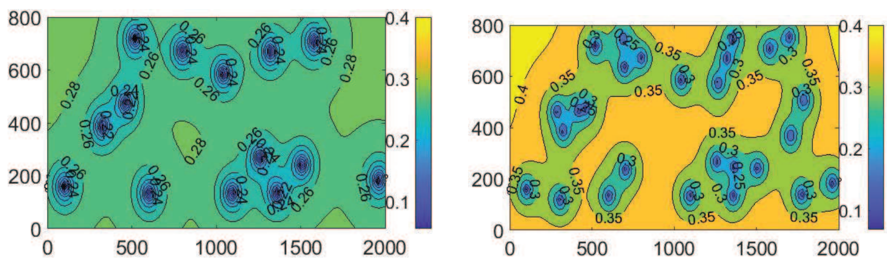
5 Assessing Uncertainty Associated with the Soil Behavior Type Index

Based on Eqs. (2), (3) and the maximum likelihood method (e.g., Juang and Zhang 2017), the random field model of I_c at a given depth can be derived. Taking 3 m below the ground surface as an example, its random field is first calibrated based on the set of 14 CPT soundings (those CPTs in Fig. 2 that have a sounding depth of 10 m). The random filed model of I_c at other depths can also be obtained using the same method. The mean, the standard deviation and the scale of fluctuation of the random field of I_c at the depth of 3 m are found to be 2.07 (dimensionless), 0.28 (dimensionless), and 226.6 m, respectively. Fig. 4(a) shows that the mean of I_c is in the range of 1.71 to 2.85, indicating that the soil is composed mainly of Type 5 soil and Type 6 soil. As mentioned previously, the uncertainty associated with I_c measures the amount of geological uncertainty at a point. Fig. 5(a) shows that the standard deviation of I_c is in the range of 0 to 0.28, indicating the geological uncertainty at each point in the ground may not be the same. As it can be seen from Fig. 5(a), the uncertainty of I_c is smallest at locations with CPT sounding. At locations that are far away from CPT soundings, the standard deviation is close to that of the random field model (i.e., 0.28). This is because that the distance between these locations and neighboring CPT soundings is greater than the scale of fluctuation and thus, the mean and the standard deviation are dominated by the mean and the standard deviation of the random field. Based on Eq. (4), the entropy associated with I_c at a depth of 3 m in the study site is -2791. Similarly, the results shown in Figs. 4(b) and 5(b) are obtained in the same way, but they are based on the 27 CPTs conducted at the study site. The significance and effect of including these additional 13 CPT soundings are discussed later. The results in this section thus shows that, through the soil behavior type index and the random field model, the results from CPT can not only be used to infer the composition of the soil at each point in the ground, but also be used to assess the magnitude of geological uncertainty at each point in the ground. The entropy of I_c measures the overall geological uncertainty in the ground. Note the procedure can also be used to infer the soil properties at other locations such as the cohesion and the friction angle of the soil, based on measurements at locations with CPT soundings. As mentioned previously, the ground model is the geological model expressed as geotechnical parameters. If such soil properties are input parameters of a geotechnical model, the uncertainties associated with such parameters can then be considered as examples of ground model uncertainty. Therefore, the random field model can also be used to assess the uncertainty associated with the ground model.



(a) (b)

Figure 4. Mean value of I_c at a depth of 3 m based on: (a) 14 CPT soundings; and (b) 27 CPTs soundings.

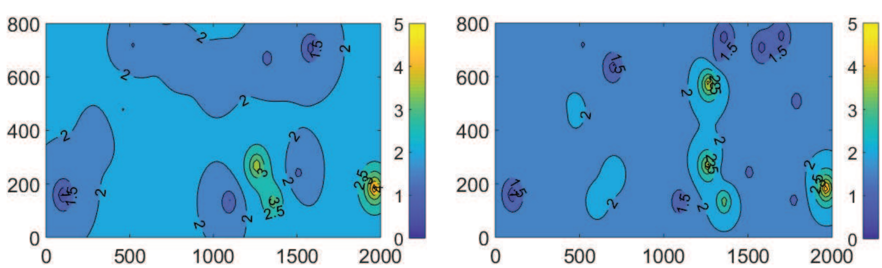


(a) (b)

Figure 5. Standard deviations of I_c at a depth of 3 m based on: (a) 14 CPT soundings; and (b) 27 CPT soundings.

