# Systematic Transformation Error in the Depth-Average Undrained Shear Strength

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**Abstract:** In this paper we show how to calibrate a site-specific transformation model to measure the undrained shear strength from CPTs. This practically unbiased transformation model can then be used to make a better estimation of the spatial average parameter, such as the depth-average of a soil layer in a homogenous deposit, using less costly indirect measurements. We show that there is a considerable difference in terms of characteristic value and probability density, between the assumptions of entirely random or systematic transformation uncertainty. Ultimately we show what the effect is of making more realistic choices for the random and systematic part in the transformation uncertainty for a case study with actual data.

Keywords: Site investigation; spatial average; undrained shear strength; transformation error; random error; systematic error.

## 1 Introduction

Geotechnical soil properties are variable in space, because of various processes during the formation process of a natural material (e.g., Lumb 1966). The natural variability in ground conditions results in an uncertain point estimate of a soil property. In most geotechnical problems though, a geotechnical failure mode considers a bigger volume of soil. Therefore, small fluctuations average over a certain volume and it is more appropriate to use the spatial average in the geotechnical design (e.g., Vanmarcke 1977).

However, uncertainty arises not only from natural variability, but also from uncertain measurements, due to measurement errors, transformation errors and statistical errors (e.g., Phoon and Kulhawy 1999a). These errors consist of random and systematic components. Random errors such as reading accuracy differ from point to point and will average if sufficient samples are taken. Systematic errors like a wrongly calibrated device will not average with an increasing number of samples.

We use transformation models to infer geotechnical properties from indirect measurements. With direct and indirect measurements from a site, we can calibrate a site-specific transformation model. By using such a model, spatial variability, measurement errors and statistical uncertainty propagate into the uncertainty of the spatial average, which is the variable of interest in most geotechnical analyses. In Van der Krogt et al. (2018) it is shown how all components enter the total uncertainty of a transformation model for undrained shear strength from cone resistance for a case with synthetic data. The main finding is that if a considerable share of the measurement and transformation errors is random or spatially variable, the uncertainty estimates can be considerably lower compared to methods proposed earlier, and hence, characteristic values can be considerably higher.

In the article a method is proposed to estimate the uncertainty in the spatial average, particularly focusing on the role of averaging of all spatially variable error components. For the sake of a good comparison, synthetic data was used for which the true but subsequently unknown values were generated. In this paper we apply the proposed method to estimate the depth-average undrained shear strength in a geological deposit (soil layer) using CPTs of a real test site in The Netherlands. For this practical example we show the effect of making more reasonable choices for the random and systematic part in the transformation error in terms of probability density and characteristic values.

## 2 Uncertainty in Spatial Average Undrained Shear Strength with a Site-Specific Transformation Model

This section quickly recaps how the uncertainty in spatial average can be estimated, when the undrained shear strength is measured using CPTs and a site-specific transformation model. The scientific background is published in Van der Krogt et al. (2018).

### 2.1 Measuring depth-average undrained shear strength using CPTs

To estimate the geotechnical parameters of interest from less costly indirect measurements such as CPTs, we often use transformation models. For instance, we can estimate the undrained shear strength (ratio) $s^I_u$ from

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normalised cone resistance $q_{net}$, through the transformation model parameter $N_{kt}$, see Eq. (1). The normalised cone resistance is the cone tip resistance ($q_c$) corrected for pore water pressures ($u_2$) through the cone factor ($a$) and normalized for the in situ vertical stress ($\sigma_{v0}$): $q_{net} = q_c + u_2 \cdot (1 - a) - \sigma_{v0}$.

