Probabilistic Bearing Capacity Evaluation for Two-Layered Soil

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Abstract: In this study, a probabilistic approach for bearing capacity evaluation for rectangular footings is discussed. The evaluation is based on a kinematical approach. The considered substrate consists of two different layers of soil: the top layer formed of medium or dense sand is followed by the layer of the soft clay. The sand layer is assumed to be homogenous, whereas the undrained shear strength of the soft clay layer is assumed to be spatially variable, described by a lognormal random field. The random field is discretized according to Vanmarcke’s spatial averaging along dissipation regions in the considered failure mechanism. The mechanism utilizes plane strain condition; however, due to the consideration of the soil spatial variability in three dimensions, the impact of the length of the foundation on the random bearing capacity evaluation is considered in the study. As a result of discretization procedure, a set of correlated random variables is obtained (each associated with individual surface of failure mechanism). A series of numerical analyses were performed for two thicknesses of the first layer and two anisotropic correlation structures for spatial variability of the undrained shear strength. The proposed method is computationally efficient and allows to consider three dimensional spatial variability in soil strength properties. The results were discussed and compared with those obtained by other methods.

Keywords: Random bearing capacity; two-layered soil; spatial variability; fluctuation scale; reliability; plane strain condition.

1 Introduction

Evaluations of bearing capacity of shallow foundations for spatially variable soil are currently used mostly for two-dimensional analysis of the engineering problems. Moreover, most of the existed methods are applied to single-layered soil. However, from the engineering point of view a two-layered soil is also a quite probable. This scenario was extensively examined in the case of deterministic analyses (Florkiewicz 1989; Michałowski and Shi 1995; Shiau et al. 2003); however there are very few studies on the probabilistic analyses of two-layered soil (Zaskórski et al. 2017). The present study utilizes a kinematical approach in conjunction with Vanmarcke spatial averaging (1979a, 1979b, 1983). Analogous algorithm for two-dimensional multi-block failure mechanism was proposed by Pula and Chwała (2018). As a deterministic background a failure mechanism proposed by Michałowski and Shi (1995) is assumed. The considered failure mechanism is dedicated for two-layered soil, i.e. top layer is a medium or dense sand and bottom layer is a soft clay. Spatial variability of soil strength is considered only in the soft clay; the sand is assumed to be homogenous. This scenario is adequate to problem of the working-platforms (Białek and Bałachowski 2015), where the top layer is a man-made structure and the assumption of its homogeneity is reasonable. The study shows that random bearing capacity of two-layered soil can be efficiently evaluated by the developed approach. Moreover, despite 2D mechanism used, the spatial variability of the undrained shear strength in the soft clay layer is considered in three dimensions. Therefore, assessment of the impact of foundation length on random bearing capacity can be estimated by the developed method. Based on the numerical procedure proposed in the study, some typical cases of the two-layered soil are analysed. The obtained results are shown and discussed in Section 4.

2 Numerical Algorithm

2.1 Random failure mechanism for two-layered soil

The failure mechanism proposed by Michałowski and Shi (1995) for two-layered soil was used in this paper. The mechanism is appropriate for considered arrangement of soil layers. It is known that providing greater number of blocks in failure mechanism results in greater accuracy of the kinematic bearing capacity assessment (Michałowski 1997). However, in this study, the number of 11 rigid blocks were assumed arbitrary according to the previous authors experience for single-layered soil layer (Pula and Chwała 2018). It is shown there that further increase in rigid blocks number provide very limited impact on final accuracy.
Figure 1. Failure mechanism for two-layered soil. The mechanism consists of 11 rigid blocks. Since in spatially variable case the undrained shear strength values associated with individual slip surfaces are various, the geometry of mechanism is not symmetrical.

For the deterministic case the mechanism is always a symmetric one, but it is not the case of spatially variable soil. The deterministic geometry of the considered failure mechanism was then modified to enable the random bearing capacity assessment, i.e. for each dissipation surface an individual value of shear strength is applied. The top layer is assumed to be homogenous. The failure geometry adjusted to the probabilistic case is shown in Fig. 1.

Note that block \( O_1P_1A_1P_2O_1 \) is a single rigid block; similarly, the block \( O_1P_2A_2P_3O_1 \) is a single rigid block, etc. Within the clay layer there are 16 slip lines (dissipation regions). Due to the homogeneity assumption of a sand layer all velocity-jump vectors within this layered are inclined to the slip lines at the same angle that is equal to the assumed internal friction angle of sand \( \varphi \). The velocities shown in Fig. 1 can be obtained from the velocity hodograph shown in Fig. 2, the hodograph is constructed according to geometrical relations shown in Fig. 1.