6 Assessing Uncertainty Associated with the Thickness of Certain Soil Strata

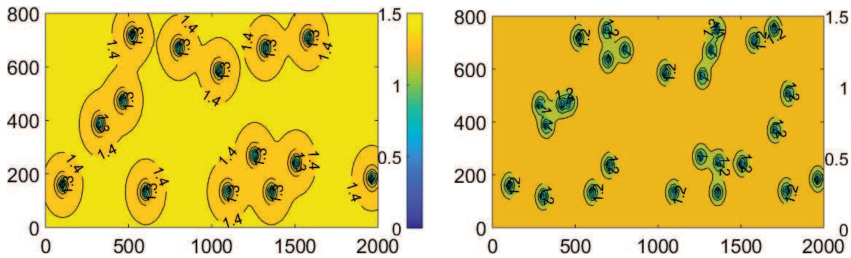
Once the uncertainty associated with the soil behavior type index is characterized, the uncertainty associated with the soil stratigraphy, another form of the geological uncertainty, may be assessed. Take the thickness of the Type-5 soil layer ($2.05< I_c <2.6$) as an example. At a location with CPT sounding, the thickness of the Type-5 soil layer can be derived. At other locations, the thickness of the Type-5 soil layer can be inferred through the random field model, and the uncertainty associated with the estimated thickness of the soil layer can be calculated through Eq. (3). Based on the data of the 14 CPT soundings, the mean, the standard deviation, and the scale of fluctuation of the thickness of the Type-5 soil layer are estimated to be 2.02 m, 1.41 m and 102.9 m, respectively. The scale of fluctuation of the thickness of Type-5 soil layer is seen different from that of I_c at a depth of 3 m, indicating that different parameters may have different scales of fluctuations. Figs. 6(a) and 7(a) show the mean and the standard deviation of the thickness of Type 5 soil layer, respectively. As can be seen from Fig. 6(a), the mean values of the thickness of Type 5 soil layer are not uniform, varying from 1 m to 3 m. Fig. 7(a) shows that the uncertainty associated with the estimated thickness is not uniform either.



(a) (b)

Figure 6. Mean value of the thickness of Type-5 soil estimated based on different CPT soundings: (a) 14 CPT soundings; and (b) 27 CPTs soundings.

Similarly, the results shown in Figs. 6(b) and 7(b) are obtained in the same way, but they are based on the 27 CPTs conducted at the study site. The significance and effect of including these additional CPT soundings are discussed later. The results in this section shows that through the soil behavior type index and the random field model, the CPT can also be used to infer the uncertainty associated with the soil stratigraphy, such as the thickness of a certain soil layer.



(a) (b)

Figure 7. Standard deviations of the thickness of Type-5 soil estimated based on different CPT soundings: (a) 14 CPT soundings; and (b) 27 CPT soundings.

7 Assessing Uncertainty Associated with the Liquefaction Potential Index

Once the uncertainty associated with the soil behavior type index is quantified, its impact on the geotechnical performance, such as liquefaction potential, may be assessed. Simplified CPT-based liquefaction assessment method is widely used to assess the liquefaction potential of the soil under the seismic loading (e.g., Seed 1979; Seed and De Alba 1986; Kayen et al. 1992; Stark and Olson 1995; Robertson and Wride 1998; Youd et al. 2001; Moss et al. 2006; Robertson 2009; Boulanger and Idriss 2014,2016). These simplified methods yield a factor of safety (F_s) as the liquefaction potential of the soil at a given depth of concern at a given CPT location. A convenient way to assess the severity of the liquefaction hazard at that location is to integrate the liquefaction potential over the depth as proposed by Iwasaki et al. (1982). The product of this integration, defined below by Iwasaki et al. (1982) as Liquefaction Potential Index I_L, has been adopted by many researchers (e.g., Sonmez 2003; Toprak and Holzer 2003; Holzer et al. 2006; Li et al. 2006; Juang et al. 2008):

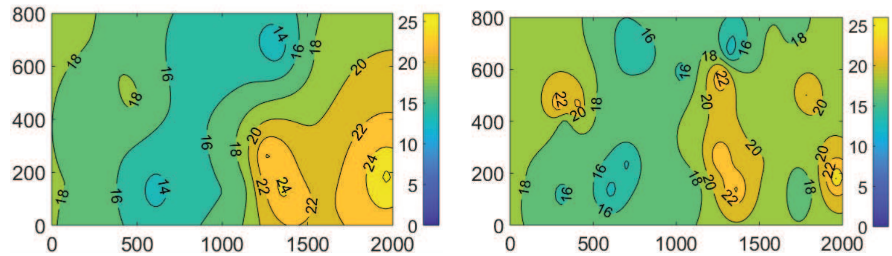
$$I_L = \int_0^{20} w(z)F(z)dz \tag{5}$$