$$s_u^I = \frac{q_{net}}{N_{kt}} \tag{1}$$

We often want to estimate the depth-average of the geotechnical parameter in a statistically homogenous layer, since failure mechanisms typically involve a vertical zone of influence which is much larger than the vertical scale of fluctuation, for instance shear planes crossing a soil layer by several meters vertically. When a CPT is used to indirectly measure the undrained shear strength, we can estimate the depth-average of a layer in CPT $j$ (denoted by a bar, $\overline{s}_{u,j}^I$ ) simply by the numerical mean of all CPT measurements $N_j$ in that layer (typically every 2 cm):

$$\overline{s}_{u,j}^I = \frac{1}{N_j} \sum_{i=1}^{i=N_j} s_{u,j,i}^I \tag{2}$$

At a random location at the site without a CPT, our best estimation for the depth-average is the mean value $\mu_{\overline{s}_u^I}$ of the depth-averages of $M$ CPTs across the entire site:

$$\mu_{\overline{s}_u^I} = \frac{1}{M} \sum_{j=1}^{j=M} \overline{s}_{u,j}^I \tag{3}$$

For both estimates of the spatial average, we are interested in the (total) uncertainty, involving both spatial variability and epistemic uncertainties. According to Phoon and Kulhawy (1999b) the total uncertainty of a spatial average parameter is a linear sum of the variances of the independent error terms:

$$\sigma^2 = \Gamma^2 \cdot \sigma_{spatial}^2 + \sigma_{measurement}^2 + \sigma_{transformation}^2 + \sigma_{statistical}^2 \tag{4}$$

where $\Gamma^2$ is the variance reduction function (Vanmarcke 1977), accounting for spatial averaging. Essentially, determining the total variance as in Eq. (4) implies the assumption that measurement uncertainty, statistical uncertainty and transformation uncertainty relate to systematic errors, which are not subject to (spatial) averaging. While this assumption is certainly conservative in the sense that it will lead to a high uncertainty estimate in engineering applications, the question is whether we should consider these error terms as entirely systematic and not subject to spatial averaging. A more differentiated approach is shown by Van der Krogt et al. (2018) (see section 2.3) and is demonstrated for a practical case in section 3 of this paper.

### 2.2 Site specific transformation model

When we use a "generic" transformation model from literature to estimate the geotechnical property from indirect measurements, it is likely that such a model is biased for the entire site (Ching et al., 2016). A site-specific transformation model calibrated using direct measurements of a soil property (e.g. laboratory results) and indirect measurements (e.g. cone resistance) at the same location, on the other hand, is expected to have on average no systematic bias. In Dutch dike design, for example, a site-specific transformation model is often used to estimate the depth-average undrained shear strength (ratio) $s^I_u$ from normalised cone resistance $q_{net}$, through the transformation model parameter $N_{kt}$, see Eq. (1).

A site-specific transformation model can be calibrated by pairing measured cone resistance with direct (laboratory) measurements from (nearly) the same location: ($q_{net,i}$ ; $s^D_{u,i}$). In Van der Krogt et al. (2018) it was shown that a nearly unbiased transformation model parameter for the entire site could be obtained with linear regression, by minimizing the standard deviation (SD), see Eq. (5).

$$\sigma_{s_u^I} = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{i=n} \left( s_{u,i}^D - q_{net,i} / \hat{N}_{kt} \right)^2} \tag{5}$$

The total uncertainty in the transformation model follows from the regression analysis. Fundamentally, transformation uncertainty is a model uncertainty, which in principle covers the model prediction errors for perfectly known model inputs. Practically spoken it is, however, impossible to determine model uncertainty in a clean fashion, nor transformation uncertainty for that matter, because such perfect conditions are not available. It is, for instance, virtually impossible to execute the CPT at the exact location as the soil sample is taken from, so distance between direct and indirect measurements will cause spatial variability to propagate into the transformation uncertainty estimate. Furthermore, both random and systematic measurement errors in the direct and indirect measurements increase the transformation uncertainty estimate. In Van der Krogt et al. (2018) it is shown that the external random errors propagate in the estimated transformation uncertainty, approximately linear, see Eq. (6). These random errors, however, average with an increasing number of independent data pairs.

$$CoV_{N_{kt}}^2 = CoV_{spatial}^2 + CoV_{\varepsilon_{s_u^D}}^2 + CoV_{\varepsilon_{q_{net}}}^2 + CoV_{\varepsilon_t}^2 \tag{6}$$