The failure geometry for a probabilistic case is no longer symmetrical (the mechanism was expanded to account for non-symmetrical cases), this is the reason why in Fig. 1 both sides of failure mechanism are plotted. Therefore, the corresponding formula for bearing capacity (for assumed lack of embedment) can be expressed as in Eq. (1).

\[
p = \sum_{i=1}^{16} \left( \sum_{j=1}^{4} c_{ui} v_{i1} l_i + \sum_{j=5}^{8} c_{ui} v_{i2} l_i + \sum_{j=9}^{12} c_{ui} v_{i3} l_i - \gamma \left( \sum_{j=13}^{15} F_j v_{j1} + \sum_{j=16}^{19} A_j v_{j1} \right) \right)
\]

where \( v_i \) is the magnitude of the velocity-jump vector along slip line, \( [v_i] \) is the vertical component of the velocity \( v_i \) (positive if downward) and \( l_i \) is the length of slip surface \( i \) (see Fig. 1). As mentioned earlier different values of undrained shear strength are assumed in Eq. (1) on each dissipation region \( (c_{ui}) \). Energy of dissipation along slip surfaces is calculated for cohesive soil \( (c_u \neq 0) \) by multiplying, undrained shear strength, magnitude of the velocity-jump vector and length of slip surface \( l_i \). \( F_i \) is the area of the specified rigid block (within the top layer, e.g., \( F_1 = \text{area of triangle } O_1P_1P_2 \)), \( \gamma \) is unit weight of sand. In this paper the soft clay is assumed to be weightless. Note that for a sand layer there is no dissipation energy \( (c_u = 0) \).

Figure 2. Velocity hodograph for failure geometry shown in Fig. 1.
2.2 Optimization procedure

The failure geometry and velocity hodograph shown in Fig. 1 and Fig. 2 are associated with the exemplary failure mechanism. The formula given in Eq. (1) provides bearing capacity for the specified geometry of the blocks. To obtain optimal failure geometry that results in the lowest possible bearing capacity for the assumed scenarios, the optimization procedure is crucial. In this study the authors utilized earlier experiences (Pula and Chwała 2018; Chwała 2018) with simulated annealing scheme (Kirkpatrick and Gelatt 1983; Kirkpatrick 1984). The objective function is the bearing capacity formula given in Eq. (1). At the beginning of the optimization procedure the starting failure mechanism geometry is assumed. For this geometry according to Eq. (1), the corresponding bearing capacity is calculated (its value is referred to as current bearing capacity \(p_c\), and corresponding failure mechanism geometry as current failure mechanism geometry). In the subsequent simulations the current failure mechanism geometry is slightly changed and a new bearing capacity is determined via Eq. (1) \(p_n\). Next the current failure mechanism geometry is updated by choosing one of the two values of bearing capacity \(p_c\) and \(p_n\). If \(p_n < p_c\), (improvement in assessment) then \(p_n\) is assigned as current bearing capacity; To overcome local extrema in the opposite case \(p_c < p_n\), worse solution \(p_n\) (greater bearing capacity) can be also accepted, but with given, limited probability \(P_g\). Initially, \(P_g\) should be about 0.5. During the simulations acceptance probability \(P_a\) decreases according to Eq. (2) (the value of \(T\) decreases during simulation, Kirkpatrick and Gelatt 1983).

\[
P_a = \exp\left(\frac{p_c-p_n}{T}\right)
\]

(2)

The above procedure was applied to optimize failure mechanism geometry for each Monte-Carlo realization of the problem. In the failure mechanism considered here, there are 18 degrees of freedom. The authors selected the following 18 parameters to govern geometry of failure mechanism, i.e.: \(x\) coordinates of points \(P_i\) (10 parameters in total), lengths \(l_2, l_3, l_4, l_{10}, l_{11}\) and \(l_{12}\) (6 parameters) and \(x\) coordinates of points \(C_1\) and \(C_2\) (see Fig. 1). Slight changes in geometry of failure mechanism (mentioned in the description of the procedure) were carried out by changing the values of those parameters. The geometry of failure mechanism obtained using—the optimization procedure for non-random case was identical to those obtained by Michalowski and Shi (1995).