$$F(z) = \begin{cases} 1 - F_s(z) & F_s(z) \le 1 \\ 0 & F_s(z) > 1 \end{cases} \tag{6}$$

where z = depth below the ground surface, $w(z)$ = 10 – 0.5z (an arbitrary weighting function), and $F(z)$ is a function of the factor of safety of the soil against liquefaction at depth z. In this study, F_s is calculated based on the method suggested in Robertson (2009), which depends mainly on the ground motion parameters, the soil behavior type index and other parameters such as cone tip resistance and sleeve friction measured during the CPT. At the study site, the local code specifies the following ground motion parameters at the level of design earthquake: the peak ground acceleration $a_{\max}$ = 0.28 g and the moment magnitude M_w = 7.1 (Shen et al. 2018).

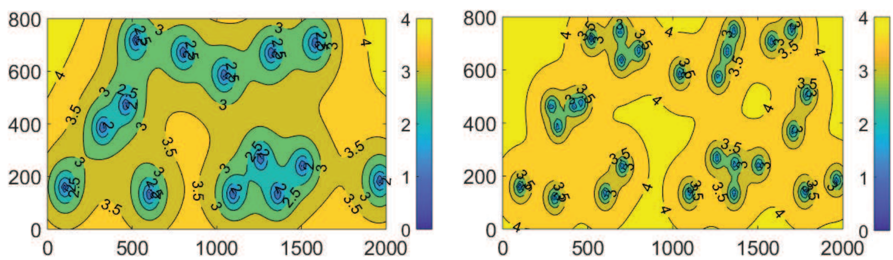
When Eq. (5) is used to assess I_L, the uncertainties due to the assumptions made during the derivation of Eq. (5), as well as the uncertainty associated with $F_s(z)$, are lumped into the geotechnical model uncertainty. In this paper, the geotechnical model uncertainty is not considered. The calculation of $F_s(z)$ depends on both the soil behavior type index, the cone tip resistance, and the sleeve friction. At a location without CPT sounding, the soil behavior type index, cone tip resistance, and sleeve friction are all uncertain. The uncertainty associated with the soil behavior type index may be considered as a measure of geological uncertainty at a point, and the uncertainties associated with the tip resistance and shaft friction are examples of ground uncertainty. Therefore, even the geotechnical model uncertainty is not considered, I_L could still be uncertain due to the existence of geological and ground uncertainties. There are two possible ways of evaluating the uncertainty associated with I_L considering the uncertainty associated with I_c. In the first approach, referring to herein as the direct approach, one may assess the soil properties at each point based on the random field method as we have done in Section 4, and then evaluate I_L at each location directly based on Eqs. (5) and (6). In the second approach, referring to herein as the indirect approach, one may first evaluate I_L at locatioins with CPT soundings as what we have done in Section 5, and then estimate I_L at other locations based on the random field method. The indirect approach is adopted in this study as it is easier to implement. As an example, the values of I_L are first calculated at locations with the 14 CPT soundings. Figs 8 (a) and 9(a) show the mean value and standard deviation of I_L estimated based on the 14 CPT soundings; here, the mean, standard deviation, and scale of fluctuation of I_L are found to be 18.51 (dimentionless), 4.34 (dimentionless), and 604.66 m, respectively. As can be seen here, the mean value of I_L ranges from 14 to 24, and the standard deviation of I_L is between 0 and 4.3. A comparision between Figs. 7(a) and 9(a) shows that if the uncertainy associated with the thickness of the soil layer is large, so is the uncertainty associated with I_L, indicating there is a strong correlation between the geological uncertainty and the uncertainty associated with the predicted performance (in this case, liquefaction potential index).

Similarly, the results shown in Figs. 8(b) and 9(b) are obtained in the same way, but they are based on the 27 CPTs conducted at the study site. The significance and effect of including these additional CPT soundings are discussed later.



(a) (b)

Figure 8. Mean value of the I_L estimated based on: (a) 14 CPT soundings; (b) 27 CPT soundings.



(a) (b)

Figure 9. Standard deviation of the I_L estimated based on: (a) 14 CPT soundings; (b) 27 CPT soundings.