Since we are only interested in the systematic uncertainty, we need to estimate the systematic part in the total transformation uncertainty. In Van der Krogt et al. (2018) it was proposed to base ourselves on estimates of the random and systematic errors involved, as indicated in Eq. (7). Here $CoV_{spatial}$ is the spatial variability due to distance between direct and indirect measurements and $CoV_{\varepsilon,su}$ and $CoV_{\varepsilon,qnet}$ the random measurement errors in the direct and indirect measurement, respectively. $CoV_{\varepsilon,t}$ is the actual spatial variable transformation error, that causes the local transformation model parameter to deviate from the site average and hence it is assumed to be systematic in a CPT (Ching et al., 2016). The actual transformation model error is most certainly spatially variable, because it is, at least to some degree, due to missing factors that are spatially variable, such as over consolidation ratio, water content and plasticity index. The systematic part of the transformation uncertainty is then given by Eq. (8).

$$r = \frac{CoV_{spatial}^2 + CoV_{\varepsilon_{s_u}}^2 + CoV_{\varepsilon_{q_{net}}}^2}{CoV_{N_{kt}}^2} \tag{7}$$

$$\sigma_{transformation} = \mu_{\bar{s}_u^I} \cdot \sqrt{CoV_{N_{kt},systematic}^2 + CoV_{N_{kt},statistical}^2} = \mu_{\bar{s}_u^I} \cdot \sqrt{\frac{1}{n} + (1-r)} \cdot CoV_{N_{kt}} \tag{8}$$

### 2.3 Uncertainty in the depth-average undrained shear strength

To estimate the uncertainty in the depth-average undrained shear strength we base ourselves on known error statistics, in line with Phoon and Kulhawy (1999b), see Eq. (4). Hence, the total uncertainty in the depth-average at a location with a CPT is the linear sum of spatial variability, statistical uncertainty, systematic measurement error (bias) and transformation error, where the error terms are assumed to be independent, see Eq. (9). The statistical uncertainty in the spatial average is dependent on the number of measurements $N_j$ and the observed variance in CPT $j$ ($\sigma_{s_{u,j}^I}^2$), from the observed scatter in one CPT.

$$\sigma_{\bar{s}_{u,j}^I}^2 = \frac{1}{N_j} \cdot \sigma_{s_{u,j}^I}^2 + \sigma_{measurement,systematic}^2 + \sigma_{transformation}^2 \tag{9}$$

The systematic part of the transformation uncertainty here is given by Eq. (8), such that:

$$\sigma_{\bar{s}_{u,j}^I}^2 = \frac{1}{N_j} \cdot \sigma_{s_{u,j}^I}^2 + \sigma_{measurement,systematic}^2 + \left( \mu_{\bar{s}_u^I} \cdot \sqrt{\frac{1}{n} + (1-r)} \cdot CoV_{N_{kt}} \right)^2 \tag{10}$$

At locations without a CPT, we can estimate the uncertainty in the spatial average from the variance of the spatial averages from all CPTs, accounting for statistical uncertainty, see Eq. (10). In Van der Krogt et al. (2018) it was shown that this estimate includes the systematic part of the epistemic uncertainties, see Eq. (11):

$$\sigma_{\bar{s}_u^I}^2 = \left(1 + \frac{1}{M}\right) \cdot \frac{1}{M-1} \sum_{j=1}^{j=M} \left( \bar{s}_{u,j}^I - \mu_{\bar{s}_u^I} \right)^2 \approx \Gamma^2 \cdot \sigma_{spatial}^2 + \sigma_{measurement,systematic}^2 + \sigma_{transformation}^2 + \sigma_{statistical}^2 \tag{11}$$

## 3 Case Study

The theory from section 2 is applied to a real case where a site-specific transformation model was used to infer the undrained shear strength at the site from CPTs. The example also demonstrates the impact of the more distinguished approach of separating random and systematic uncertainty. The site is located in Eemdijk (The Netherlands), where a ring-shaped dike was built to test the resistance of dikes with a sheet-pile construction against instability, on a 1:1 scale. For more information on the "Eemdijkproef", see Lengkeek et al. (2019). To avoid disturbances in the subsurface, e.g., due to instability during the construction of the embankment, the undrained shear strength had to be estimated across the site of approximately 100x100 m, see Figure 1.