2.3 Spatial averaging

In the previous Section the method for searching optimal bearing capacity for known soil strength parameters is discussed. However, in the case of the spatially variable soil, those parameters have to be determined in accordance with properties of random field X that describes undrained shear strength of the bottom layer. The friction angle of the sand layer is assumed to be non-random. The averaged undrained shear strength values are determined by using algorithm proposed by Pula and Chwała (2018) based on the Vanmarcke spatial averaging (Vanmarcke 1977a, 1977b, 1983). According to that algorithm generating of the random field is reduced to generating a random vector \(X\). Each component of \(X\) is an average of undrained shear strength for individual dissipation region \(V\). The average is calculated as:

\[
X_{V} = \frac{1}{|V|} \iint X(x, y, z) dx dy dz
\]

(3)

where \(V\) is the averaging domain (dissipation region) and \(X\) is the initial random field. The variance of \(X_{V}\) is given by:

\[
\text{Var}(X_{V}) = \sigma_X^2 = \gamma(V)\sigma^2 ,
\]

(4)

where the \(\sigma^2_X\) is the variance of the random field \(X\). In this study the field \(X\) is a lognormal stationary (but not isotropic) random field. As a covariance function of the random field the Gaussian covariance function was selected (see Eq. 5).

\[
R(\Delta x, \Delta y, \Delta z) = \sigma_X^2 \exp \left\{ - \left( \sqrt{\pi \frac{\Delta x}{\theta_x}} \right)^2 - \left( \sqrt{\pi \frac{\Delta y}{\theta_y}} \right)^2 - \left( \sqrt{\pi \frac{\Delta z}{\theta_z}} \right)^2 \right\}
\]

(5)

where \(\theta_x\), \(\theta_y\) and \(\theta_z\) are fluctuation scales. In the study, both horizontal fluctuation scales are assumed to be equal and denoted as \(\theta_h = \theta_x = \theta_y\); by analogy the vertical fluctuation scale is denoting as \(\theta_z\). Eq. (5) is used for determining correlated set of undrained shear strengths. Note that despite two-dimensional failure geometry the formula for covariance function include three directions (see Eq. (5)) and the averaging is calculated in three dimensions (the failure mechanism is extruded in the direction of the foundation length). The elements of the covariance matrix are derived basing on Eq. (6) (see Pula 2004; Pula and Chwała 2015) that determines covariance between two random variables \(X_{V_i}\) and \(X_{V_j}\) corresponding to dissipation regions \(V_i\) and \(V_j\).

\[
\text{Cov}(X_{V_i}, X_{V_j}) = \frac{1}{|V_i| |V_j|} \iint_{V_i} \iint_{V_j} R(x_i, y_i, z_i, x_j, y_j, z_j) dV_i(x_i, y_i, z_i) dV_j(x_j, y_j, z_j)
\]

(6)
Note that in the considered failure mechanism there are 16 dissipation regions within the clay layer; as a result the covariance matrix size is 16×16. Due to the symmetry of the matrix 136 individual elements have to be determined in one bearing capacity calculation.

2.4 Numerical procedure

In order to find probabilistic characteristics of random bearing capacity the Monte Carlo simulation is utilized. For each realization of simulation process the numerical procedures consist of three steps as follows:

Step 1. Generate 16 independent values of undrained shear strengths via a pseudo-random number generator \((c_{u,1}, \ldots, c_{u,16})\). All values are generated from the initial probability characteristics of the random field \((\mu_{c_{u}}, \sigma^{2}_{c_{u}})\). Based on the starting failure geometry, Eq. (1) and soil parameters \(c_{u,1}, \ldots, c_{u,16}\) find the optimal bearing capacity and corresponding optimal failure mechanism geometry by the optimization procedure described in Section 2.2.

Step 2. Determination of the correlated undrained shear strengths. Transform independent values of undrained shear strength obtained in Step 1 to the correlated ones by using the covariance matrix obtained for the optimal failure geometry from Step 1. For this purpose the algorithm proposed by Pula and Chwała (2015) was adopted for analyses performed in this study. This algorithm is based on the Cholesky decomposition (Horn and Johnson 1985) of the covariance matrix. Generally speaking the correlated parameters are obtained by calculating product of a vector of uncorrelated parameters and triangular matrix which is the result of Cholesky decomposition of the covariance matrix. For more details the interested reader is referred to the cited work. As a result a set of correlated values of the undrained shear strength is obtained \((c_{u,1}^{\text{corr}}, \ldots, c_{u,16}^{\text{corr}})\).