8 Effect of Introducing More CPT Soundings

8.1 Impact of more CPT data on the uncertainty of I_c at the study site

In the above analyses, only 14 CPT soundings are used. As the number of CPT soundings increases, there will be more data on the soil behavior type index at the study site. To investigate the effect of more CPT soundings on the assessment of soil stratigraphy, the random field of I_c at the depth of 3 m below the ground surface is recalibrated with measurements from the 27 CPT soundings, and the mean, standard deviation, and the scale of fluctuation of I_c at the depth of 3 m are 2.16, 0.41, and 315.4 m, respectively. The parameters of the random field model change substantially after more data are included in the analysis, indicating that 14 CPT soundings are not sufficient for a robust calibration of the random field model. Figs. 4(b) and 5(b) show the mean and standard deviation of I_c estimated from 27 CPT soundings, respectively. A comparison between Figs. 4(a) and 4(b) shows that the use of additional CPT soundings changes the distribution of the mean of I_c. A comparison between Figs. 5(a) and 5(b) shows that the uncertainty associated with I_c in Fig. 5(b) is generally greater than that in Fig. 5(a) at different locations. When 14 CPTs are used, the entropy associated with I_c is -2791. When 27 CPTs are used, the entropy is increased to -1910, indicating the overall uncertainty associated with I_c is increased. This occurs probably because the use of 14 CPTs are not enough to fully expose the uncertainty associated with I_c at this site.

8.2 Impact of more CPT data on the uncertainty of the thickness of Type 5 soil at the study site

To investigate the effect of more CPT soundings on the thickness of Type 5 soil, the random field of the thickness of the soil layer is recalibrated with measurements from the 27 soundings, and the mean, standard deviation, and the scale of fluctuation in such a case are 1.87 m, 1.28 m, and 110.00 m, respectively. Figs. 6(b) and 7(b) show the mean and standard deviation of the thickness of the soil layer estimated with 27 CPT soundings, respectively. A comparison between Figs. 6(a) and 6(b) shows that the use of additional CPT soundings changes the distribution of the mean of the thickness of the Type 5 soil. A comparison between Figs. 7(a) and 7(b) shows that the uncertainty associated with the thickness of the Type 5 soil generally decreases. When 14 CPTs are used, the entropy associated with the thickness of Type 5 soil layer is 5351.6. When 27 CPTs are used, the entropy is reduced to 5188.6, indicating the uncertainty associated with soil thickness is reduced.

8.3 Impact of more CPT data on the uncertainty of liquefaction potential index at the study site

Base on data from the 27 CPT soundings, the mean, the standard deviation, and the scale of fluction of the random field of I_L are found to be 18.22, 4.10 and 193.60 m, respectively. Figs. 8 (b) and 9 (b) show the mean and the standard deivation of I_L estimated based on data from the 27 CPT soundings, respectively. Fig. 9(b) shows that the spatial variation of the mean value of I_L is greater than that shown in Fig. 9(a). The entropy of I_L estimated based 14 CPT soundings and 27 CPT soundings are 6533.1 and 8576.3, respectively. Contrary to the results in the random field modeling of the thickness of the Type 5 soil, the entropy in I_L increases when more CPT soundings are added. Such increase in the entropy indicates that the uncertainty actually becomes greater and we are less confident in the estimated liquefaction potential. Complexity of the formulation of I_L, which involved integration over depth, might have introduced discrepancies as additional CPTs have different sounding depths. In practice, a sensitive analysis on the effect of the number of CPTs on the assessment results may help determine whether these CPTs are sufficient to characterize the uncertainty associated with the geotechnical prediction of interest.

9 Where to Conduct Additional CPT Soundings?

As discussed previously, adding more CPT soundings could help reduce the geological uncertainty at the site and hence reduce the variation of predicted performance. To this end, where to place the additional CPT soundings to achieve the maximum benefit is an important issue. The optimal location depends on the objective of such site investigation. In Fig. 9(b), the standard deviation of I_L measures the uncertainty associated with I_L. The greater the standard deviation of I_L, the greater the uncertainty associated with the I_L. At a location with CPT sounding, the standard deviation of the I_L at that location is zero. Therefore, if the purpose is to reduce the uncertainty of I_L, it is desirable to conduct additional tests at locations where the uncertainty associated with I_L is greatest (e.g., Zhao and Wang 2019). As shown in Fig. 9(b), the region with yellow color are locations with the greatest standard deviation of I_L.