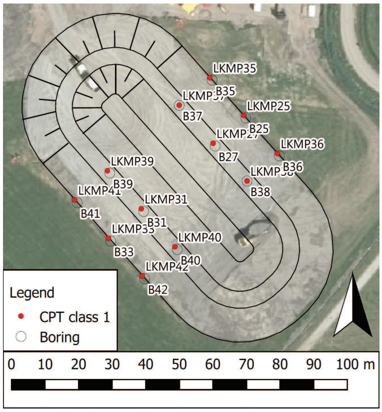


**Figure 1.** Satellite image of the test site during site preparation. The black lines indicate the intended ring dike. The markers indicate the location of the CPTs and borings.

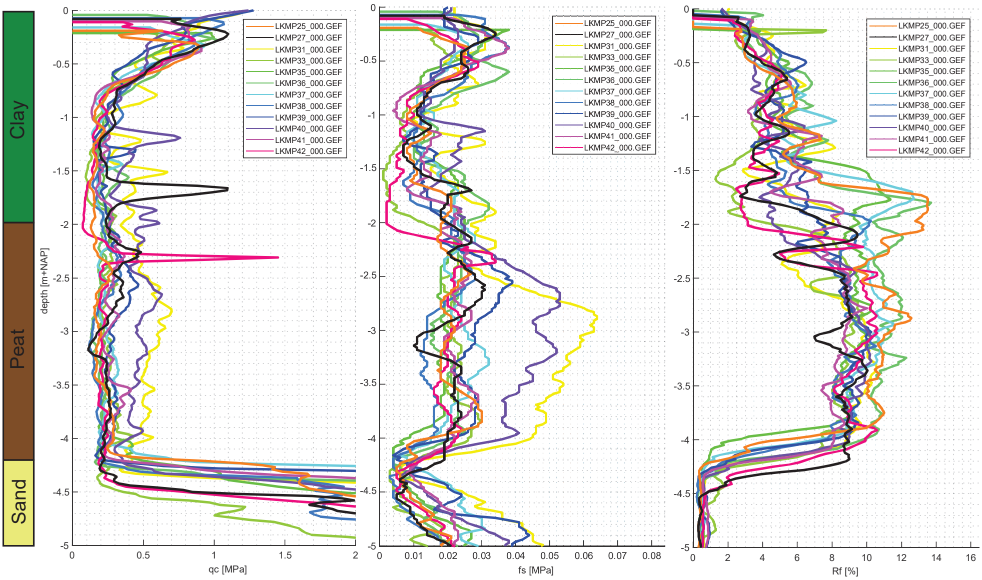


**Figure 2.** Cone resistance qc [MPa], sleeve friction fs [MPa] and friction ratio Rf [%] of 12 CPTs at the test site.

### 3.1 Site specific transformation model

On site, 12 pairs of mechanical borings and Class 1 CPTs with $u_2$ water pressure measurements were carried out and used to derive the vertical soil profile. The surface level is around the Dutch reference level N.A.P + 0.0m. The subsoil consists of a 1.5 to 2.0 m thick (organic) clay layer, an approximately 2.5 m thick peat layer and a relatively thick (6 m) Pleistocene sand layer, see Figure 2. Since the ground water table is around N.A.P + 0.75m, the top part of the clay layer is unsaturated, which can be also seen by the higher cone resistance in the top part of the CPTs.

Of seven clay and seven peat samples from the borings, the in-situ undrained (critical state) shear strength is measured in the laboratory using Unconsolidated Undrained Triaxial Compression (TXC-UU) or Direct Simple Shear (DSS) tests. These direct measurement of the shear strength ($s^D{}_u$) are paired with the normalized cone resistance $q_{net}$ of the CPT at nearly the same location and depth as the direct measurement, see Figure 3. The transformation model parameter is derived using linear regression, as shown in Eq. (5). The result is shown in Figure 3 and the uncertainty is depicted by the 90% confidence interval. Note that for the transformation model for clay both TXC as well as DSS tests are used, which is formally not correct, because the shear mechanism can differ. Because the difference between the two subsets is on the eye not significant and to avoid high statistical uncertainty, no distinction is made.