Step 3. Calculation of the final bearing capacity. For the correlated undrained shear strengths vector \((c_{u,1}^{\text{corr}}, \ldots, c_{u,16}^{\text{corr}})\) obtained in Step 2 the optimization procedure is used again. The objective is to find optimal failure geometry and the corresponding optimal value of bearing capacity that are returned as a result of the algorithm. In this study for all analyses the number of Monte Carlo simulations equals 1000. The proposed approach is computationally efficient; namely, one 3-D simulation takes about 1 minute for one core of a standard notebook.

3 Numerical Example

As mentioned earlier the soil spatial variability of the clay layer is considered in three dimensions. As a result the length of the foundation \(a\) have to be assumed in the analyses. In this study four \(a\) values are examined; namely, \(a = 1.0 \text{ m}, a = 3.0 \text{ m}, a = 6.0 \text{ m}\) and \(a = 10.0 \text{ m}\). For each foundation length two fluctuation scale scenarios are considered. By assuming vertical fluctuation scales according to Pieczyńska-Kozłowska et al. (2017) and taking horizontal fluctuation scale ten times greater than vertical one, the following combinations are selected for further analyses: \(\theta_v = 0.25 \text{ m}\) and \(\theta_h = 2.5 \text{ m}\); \(\theta_v = 0.5 \text{ m}\) and \(\theta_h = 5.0 \text{ m}\). Additionally, two thicknesses of the sand layer are examined: \(t = 0.5 \text{ m}\) and \(t = 1.0 \text{ m}\) (see Fig. 1). The following parameters are kept constant for all scenarios: angle of internal friction of sand \(\varphi = 35^\circ\), unit weight of sand \(\gamma = 18.0 \text{ kN/m}^3\), mean values of undrained shear strength \(\mu_{c_{u}} = 20 \text{ kPa}\), coefficient of variation of undrained shear strength \(v_{c_{u}} = 0.5\).

4 Results

The obtained bearing capacity mean values and bearing capacity standard deviations are shown in Fig. 3a and Fig. 3b, respectively. According to Fig. 3a an increase in bearing capacity mean values is observed with an increase in foundation length. Moreover, greater fluctuation scales provide lower mean values; however, probabilistic bearing capacity mean values are lower than those obtained for deterministic case (see dashed lines in Fig. 3a). As expected, bearing capacity standard deviation decreases with an increase in foundation length; its values differs slightly between scenarios for \(t = 0.5 \text{ m}\) and \(t = 1.0 \text{ m}\) (for the same fluctuation scales). The significance of sand layer thickness becomes visible if considering coefficients of variation of bearing capacity (see Fig. 4). One third reduction in coefficient of variation of bearing capacity in transition from \(t = 0.5 \text{ m}\) to \(t = 1.0 \text{ m}\) is observed in Fig. 4. It can be seen for probabilistic case that for a greater thickness of sand layer, a greater bearing capacity mean value is observed. Moreover, increase in thickness of top layer provide a decrease in bearing capacity coefficient of variation. Therefore, it seems that the influence of thickness of the top layer of homogenous soil suppresses the influence of spatial variability of the clay.
Figure 3. Bearing capacity mean values (a) and bearing capacity standard deviations (b) as a functions of foundation length $a$. Dashed lines plotted in Fig. 3a denote bearing capacity obtained for deterministic value ($c_u = 20$ kPa) for width of sand layer $t = 1$ m and $t = 0.5$ m.

Figure 4. Bearing capacity coefficients of variation as a functions of foundation length $a$.

5 Conclusions

The original approach for considering soil spatial variability in bearing capacity evaluations for two-layered soil is presented in the study. The proposed approach is computationally efficient (one 3-D simulation takes about 1 minute for one core of a standard notebook) and allows to include in analyses soil spatial variability in three dimensions. The proposed algorithm can be used for analyzing working-platforms. According to Fig. 4, the interesting feature is observed that the top layer of homogenous soil suppresses the influence of spatial variability of the clay. It is promising observation for further studies, that can be used in generating diagrams for indicating the width of top homogenous layer for which the impact of soil spatial variability of bottom layer can be ignored in practical applications or keep at an acceptable level.

References