In practice, instead of the exact value of I_L, one may be more concerned with the liquefaction risk level (e.g., see Table 1). When the value of I_L is uncertain, the risk level at a location may also be uncertain. Let r denote a discrete random variable describing the risk level at a location with p_i denoting the probability that the risk level is i. The uncertainty associated with the risk level at a location can be measured by the entropy $h(r)$ associated with the risk level using the following equation (e.g., Borda 2011):

$$h(r) = -\sum_{i=1}^{3} p_i \ln(p_i) \tag{7}$$

The greater the entropy, the higher the uncertainty associated with the risk level. When there is no uncertainty associated with the risk level, the entropy at this location is zero. Fig. 10 shows the contours of the entropy calculated based on Eq. (7). As shown in this figure, the entropy of the risk level at different locations is in the range of 0 to 0.7. The closer to the yellow color, the greater uncertainty associated with the risk level. Hence, it is worthwhile to conduct additional tests in the region with color close to yellow. A comparison between Fig. 10 and Fig. 8(b) reveals that the recommended region to conduct more CPTs is indeed the region with mean values of I_L around 16, which is approximately at the transition from Moderate risk level to High risk level. However, the exact risk level is uncertain. From a practical point of view, it seems reasonable to conduct additional tests at regions with greater uncertainty regarding the risk level.

Table 1. Relationship between liquefaction risk and liquefaction index [Adapted from Iwasaki et al. (1984)].

I_L	$I_L \leq 5$	$5 < I_L \leq 15$	$I_L > 15$
Risk level	1	2	3
Interpretation	Low risk	Modest risk	High risk

10 Concluding Remarks

It is difficult to characterize the geological uncertainty and to assess its impact on a geotechnical analysis. As a result, the geological uncertainty is still largely a missing element in many of the current probabilistic geotechnical analyses. This paper has illustrated the usefulness of CPT in characterizing the geological uncertainty and assess its impact on a geotechnical analysis. Based on the presented work, the following conclusions are reached:

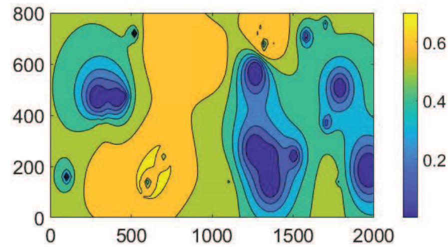


Figure 10. The contour of classification entropy.

1. The soil behavior type index derived from CPT may be used to determine the composition of the ground at a given point and assess the geological uncertainty.
2. At a location with CPT sounding, the soil behavior type index can be readily derived; at a location without CPT sounding, it may be estimated with the aid of random field theory. The uncertainty associated with the soil behavior type index can readily be quantified.
3. The scale of fluctuation is an important parameter in random field modeling, which specifies the distance within which the information gathered through CPT soundings can affect those in the neighboring area. Different soil properties may have different scales of fluctuation. When the number of CPT soundings is small, the estimated scales of fluctuation may not be robust.
4. Once the uncertainty associated with the soil behavior type index is characterized, it may be used to assess the uncertainty associated with the soil stratigraphy. The impact of the geological uncertainty on the predicted geotechnical performance may also be evaluated through an interpolation of the results obtained at locations with CPT soundings. Finally, the results from random field modeling of the geotechnical parameters could help guide the decision on where to conduct additional tests at the project site to reduce the variation of the predicted geotechnical performance.

In the case study reported herein, several assumptions were made. For example, the key soil property was assumed to be a Gaussian stationary random field. Further, the geological uncertainty presented in this paper was characterized using CPT as an example, which may not be suitable for defining the geological uncertainty in many other situations. In view of these limitations, the following are suggestions for future research.

1. The calibration of the intended random field model usually requires a large amount of data, which may not be easily obtained in a routine geotechnical practice. Further, the soil data may not be readily modelled as a Gaussian stationary random field. It is thus necessary to assess the suitability of the Gaussian stationary random field model and perhaps develop more realistic interpolation models. The Bayesian method, which often expends the capability of a geotechnical analysis when the available data is limited, could play an important role in this regard. For example, Bayesian Compressive Sampling (BCS) has been recently developed to statistically interpolate geotechnical data from sparse measurements (Wang and Zhao 2017). The BCS results may be further used together with Karhunen–Loève (KL) expansion to model random field directly from sparse measurements (Wang et al. 2018; Hu et al. 2019). The BCS-KL random field generator is non-parametric and data-driven, and no pre-determined function form is needed for the marginal probability density function or covariance function of the random field. Therefore, the BCS-KL generator appears to be well suited for modeling non-Gaussian and non-stationary random fields.
2. At sites where CPT sounding is not feasible (e.g., with deposits of gravel or rock at shallow depths), characterization of the geological uncertainty through CPT soundings will not be possible. Further research to characterize the geological uncertainty in these situations is warranted.
3. Characterization of the geological uncertainty presented in this paper is only a first step and very limited in scope. Many different forms and/or scales of the geological uncertainties exist. Further research to explore ways to identify and characterize these geological uncertainties for use in the geotechnical analysis should be pursued.

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