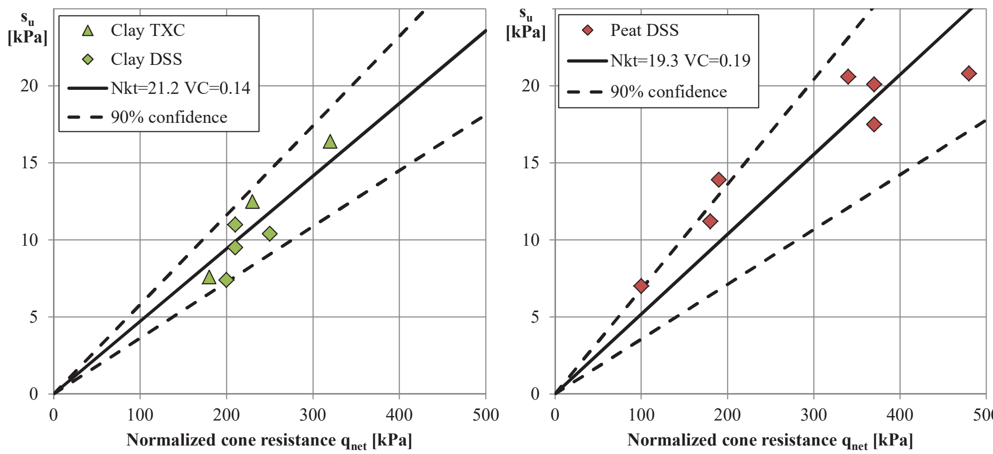


**Figure 3.** Site-specific transformation model calibrated using direct and indirect measurements for peat and clay layer.

### 3.2 Uncertainty in the depth-average undrained shear strength

The transformation models are used to measure the undrained shear strength in the clay and peat layer at the site using CPTs. Because a CPT measurement is almost continuous in depth, we can obtain the depth-average (in which random measurement errors are averaged) easily and therefore we get a better insight in the spatial average undrained shear strength in the (saturated part of the) clay and peat layer.

In Figure 4, the uncertainty in the (indirectly measured) depth-average undrained shear strength, determined using Eq. (10), is shown for CPT LKMP27. The different lines indicate the uncertainty (and 5% characteristic values) for different choices for the systematic uncertainty in the transformation model. The black lines contemplate one end of the spectrum, if the transformation error would be entirely random. The red lines indicate the conservative approach that assumes that the transformation error is entirely systematic. Since the estimated transformation uncertainty most likely consists of both random and systematic errors, neither of the two assumptions is very realistic. There is therefore a significant difference between the two assumptions and the latter is considerably conservative.

We can get a more realistic estimation of the uncertainty in the spatial average, by estimating the random error in the transformation uncertainty. If we assume that the random measurement error in cone resistance and laboratory tests is respectively 1% and 5%, and that spatial variability also causes 5% random error in the calibration of the transformation model, we can estimate the amount of random error in the transformation uncertainty using Eq. (7). It follows that $r = \sqrt{(0.05^2+0.01^2+0.05^2)}/0.19 = 0.38$ for the transformation model for peat and $r = 0.51$ for clay. With this cautious estimate of the random error in the transformation uncertainty, we obtain a more realistic estimate of the uncertainty in the spatial average, indicated with blue lines in Figure 4. The black line shows that there is much more potential if we can show that the proportion of random errors in the transformation uncertainty is larger.

In Van der Krogt et al. (2018) it is also proposed to approximate the ratio $r$ based on the ratio of local variance versus total point variance, observed in the field. This is because the total variability in point measurements at a site (inherent and epistemic) can be split in variations in depth (which average) and variations in horizontal direction (which average to a lesser extent, dependent on the failure mechanism). For the considered case study, we find a ratio of 0.64 (coincidentally) for both the clay and peat layer. The found ratio is higher than the cautious estimate of $r$ in the previous paragraph, because the approximation using the variance ratio also includes spatial averaging (falsely). In addition, the approximation based on the variance ratio does not account for random errors in direct measurements. The use of this approximation is therefore limited to cases where random measurement errors in the indirect measurement and spatial variability due to distance between paired measurements are the dominant components in the transformation uncertainty, instead of spatial averaging itself.

The estimated mean shear strength at a cross-section with a CPT is typically representative over a distance that is shorter than the horizontal scale of fluctuation. The fluctuation scale is not known for the current case, but is expected to be approximately the same as the distance between two CPTs. Hence, a CPT is representative for a (NE-SW oriented) cross-section of the dike. For sections without a CPT, such as the head sides of the ring dike, the depth mean can be estimated based on the mean and the variability of the spatial averages over the entire site, using Eq. (3) and Eq. (11). Despite the CPTs don't cover the entire site of 100x100 m$^2$, it is acceptable to assume that the CPTs are representative for the entire site, because the area has the same geology and history (apart from discrete changes such as the pre-loaded area). Based on the results of 10 CPTs, we estimate the average shear strength to be 9.4 kPa and 10.7 kPa for the clay and peat layer, respectively. The estimated uncertainty is 1.3 kPa and 1.8 kPa, respectively. As shown in Van der Krogt et al. (2018), this variability includes both variability of the spatial average and the systematic part of the epistemic uncertainties as described in Eq. (11). It is noted that two CPTs are excluded from the statistical analysis, because these locations have a different stress history, because these locations were pre-loaded by an old embankment.

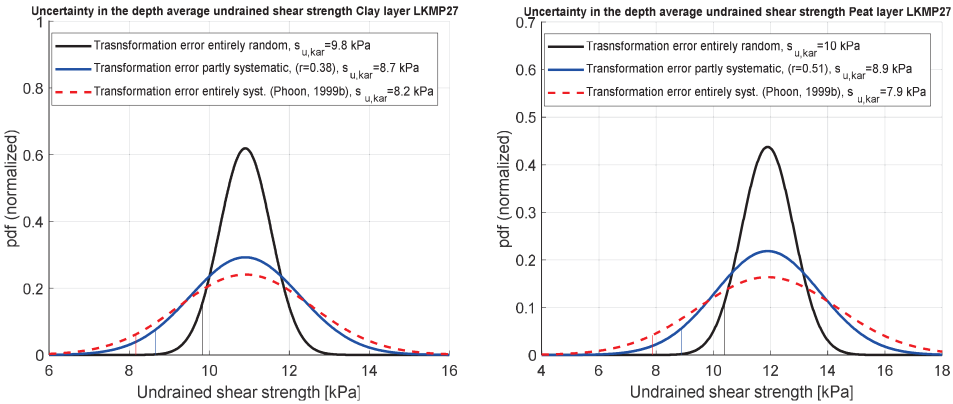


**Figure 4.** Uncertainty in the depth-average undrained shear strength in clay and peat layer at CPT LKMP27.

## 4 Conclusions

This paper shows that we can calibrate a site-specific transformation model with laboratory measurements and cone resistance measurements from a site. This site-specific transformation model can be used to estimate the undrained shear strength from less costly indirect measurements. Furthermore, we get a much better insight in the depth-average of a soil layer, contrary to when we would only use a few point measurements.

The paper also shows that there is a significant difference in the characteristic value (and probability density) if we assume the transformation uncertainty to be entirely random or systematic. Assuming that the transformation uncertainty is partly random will lead to a less conservative estimated uncertainty of the depth-average undrained shear strength, than assuming the transformation error to be only systematic. This will certainly lead to more realistic uncertainty estimates. The large difference in characteristic value also shows the importance to investigate to which extent the transformation uncertainty is ultimately systematic, for instance by focussing site investigation on e.g. repetitive laboratory measurements or analysing the spatial variability of the transformation error.

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